Deduction of Market Prices for Futures Derivatives From Projectile Physics With Effects of the Simple Harmonic Oscillations on Equilibrium Price Positions

Leonard Mushunje

Midlands State University, Gweru, Zimbabwe

We investigated the motions associated with prices for futures contracts within financial markets. We aimed to derive the market prices from the physics approach. We used the projectile motion models defined under two distinct conditions (perfect/horizontal and imperfect/drag implication) based on Newton's and Galileo's laws of motion. In addition, we applied the simple harmonic oscillatory model to present the movements of prices from the market equilibrium position. Despite the fact that it was more theoretical, we managed to derive the futures price functions and the results showed that futures prices depend largely on market forces of demand and supply and underlying assets price behaviour. Also, we managed to find the terminal prices for the securities given the initial prices, which are a worrying matter to the trading parties. The equilibrium price analysis was done and the simple harmonic model proved to be efficient in such modelling. We managed to identify the price motions to and from the equilibrium point with markets. Results suggested that it is the market frictions (market forces of demand and supply) that propel prices to move. Also, we noted that these forces are responsible for bringing back the prices at equilibrium if the market is left to operate as free. Nevertheless, from the performance comparison of the two models used, results suggested that futures price function from a drag variable is more powerful in modelling the price behaviour for options than the one solely controlled by market demand and supply forces. And the simple harmonic oscillator model is good at modelling the equilibrium movements of asset prices. Above all, we used the mean absolute deviation (MAD) to validate our futures derivative pricing model. Fortunately, the obtained MAD results supported the efficiency of our model. However, it should not be carelessly taken that the projectile models used are much good at price motions/movements within the market from time to time with a stunted ability to capture in other facts of interest, such as volatility coefficients which pave a research way for other scholars.

Keywords: projectile motion, variable drag, futures derivatives, simple harmonic oscillator, equilibrium

Introduction

Risk management is done by investors using many different approaches. One of them is through the use of financial derivatives. More commonly, investment is now practised in most financial markets within the business world and the execution of any investment contract and deal is settled with one common aim of

Leonard Mushunje, B.Sc. in Mathematics, Department of Applied Mathematics and Statistics, Midlands State University, Gweru, Zimbabwe.

Correspondence concerning this article should be addressed to Leonard Mushunje, Department of Applied Mathematics and Statistics, Midlands State University, 9055 Senga Road, Gweru, Zimbabwe. Email address: leonsmushunje@gmail.com

