Comparison of Inelastic Material Models Used to Determine Steel Frame Limit Load Conditions

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Abstract: The reduced stiffness conditions of steel W-Shape sections were evaluated and used to develop a new inelastic material model for a given axial load and bending moment. The new material model allows for direct input of parameters to adjust the stiffness reduction based on the W-shape’s dimensional properties, axis of bending, axial load and residual stress ratio. Numerous second-order, inelastic analyses up to the limit load were performed on three steel building frames using the new material model and the inelastic material model used in the AISC direct analysis method. Discussion is given regarding the two material models and their ability to match the ultimate load capacity results of the three test frames.

Key words: Nonlinear analysis, steel buildings, stiffness reduction, material model.

1. Introduction

Appendix 1 of the AISC Specification for Structural Steel Buildings [1] allows for the use any method that uses inelastic analysis to proportion members with localized yielding provided that it meets specified strength, ductility and analysis requirements. Researchers over the past couple of decades have proposed different stiffness reduction methods to account for the spread of plasticity in steel frames [2-6]. A new inelastic material model is proposed that allows for the direct input of parameters to adjust the stiffness reduction based on the W-shape’s cross-section dimensional properties, axis of bending, axial load and residual stress ratio. Chapter C of the Specification [1], with its direct analysis method, uses a simpler inelastic material model that ignores the bending moment and cross-section properties. Since both material models can be used to design steel frames, second-order inelastic analyses were performed using both material models to study the response differences of three test frames. The gravity load magnitude was varied to study the sensitivity of the two material models to predict the same lateral load magnitude at the collapse condition.

2. Stiffness Reduction Models

The stiffness reduction $\tau$ that results from yielding of the cross-section due to major-axis or minor-axis bending and axial compression was studied in detail using a fiber element model for W-shapes with an European Convention for Constructional Steel ECCS residual stress pattern [7, 8]. The normalized moment is given as $m = M / M_p$, normalized axial load $p = P / P_y$, and residual stress ratio $c_r = \sigma_r / \sigma_y$. For a given $c_r$ and $p$ condition, the maximum normalized moment $m_1$ at which $\tau = 1$ is maintained for major-axis bending is given as:

$$m_1 = \frac{S_x}{Z_x} (1 - c_r - p)$$

where $S_x$ is the major-axis elastic section modulus and $Z_x$ is the major-axis plastic section modulus. The maximum normalized moment $m_1$ at which $\tau = 1$ is maintained for the minor-axis bending and axial compression condition is determined in a similar manner and is found to be
where $S_y$ is the minor-axis elastic section modulus and $Z_y$ is the minor-axis plastic section modulus.

The stiffness reduction when $m = 0$ and $p > 1 - c_r$ is given as $\tau_p$ in Fig. 1. The relationship for $\tau_p$ for the major-axis bending condition is found to be:

$$\tau_p = \frac{\lambda \lambda_0}{1 - \left(1 - \frac{1 - p}{c_r}\right)^3 + \frac{1 - p}{c_r} [2 + 6(1 + \lambda_1)^2]}$$  \hspace{1cm} (3)$$

where $\lambda = A_w/A_T$ and $\lambda_1 = d_w/t_f$ [8]. The stiffness reduction $\tau_p$ for the minor-axis bending condition is determined in a similar manner and is found to be:

$$\tau_p = \frac{2 \left(1 - \frac{1 - p}{c_r}\right)^3 + \lambda \lambda_0 \frac{1 - p}{c_r}}{2 + 6(1 + \lambda_1)^2}$$  \hspace{1cm} (4)$$

where $\lambda_0 = t_w/b_f$ [8]. For W-shapes in which $\lambda \lambda_0^2$ is very small compared to 2, a close approximation to Eq. (4) that excludes the web effect is given as:

$$\tau_p = \left(1 - \frac{p}{c_r}\right)^{3/2} \hspace{1cm} (5)$$

For a given $p$ condition, two equations are needed to determine $m_0$ when $\tau = 0$ in Figure 1. Closed-form equations are given in [9]; however, the same results can be obtained with fewer computations using the constants $\lambda, \lambda_0$, and $\lambda_1$ [8]. For the major-axis bending and axial compression condition, one equation is needed when the plastic neutral axis is in the web, and the other equation is needed when it is inside the flange thickness:

$$m_0 = 1 - \frac{p^2(2 + \lambda)^2}{4 \lambda_0 + \lambda(4 + \lambda)}$$  \hspace{1cm} (6)$$

