Rainfall Trend Analysis in the City of Villahermosa, Mexico

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Abstract: In this paper, a study by means of linear regression of rainfall in Villahermosa is proposed, within the study area 5 meteorological stations distributed in the city were analyzed because it is one of the most populated areas of the state, once estimated the data, the non-parametric Mann-Kendall test was used to determine increasing or decreasing linear trends in precipitation. To determine the usefulness of statistical inference, the linear correlation coefficient (r) was calculated, which was 0.86 on average. Statistical efficiency (E) was also calculated, which in all the cases analyzed suggests the viability of statistical inference. As a result of this analysis, a complete rainfall database is presented and evaluated for increasing or decreasing trends for the 1980-2010 period.

Key words: Rainfall, linear regression, Mann Kendall, trend analysis.

Nomenclature

X :	independent variable
Y :	dependent variable
R :	correlation coefficient
R^2 :	coefficient of determination
S :	Mann Kendall statistic
m :	Sen's slope
H_0 :	null hypothesis
H_1 :	alternate hypothesis
$\bar{x}$:	average of x

Greek Letters

β_0 :	intercept
β_1 :	slope
ε :	random error
Σ :	summation

1. Introduction

In the state of Tabasco and the rest of the country, the National Meteorological Service (SMN) and the National Water Commission (CONAGUA) maintain climatic records. Due to the instrumentation used in the stations there are data that were not registered, being the only institutions that provide such free access data,

for any analysis it is necessary to complete the missing information.

In Mexico the distribution of precipitation varies greatly in space and time, is heterogeneous throughout the year, intimately linked to the orography of the country and is affected by its proximity to the Pacific Ocean and the Gulf of Mexico [1]. Today the science of precipitation is at the crossroads of different scientific disciplines including hydrology, numerical modeling, climate change, remote sensing, climate prediction among others [2]. Temperature and precipitation are fundamental components of climate and changes in their patterns can affect human health, ecosystems, plants and animals [3]. The trends in rainfall have a great impact on the hydrological cycle, affecting the quantity and state of water resources, their analysis also helps to see the result of variability in precipitation when droughts and floods occur [4]. Studying regional climate variability is essential for planning the management of a country's natural resources [5].

There are several methods to complete the missing values in a pluviometric series. Ref. [6] proposed several statistical methods for the filling of missing values, including linear regression, the ratio method and the normal ratio. The linear regression method is

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one of the most used when it is necessary to estimate missing temperature and precipitation data, its frequent use is due to the simplicity of the method since it gives clear and precise results [7, 8]. Linear regression is also one of the most common tests to detect trends, which assumes a normal distribution of data; on the other hand, there is the non-parametric Mann-Kendall test that can analyze the information regardless of the distribution and lost data, which is why it has been widely used in environmental sciences [9].

In this paper, linear regression is used to estimate monthly precipitation data based on the goodness of fit of the model and thus verify with statistical efficiency if the model made an adequate estimation of the data and determine if there are increasing or decreasing linear trends in precipitation, on an annual and monthly scale during the period 1980-2010, using the non-parametric Mann-Kendall test and the slope method

of Sen to quantify the trend [10]. As a result a complete rainfall database is presented and evaluated for increasing or decreasing trends for the 1980-2010 period.

The paper is organized as follows: in the second section materials and methods are presented, while in the third section experiments results are obtained. Finally, conclusions and references are given.

2. Statistical Methods

The state of Tabasco is in the tropical zone of the southeast of Mexico [Fig. 1], between 17°-18° north latitude and 91°-94° west longitude. The 95% of its territory has a flat relief and slight depressions that correspond to flood areas and swamps. Tabasco is part of the Physiographic Province called Coastal Plain of the South Gulf, in which the Humid Warm climate predominates, with average rainfall per year of 1,526.4 mm the minimum and 2,600.9 mm the maximum [11].

Fig. 1 Location of the study area.

Table 1 Data of stations used in the study.