minimising risk and maximizing returns. Investing in stock market has a lot of risks that could cause losses in the future and one way to minimise risk or protect the investment is by trading in derivatives (Rachmawati, Irene, & Budiharto, 2014). Derivative securities are financial contracts that derive their value from cash market instruments, such as stocks, bonds, currencies, and commodities (Neftci, 1996). Particular examples of such derivatives are futures, forwards, options, and swaps. All of them are subject to market expected and unexpected dynamics. We shall consider futures contracts in this study. Futures contracts are an agreement between two parties to buy (sell) an asset at a certain time in future for a certain price (Hull, 2012). Again, Neftci (1996) quoted futures as an obligation to buy or sell an underlying asset at a specified future price on a known date. Futures contracts are exchanged and traded on the market and they are market to market and this differentiates them from forwards, otherwise they are the same. This market feature makes them subject to market forces of demand and supply and other potential external forces. Normally, the movements in all the market forces are uncertainty and normally they hit the market position for any contract on trade unexpectedly. Such unexpected market movements and changes affect the market control price mechanism which brings birth to some notably frequent return volatility. Such return volatilities are a serious matter of concern to many investors and traders for it is their main drive to action in the market. When traders and investors foresee and anticipate an increase (decrease) in return of their investments they will pursue (cease) the contract or deal. Futures market behaviour was modelled using different models and approaches as found in literature. Kaldor (1939) first formalised the cash and carry model which is based on the hypothesis that futures price is equal to spot price and storage costs. Ritchken and Rob (1999) quoted that futures behave in a way that their spot and future prices gives the futures basis which converges to zero as the contract approach the maturity date and it is given as $B(t) = S(t) - F(t)$. Capital Asset Pricing Model (CAPM) pro founded by Dusak (1973) seems to be dominant in the finance world especially on pricing derivatives, such as futures. The model proves to be efficient and reliable as it considers the nexus between risk and returns which matches the central motive of every investor. In the same line, futures contract behaviour is rarely certainty implying its probabilistic nature, therefore stochastic models are of paramount use. Heston (1993) used the stochastic volatility model which appeared to operate in the same way as the black model (Brownian motion model). The models suggested jumps and randomness within the futures contracts and other derivatives, such as options. In relation to harmonic models, Meng, Zhang, Xu, and Guo (2015) investigated the behaviour of stocks in daily price-limited stock markets by purposing a quantum spatial-periodic harmonic model. They managed to derive and re-examined the effectiveness of price limit, with some observed characteristics of price-limited stock markets in China by applying the quantum model. However, this paper tackled and investigated the futures market behaviour in the context of financial physics. Literature seems to be lightly appearing on the subject. Stadnik (2014) explored the spring oscillations within financial markets by investigating volatility clustering under momentum trading techniques and results showed some springs associated with financial markets. Zhang and Huang (2010) developed a quantum model for stock markets which enabled them to define wave functions and operations of the stock market. Yura, H. Takayasu, Sornette, and M. Takayasu's (2014) molecular fluid dynamics in financial markets uses harmonic oscillations together with Hildalgo and Ricardo (2011) and Chowd, Hurry, and Das (2018) who looked at statistical physics in modelling financial markets using numerical analysis that is Runge Kutta and Euler methods. All these greater works paved a way to the solid and effective application of physics to financial studies. As such, this paper aims to investigate and model the market behaviour and performance for futures contracts using projectile motions and variable drag implications under

two distinct cases corresponding to perfect and imperfect market conditions respectively. In this study, we defined perfect market as the one controlled by market forces only where as imperfect as being a condition where the markets contracts prices are affected and controlled by market forces and the change in other powerful factors, such as market volatility. In addition, harmonic models (main and simple) are implemented to derive and explain the equilibrium positions of the prices with time. The price motions are explained from the time of release to the time of maturity of the contracts. However, this study is just but an extension of the paper by Mushunje (2019) on the determination of market prices for futures contracts using the projectile physics and variable drags. The novelty of this paper is on the consideration of the equilibrium effects of the futures prices being modelled using the harmonic oscillatory models.

Problem Formulation

Risk mitigation is actually an important aspect in finance and investments. Interestingly, derivatives are a special form of managing investment risks. Among such derivatives, as forwards, options, and vanilla swaps, this study focuses on the futures derivatives. Futures contracts behaviour varies from market to market depending on the majoring and topping conditions in each market. We formulate that contracts in the perfect markets perform differently from those in the imperfect markets, even if they are introduced at same time in the market. The major reason being identified as the external within and outside market forces associated with imperfect market platform. So, as long as market conditions are different, the futures performance likewise differs. So, the projectile models are to be used with a variable drag implication assumed to be available in the imperfect market situation. In addition, market traders and investors aim to operate at an equilibrium point. This gives an insight on the importance of a knowhow on the equilibrium position of market prices. As such, this study aims to contribute physics based model to explain the equilibrium prices for futures contracts.

Projectile Motions and Variable Drag Motions Background

Projectile motions are commonly used in the fields of physics where it is used to model the motion of objects with time horizontally and vertically into air subject to force of gravity “real life projectile motion” (2018) and “physics of projectile motion” (2001). In the same line, drag implications are used in fluid mechanics where the motion of the context object is modelled with the inclusion effect of the drag and gravitational force. However, all these complex and non-linear object motion models are built from the Galileo’s laws of motion where gravity is the only external force with other resistance forces assumed negligible, together with the Newton’s laws of motion. Of interest projectile motions are described into forms, that is the horizontal and inclined projectiles “projectile motion” (Murewi, 2018). All of these set ups arise or are centred on the laws short listed below.