when $p < \frac{\lambda}{2 + \lambda}$

$$m_0 = \frac{(2 + \lambda)^2 - [p(2 + \lambda) - \lambda + \lambda_1]^2}{4 + \lambda_1(4 + \lambda)}$$  \hspace{1cm} (7)$$

when $p \geq \frac{\lambda}{2 + \lambda}$

For the minor-axis bending and axial compression condition, one equation is needed when the plastic neutral axis is inside the web thickness, and the other equation is needed when it is outside the web thickness:

$$m_0 = 1 - \frac{p^2(2 + \lambda)^2}{(2 + \lambda \lambda_0)(2 + \lambda)}$$  \hspace{1cm} (8)$$

when $p < \frac{2 \lambda_0 + \lambda}{2 + \lambda}$

$$m_0 = 4 - \frac{[p(2 + \lambda) - \lambda]^2}{2(2 + \lambda \lambda_0)}$$  \hspace{1cm} (9)$$

The new proposed inelastic material model takes advantage of the closed-form equations for the perimeter conditions given by $m_1, \tau_p$ and $m_0$. The three-dimensional surfaces in Fig. 1 were developed from Eqs. (10) and (11). In the absence of any effort to determine the $n$ value for a given W-shape, it is recommended to use $n = 4$ for major-axis bending and $n = 2$ for minor-axis bending [10]:

$$\tau_A = 1 - \left(1 - \frac{m - m_1}{m_0 - m_1}\right)^n$$  \hspace{1cm} (10)$$

when $p < 1 - c_r$

$$\tau_A = \tau_p \left[1 - \left(\frac{m}{m_0}\right)^n\right]$$  \hspace{1cm} (11)$$

when $p \geq 1 - c_r$

For a given axial compression $p$ condition, and a W-shape with its $\lambda, \lambda_0$, $\lambda_1$, and $c_r$ constants, the $m_1$, $\tau_p$ and $m_0$ values are evaluated from Eqs. (1), (3), (6) and (7) for major-axis bending, and Eqs. (2), (5), (8) and (9) for minor-axis bending. As illustrated in Fig. 1, for a given $p$ and its corresponding $m_1$, $\tau_p$ and $m_0$ values, $\tau$ is evaluated based on the magnitude of the normalized moment $m$. If $p < 1 - c_r$ and $m \leq m_1$, there is no stiffness reduction and $\tau = 1$. If $p < 1 - c_r$, and $m > m_1$, there is stiffness reduction between 1 and 0 using Eq. (10) depending on the magnitude of $m$. If $p \geq 1 - c_r$, then stiffness reduction is between $\tau_p$ and 0 using Eq. (11) depending on the magnitude of $m$. If $m \geq m_0$ for any given $p$ condition, then $\tau = 0$.

Eqs. (12) and (13) have been frequently used in the stability analysis of steel framed structures [11], and
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3. Test Frame Analysis Results

System limit load analyses on three benchmark frames were conducted using the new $\tau_A$ material model and the traditional $\tau_B$ material model. All structural models used a load increment size of 0.02 and the second-order inelastic analysis capabilities of MASTAN2 [12]. All member cross-sections were assumed to be fully-compact and out-of-plane behavior fully restrained. Gravity loads were first applied, then lateral loads were applied up to the collapse condition. Initial geometric imperfections were directly modeled using $L/500$ in the compounding direction. Analyses were conducted using the number of line elements per member as indicated in the accompanying figures. All girders were modeled with major-axis bending, and columns were modeled with either all minor-axis bending or all major-axis bending. In order to compare only the material models, analyses did not include the stiffness reduction factors in the Specification [1].

The frame in Fig. 2 was modeled with all columns in minor-axis bending. The given gravity loads produce a collapse condition based on a detailed finite element model analysis [13]. In Fig. 5, when the applied load ratio $= 1$, the gravity loads in Fig. 4 are fully applied. Notice the frame with $\tau_A$ requires the same loads for the collapse condition, while the frame with $\tau_B$ requires additional load for collapse. The differences in results can be partially explained by examining the shaded regions in Fig. 6 where the location and relative extent of stiffness reduction are illustrated at the collapse condition [14]. For each member where there is no shaded portion $\tau = 1$, and where the shaded portion touches in the middle $\tau = 0$.

Partial yielding of several columns in Fig. 4a illustrates the ability of $\tau_A$ to model the stiffness reduction of the members compared with the results in Fig. 4b where there is stiffness reduction due to $\tau_B$ and hinges form only because of the existence of Eq. (14).