ID	Station (Centro, Tabasco)	Longitude	Latitude	Altitude (MSNM)
27055	VILLAHERMOSA (SMN)	-92.928	17.997	6
27054	VILLAHERMOSA (DGE)	-92.928	17.997	24
27065	DOS MONTES (DM)	-92.767	17.978	14
27096	PORVENIR (PRVN)	-92.881	18.008	5
27014	ESCUELA DE INGENIERIA (EI)	-92.933	18.017	7

Fig. 2 Location of stations.

The information used in this work is part of the data record monitored by the existing stations in the CLICOM database. Fig. 2 and Table 1 show the information of the stations used in the study.

2.1 Linear Regression by Least Squares

According to Ref. [12] regression analysis is a statistical technique for the modeling and investigation of the relationship of two or more variables. The case of simple linear regression considers only a regressor or predictor x , and a dependent variable or answer Y . Suppose that the true relation between Y and x is a straight line, and that the observation Y in each level x is a random variable.

$$Y = \beta_0 + \beta_1 x + \epsilon \quad (1)$$

where the intercept β_0 and the slope β_1 are the unknown coefficients of the regression, ϵ is a random error with zero mean and variance σ^2 .

The estimates of β_0 and β_1 should result in a line that fits the data as best as possible. Estimating the parameters β_0 and β_1 of the equation so that the sum of the squares of the vertical deviations is minimized is known as the least squares method. The least squares estimates, of the ordinate to the origin and the slope, of the simple linear regression model are:

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \quad (2)$$

$$\hat{\beta}_0 = \frac{\sum_{i=1}^n y_i x_i - \frac{(\sum_{i=1}^n y_i)(\sum_{i=1}^n x_i)}{n}}{\sum_{i=1}^n x_i^2 - \frac{(\sum_{i=1}^n x_i)^2}{n}} \quad (3)$$

where $\bar{y} = (1/n) \sum_{i=1}^n y_i$ and $\bar{x} = (1/n) \sum_{i=1}^n x_i$.

2.2 Coefficient of Determination R^2

According to Ref. [13], the coefficient of determination is the most used factor to verify the goodness of fit of the model, since it measures the percentage of the total variation in the dependent variable that is explained by the estimated model, the value is contained between 0 and 1. The closer to 1, the greater the adjustment of the proposed model, on the contrary, when the value of R^2 is close to 0, then the constructed model shows in a low percentage the variability of the dependent variable Y . The coefficient of determination has the following structure:

$$R^2 = \frac{\sum (\hat{y}_i - \bar{y})^2}{\sum (y_i - \bar{y})^2} \quad (4)$$

2.3 Statistical Efficiency

The statistical efficiency (E) helps to determine if it improves the average value of the record y and, therefore if it is convenient to make the inference. If the

value of (E) is greater than one, it will not improve the record of the y_i , on the contrary, if (E) is less than the unity, it is convenient to make the inference of the missing values from the x_i . The statistical efficiency is calculated in the following way according to Ref. [14]:

$$E = 1 - r^2 \left(\frac{n-m}{n} \right) + \frac{n-m}{n(m-3)} (1 - r^2) \quad (5)$$

where m = total data number of y ; n = total data number of x ; r = correlation coefficient.

2.4 Student's t -Test

If you want to test the hypothesis that the slope is equal to a constant, for example $B_{1,0}$. The appropriate hypotheses are:

$$H_0: B_1 = B_{1,0} \quad (6)$$

$$H_1: B_1 \neq B_{1,0} \quad (7)$$

The statistic T is calculated as follows:

$$T_0 = \frac{\hat{B}_1 - B_{1,0}}{\sqrt{\hat{\sigma}^2 / S_{xx}}} \quad (8)$$

A very special case is $H_0: B_1 = 0$ and $H_1: B_1 \neq 0$. These hypotheses are related to the significance of the regression. The failure to reject $H_0: B_1 = 0$ is equivalent to concluding that there is no linear relationship between x and Y [12].