Newton’s laws of motion. He put forward three distinct laws with the first one being:

- (1) The object remains at rest or in motion unless acted upon by other external forces.
- (2) The change of motion is proportional to the exerted force and it occurs in the straight line direction along which that force acts, i.e., $F = ma$.
- (3) To every action, there is always an equal and contrary reaction such that $F_p = -F_q$ (Nelkon, 1956).

Galileo’s laws of motion.

- (1) There is horizontal motion where the object would continue to move at constant velocity unless pushed.

- (2) There is vertical motion where a falling object decelerates at an uniform rate.
- (3) There is an added vertical motion under projectile motions.

These laws are useful in model building which we shall use in this study. Before advancing to model building, we shall first highlight some important assumptions from which our models and their applicability rest.

Simple Harmonic Oscillator Motions (SHM)

From the quant's perspective, this refers to any motion where a restoring applied force is proportional and opposite of the particle displacement. This means that as the particle is pulled out or in other direction, it will strive to bring itself back to the equilibrium. This study shall employ the same concept to model the price motions and associated equilibrium points of the futures prices. The model is built as well from the Newton's laws of motion, Galileo's and the harmonic laws of oscillations (implied). We defined the simple harmonic motion model in the below section. But basically, the idea is all about the motion forces (friction in physics) that bring back the futures prices to the stable and equilibrium points. We shall model this problem using the SHM model.

Basis of Assumptions

Firstly, we assumed that the unit vector directions denoted by i , j , and k are all equal to one. Also, we assumed that mg is the only market force for the horizontal projectiles with additional forces in the inclined version. On the simple harmonic motions, we assumed a small angular market occupation by the assets and we assumed that the market is free (no government interference). Last but not least, we assumed that there is null relationship between the horizontal and vertical projectile environments where we have $i.k = 0$, and there is a perfect and direct linear relationship within each market environment and itself indicated in physics as $k.k = 1$, $i.i = 1$. From all these assumptions, we shall build and solve our models as in the following section.

Methodology

We used two distinct models with each explaining the how the price of futures instruments are built from the projectile motions of underlying asset's prices and we considered stock assets. So, we considered the futures contracts and corresponding stock assets, under two different market conditions and the case models are as below.

Case 1: Perfect Market

We used a projectile motion model constructed using the Newton's second law of motion. The essence of the model is that we start with the difference equation for asset prices, and after several formal integrations, we will identify the price function of the futures instrument. We start with a second order difference equation as below,

$$F = ma \quad (1)$$

which can be re-expressed as,

$$F = VS\ddot{t} \quad (2)$$

where V is the volume of assets in the market and $F = -M$, the classical market forces. Thus,

$$VS\ddot{t} = -M \quad (3)$$

Equation (3) will be solved to extract the asset's price function and finally for futures contracts which is our focus of study.

Case 2: Imperfect Market

Secondly, we considered a second order difference equation in another different market condition that is imperfect market. The idea comes from the Newton's first law of motion in which we argue that futures prices remain at rest or in motion (decreasing or increasing) unless acted upon by other factors (forces). The idea is mathematically presented as below,

Using (1) and (2) in our first case, we get,

$$F = VS''(t) \quad (2)$$

Now,

$$F = -F'(S(t)) - M \quad (4)$$

thus, $VS''(t) = -F'(S(t)) - M$, this will be solved further in order to derive the underlying asset's price function and for the futures contracts (see "Main Results and Findings" section).

Simple Harmonic Oscillator Model

To deduce and present the market price equilibrium, we used the SHM described as below. There are two different forms of such harmonic motions. These are the simple harmonic motion model and the main harmonic motion model. We shall restrict our focus to the simple harmonic motion model. However, both of them shall be presented as follows.

Main harmonic motion model. The equation is given as:

$$X = A \sin \omega t \quad (5)$$

where,

X is the market displacement for the assets;

ω is the angular measure of market penetration and occupancy of the assets;

t is the time in seconds;

A is the velocity (change in market prices).

This is the main model and it shall assist to explain the movements of the prices together with the simple model.

Simple harmonic motion model.

$$T = 2\pi \sqrt{\frac{V}{k}} \quad (6)$$

where,

T is the time of oscillation;

K is the spring constant;

V is the assets (futures derivatives) volume.

This is our main focus model and the full interpretations are given in the following section.

