Quantum Space-Time as the Set of Shapeless Finite Fundamental Elements

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Abstract: The idea of space-time as a combination of shapeless fundamental elements is proposed. The history of the development of ideas about the discrete space-time structure is analyzed. Quantum space-time is considered as a set of quantum states defined on a set of discretizations with an arbitrary shape of the boundaries of regions. The fundamental element of such space-time is described by the totality of its probabilistic characteristics. We consider a concept in which space-time is the only quantum object, and all material particles and interaction carriers are described as excited states of the fundamental elements of this quantum object.

Key words: Space-time, shapeless fundamental elements, quantum.

Consider the conceptual structure of physics. There are six basic concepts in the language of which the basic concepts, laws of physics, and basic equations are formulated. These include: classical physics, classical relativistic physics, nonrelativistic quantum theory, relativistic quantum theory (quantum field theory), Einstein's theory of gravity, and the theory of superstrings. We will characterize the concepts by the number of entities that define the imaginative structure of the concept. In classical physics, there are four such entities: it is space, time, matter and field. The same four entities underlie the imaginative structure of the nonrelativistic quantum theory, while the nonrelativistic quantum theory is based on the deeper quantum principle of describing natural phenomena. The basis of classical relativistic physics are three entities — space-time (ST), matter and field. These three entities define the imaginative structure of quantum field theory. In principle, the same three entities underlie Einstein’s theory of gravity, but due to the basic equations of the concept, the properties of space-time are rigidly connected with the energy-momentum tensor of matter and field. Finally, the superstring theory is two-entity, its imaginative structure is determined by space-time and the string world surface. Both particles of matter (fermions) and interaction carriers (bosons) are some vibrations of the string world surface.

In this case, it can be noted that each deeper concept has a smaller number of entities. Thus, classical physics, which is the limit of both nonrelativistic quantum theory and relativistic classical physics, is four-fold, and relativistic classical physics has three entities. In turn, quantum field theory is three-entity, and non-relativistic quantum theory is four-entity, and is the limit of quantum field theory as \( c \rightarrow \infty \). Finally, the superstring theory, which arose as an attempt to synthesize quantum field theory and Einstein’s theory of gravity, is two-entities, whereas both of these concepts are three-entities. At the moment of the conceptual development of physics, the theory of superstrings is the deepest physical concept.

A natural question arises: is there a deeper, than the theory of superstrings, a single-entities physical concept? The author answers this question: YES. To the question: what is this entity? The author answers: space-time. Space-time naturally is quantum. This
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This report proposes a general view of the quantum theory of space-time as the most profound physical concept. The only entity in this concept is quantum space-time (QST), and all particles are excited states of the fundamental elements (FE) of space-time. For the first time the idea that more fundamental carriers of properties than particles are elements of the volume of space-time is found in Schwinger [1]. Schwinger raises the question of the measurability of arbitrarily small distances and comes to the conclusion that the operator theory he developed does not give an unambiguous answer to this question.

It is understood that at distances of the order of the Planck length, space-time exhibits quantum properties. Analysis of the procedure for measuring spatial and temporal coordinates in the quantum theory of gravity leads to inequality similar to the Heisenberg uncertainty principle:

\[ (\Delta L)^2 > L_{Pl}^2 \]  

This means that distances smaller than the Planck length are immeasurable. One of the ways to represent the space-time, in which the condition (1) is fulfilled, is to endow the space-time with a discrete structure [2, 3]. In this consideration, space-time consists of close-packed fundamental elements having sizes of the order of \( L_{Pl} \).

In the literature the discretization of space-time by the fundamental elements of different shapes are investigated. Below we will analyze some approaches to the discretization of space-time. This report proposes an idea that states that the fundamental elements of quantum space-time do not have certain shape and sizes.

We will consider the fundamental element of quantum space-time as a quantum object. The parameters describing this quantum object are its volume, surface area, and other geometrical and topological characteristics. Since it is the quantum object, it cannot have exactly defined sizes. As characteristics of the quantum object, the sizes of the FE should be described by a probability distribution. The same goes for the shape of the FE. Being the quantum geometrical object, the FE cannot have the definite shape.

In the QST with the shapeless fundamental element, the fundamental length plays the role of a certain geometrical characteristic of the FE of the dimension of length. Obviously, it should be the average value of a geometrical value of the dimension of length. It can be magnitude \( \langle V^{1/n} \rangle \), or magnitude \( \langle V \rangle^{1/n} \), or magnitude \( \langle 2n \frac{V}{S} \rangle \), where \( n \) is the dimension of space-time, \( V \) is the volume of FE, \( S \) is the surface area. Thus, when describing the properties of the QST, the fundamental length loses its value as the only fundamental geometrical characteristic of the FE.

Let us analyze how the idea of the discrete structure of the ST has historically developed. Initially, a lattice space-time densely packed with cubes of the same size was considered [2-5]. Redge [6] and other authors [7] considered ST densely packed with simplexes. Somewhat later, some authors began to consider the random lattice as a way of endowing the ST with the discrete structure [8-10]. Finally, in the 90s of the 20th century, ST began to be considered densely packed with polyhedra [11-13]. Closest to the concept of the QST with the shapeless FE is the concept in which the assignment of a wave function on the topology is assumed [14]. As it can be seen, the development of ideas about the shape of the fundamental elements of the discrete ST goes from simple shapes to more complex ones, and the natural limit of this ideological development is the idea of the shapeless fundamental element.