2.5 Mann-Kendall Test

In this test each data value in the time series is compared to all subsequent values. Initially, the Mann-Kendall (S) statistics are zero, and if a data value in subsequent periods of time is greater than a data value in previous time periods, S is increased by 1 and vice versa. The net result of all these increases and decreases gives the final value of S . The Mann-Kendall statistics (S) are given as:

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^n \text{sign}(x_j - x_i) \quad (9)$$

where $\text{sign}(x_j - x_k) = 1$.

If $(x_j - x_k) > 0$; 0, if $(x_j - x_k) = 0$; -1 if $(x_j - x_k) < 0$.

A positive value of S indicates a rising trend and a negative value indicates a decreasing trend. However, it is necessary to perform the statistical analysis for the

importance of the trend. The test procedure using the normal approach test is described in Ref. [15]. The variance (S) is calculated by the following equation:

$$\text{Var}(S) = \frac{1}{18} [n(n-1)(2n+5) - \sum_{p=1}^g t_p(t_p-1)(2t_p+5)] \quad (10)$$

where n is the number of data, g the number of grouped data and t_p the number of data in the group p .

2.6 Estimator of Sen's Slope

The Sen's slope is estimated as the average of all the slopes between each pair of points in the database [16], each slope is estimated using the following equation:

$$m_{ij} = \frac{(Y_j - Y_i)}{(j - i)} \quad (11)$$

where $i = 1$ to $n-1$, $j = 2$ to n , Y_j and Y_i are data values at time j and i ($j > i$), respectively. If there are n values of Y_j in the time series, there will be $N = n(n-1)/2$ estimated slopes. The slope of Sen is:

$$m = m_{\left[\frac{N+1}{2}\right]}, \text{ if } n \text{ is odd}$$

$$m = \frac{1}{2} \left(m_{\left[\frac{N}{2}\right]} + m_{\left[\frac{N+2}{2}\right]} \right), \text{ if } n \text{ is pair}$$

A positive Sen slope indicates an increasing trend while a negative indicates a decreasing trend.

3. Results

For the stations of the city of Villahermosa, from the climatological rainfall variable, a linear regression analysis was carried out between nearby stations in order to calculate the correlation and determination coefficients, as well as the statistical efficiency and the p -value for the intercept and the slope of the lines generated for the monthly models, these results are presented below.

Table 2 shows the results of the statistical parameters that suggest that it is feasible to make the statistical inference of the missing data by linear regression analysis. Fig. 3 shows the scatter diagrams of the precipitation data reported by a pair of stations and their corresponding regression lines, in all cases the linear

Table 2 Calculated data of inference and statistical efficiency.

Months	X_i	Y_i	R	R^2	p -value intercept/slope	Statistical efficiency
January	DGE	PRVN	0.9952	0.9905	0.0011/0.0000	0.54
February	DGE	PRVN	0.9911	0.9823	0.0000/0.0000	0.54
March	DGE	PRVN	0.9739	0.9484	0.0000/0.0000	0.56
April	DGE	PRVN	0.9484	0.8995	0.0014/0.0000	0.58
May	DGE	PRVN	0.9850	0.9703	0.0002/0.0000	0.55
June	DGE	PRVN	0.9981	0.9963	0.0000/0.0000	0.54
July	DGE	PRVN	0.9910	0.9821	0.0000/0.0000	0.54
August	DGE	PRVN	0.9891	0.9783	0.0000/0.0000	0.54
September	DGE	PRVN	0.9961	0.9923	0.0000/0.0000	0.54
October	DGE	PRVN	0.9981	0.9963	0.0000/0.0000	0.54
November	DGE	PRVN	0.9922	0.9845	0.0001/0.0000	0.54
December	DGE	PRVN	0.9899	0.9800	0.0001/0.0000	0.54

Fig. 3 Scatter diagrams and regression lines of the pair of DGE-Porvenir stations, with which the missing data was filled.

adjustment has values of $m > 0$, this is justified thanks to the p -value obtained for the slope, the above shows that there is a strong positive linear correlation between the variables. An average coefficient of determination of 0.97 was obtained for the generated models and an

average statistical efficiency is 0.54.

Table 3 shows the data of the monthly rainfall observed in the city of Villahermosa during 1980-2010, monitored by the Porvenir station, it also shows the estimations that were made.