Main Results and Findings

From Case 1,

$$S''(t) = -\frac{M}{V}$$

$$\int (S''(t)) dt = \int -\frac{M}{V} dt.$$

$$S(\dot{t}) = -\frac{M}{V}t + C \quad (7)$$

But at introduction of the asset in the market at $t = 0$, we have asset price, $S(0) = S_0(|\ln(\cos\emptyset + \sin\emptyset)|)$, so if $\emptyset$ is a measure of market size occupied by the asset, then $S_0(|\ln(\cos\emptyset + \sin\emptyset)|)$ will become the newly transformed per asset price. Now, $S_0(|\ln(\cos\emptyset + \sin\emptyset)|) = C$. Thus,

$$S(\dot{t}) = -\frac{M}{V}t + S_0(|\ln(\cos\emptyset + \sin\emptyset)|) \quad (7)$$

The above equation gives the price of the asset at any time t given the market force and the asset volume M and m respectively per market area occupied($\emptyset$).

Now, integrating secondly equation (7),

$$\int S(\dot{t})dt = \int \left(-\frac{M}{V}t + S_0(|\ln(\cos\emptyset + \sin\emptyset)|)\right)dt$$

$$F_s(T) = -\frac{Mt^2}{2V} + t(S_0(|\ln(\cos\emptyset + \sin\emptyset)|) + B \text{ where } F_s(T) \text{ is the price function for the futures contract}$$

which is equivalent to $S(t)$. but identically $B = 0$ when $t = 0$. Thus, the futures price function will be

$$F_s(T) = -\frac{Mt^2}{2V} + tS_0(|\ln(\cos\emptyset + \sin\emptyset)|) \quad (8)$$

$$F_s(T) = t(S(\dot{t})) \quad (9)$$

Before futures contracts are traded, a future maturity date is known and preset and investors maybe more concerned with the total time their contracts can take to reach the expiration which is calculated as,

$$F_s(T) = 0$$

$$-\frac{Mt^2}{2V} + tS_0(|\ln(\cos\emptyset + \sin\emptyset)|) = 0$$

$$t = 0 \text{ or } t = \frac{2V(S_0(|\ln(\cos\emptyset + \sin\emptyset)|))}{M}$$

This gives the initial time when we introduce our futures and the total time in the market till it reach the expiration date, so total market life span for the futures contract is given as,

$$T = \frac{2V(S_0(|\ln(\cos\emptyset + \sin\emptyset)|))}{M} \quad (10)$$

Case 2: Imperfect Market Condition

Now, our second case is made up and it consists of a rough market condition. The price would be supposed to remain moving up, downward, or to remain at rest, if the market condition is smooth. This means that the futures contract prices depend on the underlying asset which greatly depends on some market environmental factors. Thus, we shall introduce more than one force that has also an effect on the futures price life. Now, we know that,

$$F = VS(\ddot{t}) \quad (2)$$

Now,

$$F = -F(\dot{S}(t)) - M \quad (11)$$

where F is just a constant of proportional relations between the market drag and the price for both the underlying asset and futures securities and M is the market forces of demand and supply for the derivative in

question, thus,

$$VS(\ddot{t}) = -F(\dot{S}(t)) - M$$

thus,

$$S(\ddot{t}) = -\frac{F(\dot{S}(t))}{V} - \frac{M}{V} \quad (12)$$

This is a second order linear equation which can be made linear through a complete exponential transformation of the whole equation as follows.

Let $K(t) = S(\dot{t})$, so that,

$$\begin{aligned} e^{\frac{F}{V}t} d. \frac{K(t)}{dt} + \frac{e^{\frac{F}{V}t} F}{V} K(t) &= e^{\frac{F}{V}t} \left(-\frac{M}{V}\right) \\ \int \frac{d. e^{\frac{F}{V}t} F K(t)}{dt} dt &= \int e^{\frac{F}{V}t} \left(-\frac{M}{V}\right) dt \\ K(t) &= -\frac{M}{F} + C e^{-\frac{F}{V}t} \end{aligned} \quad (13)$$