We now analyze the problem of the shape of the FE of ST from the point of view of the theory of deformation of mathematical structures. As is known, within the framework of this theory there is the concept of sustainable structure. It is obvious that FE with a certain shape is not stable with respect to a small strain of its shape. The shapeless FE will be a stable structure, since any deformation of its shape...
will also lead to a shapeless FE.

Generally speaking, the QST with the shapeless FE must be set axiomatically. So far, one axiom is clear - an analogue of the axioms of separability. It can be called the inseparability axiom, because it lies in the fact that the intersection of two fundamental elements is not empty and forms the third fundamental element. From this, it obviously follows that the FE of the QST is a probabilistic structure. That is, the FE of QST is described as the geometric object with the probability of its realization.

Consider a natural way of describing the QST with the shapeless FE. To do this, consider the set of discretizations with an arbitrary shape of the boundaries of the regions. By discretization, we mean the partition of the region \( \mathcal{R}^n \) into a set of open regions and their boundary points. Moreover, the set of boundary points is assumed to be connected. We will denote the set of discretizations with arbitrary boundaries of the regions \( D \). We denote individual discretizations by the macroindex \( \{ l \} : D_{(l)} \). Discretization is parametrized by specifying a set of boundary points of the discretization regions. The set \( D \) can be provided with a metric. Note that we can introduce the concept of converting one discretization to another, allowing the boundaries of regions to move in an arbitrary way. To convert discretizations with a different number of regions, we can introduce the concept of generators of birth and destruction of an infinitely small region. In the two-dimensional case, the distance between two discretizations is equal to the infimum of the area swept up by the boundaries when converting one discretization to another. This metric is fairly generalized to the n-dimensional case.

On the set \( D \), we can define the wave function. The squared modulus of the wave function gives the probability density to find the ST in the implementation described by this discretization. The continuity of the wave function is achieved by the fact that when passing from one discretization to an infinitely close to it, the module and phase of the wave function also changes infinitely small. The FE of such the QST is characterized by the probability distribution of its geometric characteristics. We assume that on the set \( D \) one can define the measure \( d\Sigma \). Then the wave function defined on \( D \) satisfies the equality.

\[
\int_D |\Psi(D_{(l)})|^2 d\Sigma = 1
\]  

(2)

Probability of finding the FE containing the point \( r_0 \) in the implementation with geometrical characteristics (volume, surface area, etc.) \( \Gamma \)

\[
P = \int_{D_{(r_0)}} |\Psi(D_{(l)})|^2 d\Sigma,
\]

(3)

where \( D_{(r_0)} \) is the set of discretizations in which the FE containing the point \( r_0 \) has the geometrical characteristics \( \Gamma \).

Probability of finding the FE containing the region \( O \) of the continuous space in the implementation with geometrical characteristics \( \Gamma \)

\[
P = \int_{D_{(\Gamma)}} |\Psi(D_{(l)})|^2 d\Sigma,
\]

(4)

where \( D_{(\Gamma)} \) is the set of discretizations in which the FE containing the region \( O \) has the geometrical characteristics \( \Gamma \).

The average value of a certain function of the geometric characteristics \( \Gamma \) of the FE containing the point \( r_0 \)

\[
\langle f(\Gamma, r_0) \rangle = \int_D f(\Gamma, r_0) |\Psi(D_{(l)})|^2 d\Sigma
\]

(5)

In particular, the average value of the function of the volume \( V \) and the surface area \( S \) of the FE containing the point \( r_0 \)

\[
\langle f(V, S, r_0) \rangle = \int_D f(V, S, r_0) |\Psi(D_{(l)})|^2 d\Sigma
\]

(6)

Different wave functions \( \Psi(D_{(l)}) \) correspond to different states of the QST. The QST with this consideration is defined as the set of all quantum states given on \( D \).

QST acts in this concept as the only object. All fundamental particles are excited states of the QST. Different states of QST correspond to different
numbers of particles filling and different positions of particle localization.

\[ \Psi(D_{(l)}) = \Psi(n_1, r_1, n_2, r_2, ..., n_N, r_N) \]  

(7)

The characteristics of the particles in this consideration, obviously, will be some probabilistic characteristics of the fundamental elements of the QST. The localization of the particle in a certain region of space at some moment in time means that the FEs corresponding to the space-time coordinates of this event will have such probabilistic characteristics that describe the particle. The dynamics will be that the region of localization of the particle will shift in the direction of the time axis and in the direction of movement of the particle.

Vacuum state, as it is understood in quantum field theory, corresponds to a uniform scalar field set in the whole space. In the language of the QST, this means that all FE of the QST will have some given probabilistic characteristics describing a scalar field. This is achieved by a certain dependence \( \Psi(D_{(l)}) \).

It was noted above that the developed concept of QST is the most profound one-entity physical concept. Two-entity physical concept — the theory of superstrings should be a limiting case of the concept of a quantum description of the ST. It can be seen that the string world surface is a two-dimensional excitation of the FEs of a QST. The probabilistic characteristics of the FEs, which form the string world surface, will differ from the probabilistic characteristics of the FEs, which do not lie on the string world surface. Otherwise, we can say that the string description is a description of a two-dimensional basis in the space of excitations QST. Similarly, p-branes can be described in terms of QST. It is obvious that the string description is suitable in the case when the length of the string is much greater than the Planck length.

In a certain sense, specifying a set of quantum states on a set \( D \) is a semiclassical picture, since the classical continual space (space-time) is discretized in this case. In a sequential quantum consideration, the set of realizations is set axiomatically and, apparently, the regions in the implementation will have a scatter of the values of the boundary points.

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**References**