It is important to mention that only one table is shown with the inference and statistical efficiency since the obtained results had a similar trend in the other analyzed stations, the same applies for the linear regression models generated monthly for each station.

Table 4 shows the results of applying the Mann-Kendall test and the Sen slope estimator, for the 12 months of the study period, to an average of the 5 stations. Being the null hypothesis $H_0 = 0$ (there is no trend in the series) and alternative $H_1 \neq 0$ (there is a trend), with level of significance ($\alpha = 0.05$) the obtained p values show that no trends are detected in

the analyzed period. The statistic (S) of MK is negative for all months except April (3), May (15) and August (29), in turn the slope of Sen obtained is 0.007 for the first, 0.348 for the second and 0.510 for the third indicating a growing trend. Despite the insufficient statistical evidence of a significant trend, the slope of Sen is negative for all months except those mentioned above, which indicates a decreasing trend in precipitation over time. Table 5 shows the results of the annual total for the same test and according to the p -value obtained no trend is presented, the value of the MK test is -53.000 which points to a decrease in precipitation along with the Sen slope (-0.702).

Table 3 The complete monthly precipitation record of the Porvenir station is presented, for the period 1980-2010.

Año	Enero	Febrero	Marzo	Abril	Mayo	Junio	Julio	Agosto	Septiembre	Octubre	Noviembre	Diciembre
1980	79.3	137.1	57.3	88.6	41.5	169.6	133.3	109.0	363.5	888.0	319.9	101.1
1981	31.0	307.2	169.2	83.3	82.5	534.7	418.6	585.2	502.2	439.8	153.2	299.1
1982	125.1	153.1	57.6	56.3	189.7	228.7	132.6	130.7	474.2	241.5	184.8	174.5
1983	120.5	70.8	28.9	54.2	33.9	164.7	155.2	194.8	374.2	298.4	149.9	206.3
1984	176.8	39.4	37.3	28.6	403.3	205.6	183.0	328.3	377.3	137.5	212.3	95.8
1985	190.9	94.2	67.8	158.8	102.2	237.0	351.2	392.7	234.3	401.0	79.7	397.7
1986	136.5	68.8	58.6	28.6	231.3	122.6	85.0	128.5	103.0	102.5	130.3	51.3
1987	132.1	169.1	133.1	89.2	58.6	509.6	480.9	411.0	328.6	320.3	204.7	105.2
1988	184.3	136.5	75.9	58.3	21.2	351.5	262.8	576.7	350.6	606.1	297.3	92.7
1989	32.8	11.2	8.6	5.5	54.1	120.2	90.2	148.4	196.1	100.6	17.9	8.2
1990	106.2	183.2	91.2	41.6	110.7	173.4	131.8	207.7	214.8	245.7	117.3	290.8
1991	207.5	206.2	9.9	19.6	65.6	233.8	129.2	123.1	266.2	362.1	227.8	190.1
1992	274.7	125.2	88.6	59.8	105.5	334.8	55.3	294.3	313.9	275.8	319.5	67.1
1993	137.4	28.8	63.3	58.5	146.4	350.5	182.7	276.1	376.9	270.2	116.5	77.7
1994	132.3	75.7	49.7	97.0	92.0	85.3	98.9	275.3	241.3	134.1	95.2	148.0
1995	100.6	73.5	56.6	95.8	12.5	247.9	220.7	286.9	366.6	602.4	136.1	177.0
1996	92.2	23.6	92.9	110.0	137.0	148.2	287.3	212.5	150.5	346.9	174.8	99.5
1997	72.2	155.2	48.7	65.3	125.1	136.9	188.1	189.0	208.6	247.5	235.1	99.9
1998	13.8	27.6	25.3	33.6	12.6	116.1	466.9	124.8	146.3	141.2	87.3	68.3
1999	67.0	92.0	13.9	48.5	41.0	138.6	172.4	167.4	384.8	455.3	112.8	99.3
2000	69.3	40.2	40.2	35.5	289.0	167.1	96.6	92.8	131.6	126.7	78.9	62.1
2001	47.5	159.9	11.7	57.2	130.1	118.0	147.7	284.2	268.8	480.2	132.7	179.4
2002	99.3	275.1	100.5	56.4	98.8	357.8	105.4	181.2	350.4	145.0	371.8	118.4
2003	49.2	17.7	9.7	27.6	62.2	276.2	173.9	239.4	217.4	426.4	232.0	128.9
2004	149.5	93.8	91.1	98.8	215.9	134.2	150.2	210.3	293.3	218.8	109.3	103.3
2005	28.6	26.8	16.7	11.0	56.7	402.8	221.9	270.9	372.0	277.0	145.1	142.5
2006	195.2	102.7	75.7	44.4	168.6	375.5	234.7	246.9	282.2	404.1	173.6	212.8
2007	285.2	122.5	63.0	17.8	12.2	90.7	115.4	316.8	258.3	433.8	193.7	55.0
2008	185.9	105.7	62.0	131.6	170.1	339.5	227.8	182.1	503.4	238.9	58.7	80.7
2009	71.5	30.8	25.2	23.4	46.2	114.5	133.4	181.0	225.3	82.2	232.2	60.0
2010	108.7	36.4	16.6	24.7	46.8	233.0	230.9	302.0	354.3	103.7	144.1	68.0