But we know that $S(\dot{0}) = 0 = S_0(|\ln(\cos\emptyset + \sin\emptyset)|)$, thus we get,

$C = S_0(|\ln(\cos\emptyset + \sin\emptyset)|) + \frac{M}{F}$, which gives

$$S(\dot{t}) = -\frac{M}{F} + e^{-\frac{F}{V}t} \left(S_0(|\ln(\cos\emptyset + \sin\emptyset)|) + \frac{M}{F}\right) \quad (14)$$

This gives the asset price behaviour at any time and we shall derive the futures price function from (12). We shall perform our calculus integration as:

$$\begin{aligned} \int S(\dot{t}) dt &= \int -\frac{M}{F} + e^{-\frac{F}{V}t} \left(S_0(|\ln(\cos\emptyset + \sin\emptyset)|) + \frac{M}{F}\right) dt. \\ S(t) &= -\frac{M}{F}t - \frac{V}{F} \left(S_0(|\ln(\cos\emptyset + \sin\emptyset)|) + \frac{M}{F}\right) e^{-\frac{F}{V}t} + B. \end{aligned}$$

But, $S(0) = -\frac{V}{F} \left(S_0(|\ln(\cos\emptyset + \sin\emptyset)|) + \frac{M}{F}\right)$, so we get,

$$S(t) = -\frac{M}{F}t - \frac{V}{F} \left(S_0(|\ln(\cos\emptyset + \sin\emptyset)|) + \frac{M}{F}\right) e^{-\frac{F}{V}t} + \frac{M}{F} \left(S_0(|\ln(\cos\emptyset + \sin\emptyset)|) + \frac{M}{F}\right) \quad (13)$$

Thus, the price function for futures contracts is given by:

$$F_s(T) = -\frac{M}{F}t - \frac{V}{F} \left(S_0(|\ln(\cos\emptyset + \sin\emptyset)|) + \frac{M}{F}\right) e^{-\frac{F}{V}t} + \frac{M}{F} \left(S_0(|\ln(\cos\emptyset + \sin\emptyset)|) + \frac{M}{F}\right)$$

Using Equation (14) and making proper substitutions, we get,

$$F_s(T) = -\frac{M}{F}t - \frac{V}{F} (S(\dot{t}) + \frac{M}{V}t) e^{-\frac{F}{V}t} - \frac{M}{F} (S(\dot{t}) + \frac{M}{V}t) \quad (15)$$

Equation (14) clearly shows that the price for futures contracts depends on the underlying asset's price function, $S(\dot{t})$ as required.

Simple Harmonic Oscillator Model Interpretations

The model in its nature is sinusoidal. In financial terms, it is measuring the time through which the derivative price swings from any point within the market whenever subjected to contradicting forces. This

model was greatly in agreement with the imperfect market (drag implications model). This model enabled in addition to the drag implication to derive and explain the price movements of the derivatives up until equilibrium is reached. It is well known that as assets and corresponding derivatives exist in the markets different price positions are seen and eventually an equilibrium point is obtained and achieved. The time to time movements are explained well and derived well from the simple harmonic motion model. The circular and periodic movements of prices from time to time are well explained by the main harmonic model. The model has a sine function. This implies that assets and derives prices start at zero value as they are released into the market and attain different values along their way to maturity. In addition, the simple harmonic model can give us the time taken by the contracts to maturity.

$$T = 2\pi \sqrt{\frac{V}{k}}. (\pi = \frac{22}{7}) \text{ and } V \text{ is a known volume of the assets or derivative in the market.}$$

This is an important aspect to investors. By knowing the time required for the contract to mature, risk management is made easy. This was in support with the above models which enabled us as well to derive such functions as below.

Initial and Terminal Prices for Futures Contracts

Normally, derivatives are introduced in a market with an initial price attached, though it changes with time (see Corollary 1). Futures are traded for a definite life horizon therefore investors are much worried about the terminal price for the security so as to ensure a positive future basis. So, terminal price for the futures contract at the maturity date can be determined from its price function in Equation (14) by evaluating its limit as time approaches the expiration. Below is a brief illustration of getting the terminal price for the futures.