Fig. 1 Inelastic material model $\tau_A$ for: (a) major axis bending; and (b) minor axis bending.

excluding any slender-element effects, it is used in Chapter C of the Specification [1].

\begin{align*}
\tau_B &= 1 \quad \text{when } p \leq 0.5 \quad (12) \\
\tau_B &= 4p(1-p) \quad \text{when } 0.5 < p \leq 1.0 \quad (13)
\end{align*}

The inelastic material model in Eqs. (12) and (13) are used in combination with the yield surface in Eq. (14) to account for full cross-section yielding at the end of the element in MASTAN2 [12]:

\begin{equation}
\Phi = p^2 + m_z^2 + m_y^2 + 3.5p^2m_x^2 + 3p^4m_y^2 + 4.5m_z^2m_y^2 = 1 \quad (14)
\end{equation}

where $m_x = M_x / M_p$ is the normalized major-axis moment and $m_y = M_y / M_p$ is the normalized minor-axis moment.
The 6-story frame in Fig. 5 was proportioned with member sizes that induce loss of stiffness primarily in the columns such that limit loads were not governed by girder mechanisms. The frame was studied over a range of gravity loads $w$, and the lateral loads $H$ were increased up to the collapse condition. The results are given in Fig. 6 for the two material models with the columns oriented in major-axis bending and in minor-axis bending. The $\tau_B$ model gives more conservative results when the columns are in major-axis bending, but the $\tau_A$ model gives more conservative results as the gravity loads increase. When the columns are in minor-axis bending, the $\tau_A$ model gives the more conservative results throughout the full range of gravity loads. A review of the conditions at the limit load indicates extensive loss of stiffness at the top and bottom first floor columns and several leeward columns for the frames with the $\tau_A$ material model. However, this first floor column loss of stiffness only occurred with the higher gravity load conditions of the $\tau_B$ material model.
The effect of the gravity load magnitude on the percent difference in the results is illustrated in Fig. 7. The abscissa is normalized by dividing the total gravity load by the total yield load capacity of the first floor columns ($\sigma_y = 345$ MPa). The ordinate values were determined using

$$\frac{\text{Total Lat. Loads}_B - \text{Total Lat. Loads}_A}{\text{Total Lat. Loads}_B} \times 100\% \quad (15)$$

As the gravity loads increase, the lateral load relative percent difference increases to about 30% for column major-axis bending and to about 50% for column minor-axis bending. The modeling differences in Fig. 6 appear to be relatively insignificant, but Fig. 7 clearly demonstrates that this is not the case, especially for the frames with the higher gravity loads.

Fig. 7 Lateral load relative percent difference for 6-story building.
To determine if the modeling differences of the 6-story frame would be similar to that of a 2-story frame, the members in Fig. 8 were proportioned with W-shapes that would induce loss of stiffness primarily in the columns and reduce the potential for girder failure mechanisms. The results as illustrated in Fig. 9 indicate that the $\tau_B$ model consistently gives conservative results for columns with major-axis bending. At the collapse conditions, the $\tau_A$ model provides for yielding over the top and bottom first floor columns and a greater capacity to resist lateral loads at the lower gravity loads. Both model results tend to converge as the gravity loads increase and $\tau_A$ stiffness reduction begins to take effect in the interior first floor columns. For column minor-axis bending, the $\tau_B$ model gives conservative results with the lower gravity loads, but the $\tau_A$ model gives more conservative results as the gravity loads increase. Comparing the results in Fig. 6 and with those in Fig. 9 indicates that generalizations cannot be made regarding which material model will consistently give the more conservative results for a given frame and loading condition.

The effect of the gravity load magnitude on the differences in the modeling results is given in Fig. 10. As with the 6-story frame, the 2-story frame has significant differences in the modeling results. The differences change significantly as the gravity load increases, but the curves begin and end at different values than those in Fig. 7. Thus again emphasizing that generalizations cannot be made regarding which material model will consistently give the more conservative results.

4. Conclusions

This study investigated the degree to which two material models provide limit load results that are
consistent with one another when all factors that adjust the stiffness are limited to just the material models. The $\tau_A$ material model allows for direct input of parameters to adjust the stiffness reduction based on the W-shape’s dimensional properties, axis of bending, axial load magnitude and residual stress ratio. Whereas the $\tau_B$ material model only provides stiffness reduction when the axial load is above 50% of the yield load and requires the use of an empirical $\Phi$ equation to apply a hinge when the $m$ and $p$ reaches the yield surface. Since the $\tau_B$ material model is included in Chapter C and the $\tau_A$ material model meets the requirements for use in Appendix 1 of the Specification [1], it is important for design engineers to recognize that material models can give significantly different limit load results, especially under high gravity load conditions, and to be aware of the material model’s limitations when using them to design steel building frames.

References