Table 4 Mann-Kendall test on an average of the 5 stations in the 1980-2010 period.

Series/test	Mann kendall statistic (S)	<i>p</i> -value	Sen's slope
Enero	-17.000	0.788	-0.336
Febrero	-55.000	0.362	-1.132
Marzo	-47.000	0.438	-0.396
Abril	3.000	0.973	0.007
Mayo	15.000	0.814	0.348
Junio	-39.000	0.522	-1.127
Julio	-3.000	0.973	-0.068
Agosto	29.000	0.637	0.510
Septiembre	-7.000	0.920	-0.218
Octubre	-53.000	0.380	-2.962
Noviembre	-41.000	0.500	-1.090
Diciembre	-105.000	0.077	-1.920

Table 5 Mann-Kendall test of the 5 stations of the annual total 1980-2010.

Series/test	Mann kendall statistic (S)	<i>p</i> -value	Sen's slope
Total anual	-53.000	0.380	-0.702

Fig. 4 Annual rainfall of 30 years.

In Fig. 4 it is observed that the year with less precipitation was 1989 with 793.8 mm and the maximum 1981 with 3,605.9 mm, the average for the 30 years is 2,027.6 mm.

4. Conclusions

In our analysis of the *t*-test for the slope, the test statistic depends on the proposed null hypothesis ($H_0 = 0$) and alternative hypothesis ($H_1 \neq 0$), as a result a

p-value was obtained that on average was lower than the level of significance ($\alpha = 0.05$), this allows us to reject H_0 and accept H_1 .

The statistical tests were performed, that is, the average linear correlation coefficient ($\bar{r} = 0.9873$), the average observed value of the coefficient of determination ($R^2 = 0.9751$) and the average statistical efficiency ($\bar{E} = 0.54$) confirm the goodness of the results, this applies in the same way for the remaining

analyzed stations. Although in the Mann-Kendall test the p values did not allow us to reject the null hypothesis (there is no trend in the series), the values of the MK (S) statistic are negative for all months except April (3), May (15) and August (29), together with the values of the slope of Sen (0.007), (0.348) and (0.510) respectively, which indicates an increase in precipitation although nothing significant.

The general trend shows decreasing precipitation in the values of the Sen slope, both for the 12 months of the 30 years and for the annual total of the 1980-2010 period. The above coincides with that described by Ref. [9] who report that the wettest regions of the country have an average rainfall decrease of 1.2% in total annual precipitation. This study was motivated by the discrepancy in opinion regarding how precipitation has changed in recent years at the study site.

Based on these results this work provides a complete, homogeneous and statistically reliable rainfall database, as well as an analysis of precipitation trends for the 1980-2010 period, due to a slight downward trend future investigations could reveal more aspects that could be of help for the best management of resources in the city of Villahermosa.

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