Simulation of the Rayleigh–Benard Convection in a T-Shaped Cavity

E. Evren Selamet, A. Selamet and R. Dehner
Ohio State University, Mechanical and Aerospace Engineering, Columbus, OH 43210, USA

Abstract: Rayleigh-Benard convection driven by the temperature difference between top and bottom surfaces of a T-shaped cavity with two different boundary conditions is studied numerically. Steady or unsteady cellular flow structures and temperature patterns are illustrated along with the evolution of heat transfer rates (Nusselt number, Nu). The cavity is filled with fluids of various Pr (Prandtl number), including 0.024, 0.71, 6, and 450. The effect of Pr and Ra (Rayleigh number) on the flow regime and heat transfer is established, while comparing the results also with those of the square cavity.

Key words: Convection, Prandtl number, Rayleigh number, instability.

1. Introduction

Natural convection heat transfer induced by temperature differences across the walls of cavities is important in a large number of engineering applications. A variety of patterns of fluid flow and heat transfer forms inside the cavity for different values of Ra and Pr are the primary dimensionless numbers in natural convection (besides Nu). Many studies have been carried out for rectangular enclosures where the flow is usually unidirectional. Among them, Marshall et al. [1] and Jones [2], for example, performed a numerical analysis of natural convection due to a horizontal temperature difference for Pr = 1 and 0.7, respectively. Several studies have also been conducted on natural convection caused by the vertical temperature differences as a result of the heated bottom wall. For example, Shiralkar and Tien [3] studied a rectangular cavity with Pr = 0.7, and Ganzarolli and Milanez [4] with Pr = 0.7 and 7; Corcione [5] examined the effect of thermal boundary conditions in an air-filled enclosure with different width to height ratios; Basak et al. [6] investigated natural convection in a square cavity for Pr = 0.7 and 10 by insulating the top wall and cooling the vertical walls. In practical applications, yet the shape of cavities may vary widely. Therefore, the present study chooses a buoyancy-driven T-shaped cavity (hereafter referred to as “T-cavity”) and performs a numerical investigation of fluid flow and heat transfer at varying Ra. Four different fluids are considered with Pr = 0.024, 0.71, 6, and 450 to fill the cavity. The specific focus is on understanding the effect of Pr and Ra along with two different thermal boundary conditions on flow structure and instability.

2. Definition of the Problem

The T-cavity considered here is shown schematically in Fig. 1. The height \( h \) and width \( w \) of the narrow bottom domain are \( h/L=0.4 \) and \( w/L=0.5 \), respectively, with \( L \) being the overall height of the enclosure. Two different boundary conditions are considered for the bottom region: in case A (hereafter called “T-A”), all bottom surfaces of the narrow region (red) are kept at constant hot temperature \( T_{H} \), while in case B (hereafter called “T-B”), only the very bottom surface (red) is maintained at \( T_{H} \). Side walls of the wide upper region (black) are insulated in T-A, while all the side walls of both regions (black) are insulated in T-B. Top surface (blue) of both T-cavities...
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is kept at cold temperature $T_C$.

Unsteady governing equations are employed here since the evolution of flow pattern and the associated temporal behavior can only be captured by using time-dependent computations. Nondimensionalized governing equations involve two important and well-known dimensionless numbers, including the Rayleigh number $Ra = g\beta \Delta T L^3/\nu\alpha$ and the Prandtl number $Pr = \nu/\alpha$; $g$ being the gravitational acceleration, $\beta$ the coefficient of thermal expansion of the fluid, $\Delta T$ the temperature difference between bottom and top walls, $\nu$ the kinematic viscosity, and $\alpha$ the thermal diffusivity.

The governing equations with primitive variables are solved on a staggered grid (99 $\times$ 99) by two-dimensional numerical scheme developed by the lead author applying the Godunov scheme to convective terms and centered finite difference to diffusive terms. Formulations of the problem and details of the numerical method have been described in Ref. [7]. The scheme has already been validated for a number of cases through comparisons [7, 8].

3. Results and Discussion

Results are first obtained in a square cavity of $L \times L$ heated from the bottom wall and cooled from the top, while retaining the side surfaces adiabatic for comparison purposes. Flow characteristics of the square and T-cavities are investigated in terms of streamlines and temperature profiles (isotherms) as well as the rate of heat transfer from hot surface (as $Nu_H$, the dimensionless temperature gradient integrated over the hot walls) for $Pr = 0.024, 0.71, 6$, and $450$ at $Ra = 5 \times 10^4$-$10^6$. The time-evolution of $Nu_H$ dictates if the solution is steady or unsteady.