This can be illustrated as:

For Initial price, $\forall X(s) \in M_d, \text{ att} = t_0 \exists S_0 : F_s(T) > 0$. And

Terminal price is found as:

$$\lim_{t \rightarrow t_n} F_s(T) \text{ where } t_n = \frac{2m(|\cos\phi + \sin\phi| + \frac{Mg}{m})}{Mg} \text{ the total market life time of the futures.}$$

$$\lim_{t \rightarrow t_n} \left(-\frac{Mg}{F}t - \frac{M}{F} \left(S(\dot{t}) + \frac{Mg}{m}t \right) e^{-\frac{F}{m}t} - \frac{M}{F} \left(\frac{Mg}{m}t \right) \right)$$

$$\text{This gives } -\frac{Mg}{F}t_n - \frac{M}{F} \left(S(t_n) + \frac{Mg}{m}t_n \right) e^{-\frac{F}{m}t_n} - \frac{M}{F} \left(\frac{Mg}{m}t_n \right)$$

This means that we get the terminal price as we approach the maturity date as above.

Model Validation and Testing

We used daily price data for top 40 futures and stocks traded at Johannesburg Stock Exchange. We then estimated futures prices for perfect market case using available stock price data. We then used the mean absolute error (MAD) for validation. MAD simply refers to a method which estimates forecasting errors by spreading absolute errors over available units making up the sampling frame. Below is a computed table with actual, estimated and the MAD values for futures instrument's prices.

Comments

We used MAD values to validate our model. The MAD values are found by using the formula

$\frac{1}{n} \sum_{i=1}^n |x_i - x|$. This works based on selected limits. If the values lie within the MAD limits, then the forecast is

concluded to be strong and efficient one. Our aim whenever we make forecasting is to minimize errors as small as possible. We found favourable individual MAD values and a final supportive value of 1.204675. These suggest the efficiency of our model. We neglected four extreme values that fell out of the MAD limits that is $\pm 2MAD$. We suggested that these extremes were from market instabilities and uncertainty effects and as well misbehaviour of junk stocks within financial markets.

Table 1

Model Validation Results

Period (month) Date	Stock prices ($S(t)$)	Futures values (estimated)	Actual futures prices	MAD values (individual)	Final MAD value
Jan. 22 2019. (1)	47.925	47.925	48.182	0.006425	1.204675
Jan. 21 2019. (1)	48.037	48.037	48.274	0.005925	
Jan. 18 2019. (1)	47.585	47.585	47.960	0.009375	
Jan. 17 2019. (1)	47.298	47.298	47.605	0.007675	
Jan. 16 2019. (1)	47.665	47.665	47.975	0.00775	
Jan. 15 2019. (1)	47.382	47.382	47.575	0.004825	
Jan. 14 2019. (3)	47.335	142.005**	47.828	2.354425**	
Jan. 11 2019. (1)	47.491	47.491	47.559	0.0017	
Jan. 10 2019. (1)	47.164	47.164	47.411	0.006175	
Jan. 09 2019. (1)	47.140	47.140	46.466	0.01685	
Jan. 08 2019. (1)	46.116	46.116	46.263	0.003675	
Jan. 07 2019. (3)	45.875	137.625**	46.457	2.2792**	
Jan. 04 2019. (1)	46.156	46.156	45.898	0.00645	
Jan. 03 2019. (1)	45.622	45.622	45.642	0.0005	
Jan. 02 2019. (2)	45.310	90.62	47.058	1.08905	
Dec. 31 2018. (3)	46.726	140.178**	46.908	2.33175**	
Dec. 28 2018. (1)	46.493	46.493	46.065	0.0107	
Dec. 27 2018. (3)	45.655	136.965**	46.656	2.257725**	
Dec. 24 2018. (1)	46.203	46.203	45.099	0.0276	

Note. **outliers. The cause was mainly from stock volatility effects.

Conclusion and Discussion

We theoretically derived the futures contracts price functions under two distinct market conditions using projectile mechanics and variable drag. We showed that futures contracts prices depend on the price functions of the underlying assets and market forces. Also, the projectile models used were mainly based on the Newton's second law of motion. The models allowed us to derive the terminal prices for futures and also the total market life span given the initial/release price. We showed that as we approach the expiration of the contract, we are bound to reach the terminal price which is useful on contract valuation. Also, from the Newton's first law, we saw that the futures prices remain unchanged unless acted upon by other factors, such as the underlying assets and market forces of demand and supply. However, the models were very much limited on multi-variable capturing that is other important factors, such as volatility were not considered. Therefore, projectile mechanics alone cannot be used to make powerful conclusions implying the chance rise for use of other competing and valid models. Also, for powerful analysis of market price movements and motions for futures contract, the projectile motion model with a variable drag is considered powerful compared to the one for perfect condition/horizontal motion. As a complimentary analysis, a simple harmonic oscillation model and the main harmonic model were consulted. The models were used to derive and provide a basic analysis of the

equilibrium position of the prices for futures contracts and (or) underlying assets. The models enabled us to derive the oscillatory behaviour and time taken before reaching maturity and time to get back to the equilibrium point. This is of noble importance as it gives direction and knowhow plan on risk management prior the maturity date. The results in overall eyes suggested and supported the efficiency of physics motion models when modelling finance problems. This gave power and ever promising rise of this interdisciplinary aspect of finance and physics.

References

- Dusak, K. (1973). Futures trading and investor returns: An investigation of commodity market risk premiums. *Journal of Political Economy*, 81(6), 1387-1406.
- Heston, S. L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies*, 6(2), 327-343.
- Hull, J. C. (2012). *Options, futures and other derivatives*. New Jersey: Pearson Prentice Hall.
- Kaldor, N. (1939). Speculation and economic stability. *The Review of Economic Studies*, 7(1), 1-27.
- Meng, X, Zhang, J. W., Xu, J., & Guo, H. (2015). Quantum spatial-periodic harmonic model or daily price-limited stock markets. *Physica A: Statistical Mechanics and its Applications*, 438(C), 154-160.
- Murewi, C. (2018). Lecture notes on mechanics. www.msu.ac.zw/elearning, (class circulars).
- Naughton, S. (2018). Seminar 7: Transforming organisations: Strategy, structure & design, lecture notes. *Organisation Change Management BMO6624*, Victoria University.
- Nefcti, S. N. (1996). *An introduction to the mathematics of financial derivatives* (1st ed.). San Diego, Calif.: Academic Press.
- Nelson, M. (1956). *Principles of physics*. London: Christophers.
- Rachmawati, R. N., Irene, & Budiharto, W. (2014). Oscillatory reduction in option pricing formula using shifted poisson and linear approximation. *EPJ web of conferences* 68, 00006(2014). Retrieved from <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.949.6933&rep=rep1&type=pdf>
- Ricardo, H., & Cesar, A. H. (2011). The network structure of economic output. Papers 1101.1707, arXiv.org, revised Feb 2012.
- Ritchken, P., & Rob, T. (1999). Pricing options under generalized GARCH and stochastic volatility processes. *The Journal of Finance*, 54(1), 377-402.
- Science Clarified. (n.d.). *Real-Life Chemistry Vol. 3: Physics Vol. 1*. Retrieved from <http://www.scienceclarified.com/everyday>
- Simoes, A. G., & Hidalgo, C. A. (2011). The economic complexity observatory: An analytical tool for understanding the dynamics of economic development. *Scalable Integration of Analytics and Visualization: Papers from the 2011 AAAI Workshop (WS-11-17)*. Retrieved from <https://pdfs.semanticscholar.org/7733/68ce1faa36d9ac833b3c3412d136033b91c1.pdf>
- Stadnik, B. (2014). Spring oscillations within financial markets. *Social and Behavioural Sciences*, 110, 1176-1184.
- The physics of projectile motion*. (2001). Retrieved from <http://library.thinkquest.org/2779/>
- Yura, Y., Takayasu, H., Sornette, D., & Takayasu, M. (2014). Analysis of financial markets using laws of molecular fluid dynamics. *Physical Review Letters*, 112(9), 098703.
- Zhang, C., & Huang, L. (2010). A quantum model for the stock market. *Physica A: Statistical Mechanics and Its Applications*, 389(24), 5769-5775.