3.1 $Pr = 0.024$ (Liquid Gallium)

For $Pr = 0.024$, Fig. 2 shows the streamlines (upper row) and isotherms (lower row) at $Ra = 10^5$ (as a representative result) for square and two T-cavities. A steady single-cell flow structure is observed in the square cavity, whereas two cells rotating opposite to each other are present in the upper (wide) regions of both T-cavities along with weak cells at the lower (narrow) region. Same observations apply to all three cavities chosen here at all Rayleigh numbers, with the exception of a dominant single cell in T-B at $Ra \geq 3 \times 10^5$. Fig. 3 compares the evolution of $Nu_H$ as a function of dimensionless time $\tau$ for all three cavities at all Rayleigh numbers considered in this study. Increasing $Ra$ to $3 \times 10^5$ results in oscillatory convection in the square cavity, whereas both T-cavities exhibit nearly a steady behavior. Higher $Ra$ leads to unsteadiness in both square and T-A cavities, while slightly perturbing the steady behavior in T-B. As expected, the heat transfer rate is much smaller for T-B. Unsteady irregular behavior is observed in T-A.

![Fig. 1 Schematic of the T cavity for two cases: T-A and T-B.](image-url)
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3.2 Pr = 0.024

For Pr = 0.024, Fig. 2 shows the streamlines (top) and isotherms (bottom) for Pr = 0.024 at Ra = 10^5.

![Streamlines and Isotherms](image)

<table>
<thead>
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<th>Ra</th>
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<th>Isotherms</th>
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![Temporal Variation of Nu_H](image)

Fig. 3 Temporal variation of Nu_H for Pr = 0.024 at various Ra.

For Ra ≥ 5 × 10^5, whereas periodic oscillations are present with larger amplitudes for Ra ≥ 3 × 10^5 in the square cavity.

3.2 Pr = 0.71 (Air)

For Pr = 0.71, Fig. 4 shows the streamlines (upper row) and isotherms (lower row) at Ra=3 × 10^5 and 10^6 (as examples) for square and both T-cavities. In general, a single cell is present in the square cavity for Ra = 5 × 10^4, 10^5, 3 × 10^5, and 7.5 × 10^5, as opposed to two counter rotating cells on top of each other for Ra = 5 × 10^5 and 10^6. In T-A, two cells are observed in the upper region along with two weak ones in the lower region for Ra = 5 × 10^4-5 × 10^5, whereas a single cell exists in the upper region and a weaker one at the bottom for Ra = 7.5 × 10^5 and 10^6. In T-B, two cells of
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Fig. 4 Streamlines (top) and isotherms (bottom) for Pr = 0.71 at Ra = 3 × 10^5 and 10^6.

unequal size are present for all Ra with the exception of Ra = 3 × 10^5 when only a single cell is observed in each of the upper and lower regions. This Pr gives the most stable heat transfer results for the Ra range considered here as Fig. 5 illustrates in terms of the time evolution of Nu:H for both square and T-cavities.

3.3 Pr = 6 (Water)

For Pr = 6, Fig. 6 presents the streamlines (top row) and isotherms (bottom row) for the square as well as T-cavities specifically at Ra = 10^5. In the square cavity, a steady pair of counter rotating cells appear at Ra = 5 × 10^4 and 10^5 whose transition to unsteady two cells rotating in the same direction coupled with two weaker vortices at Ra = 3 × 10^5 and 5 × 10^5, and finally to an unsteady central one with additional smaller vortices at Ra = 7.5 × 10^5 and 10^6. At Ra = 5 × 10^5 and 10^6, a steady two-cell structure is present in T-A
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Fig. 5 Temporal variation of NuH for Pr = 0.71 for various Ra.

Fig. 6 Streamlines (top) and isotherms (bottom) for Pr = 6 at Ra = 10^5.

as opposed to three cells in T-B. Higher Ra in T-cavities promotes unsteadiness in the form of two cells in the upper region combined with weaker one or two cells in the lower region. Fig. 7 compares the time evolution of NuH illustrating that the behavior is steady at Ra = 5 \times 10^4 and 10^5, unsteady with regular oscillations at Ra = 3 \times 10^5 and 5 \times 10^5, and chaotic at higher Ra.

3.4 Pr = 450 (Silicon Oil)

For Pr = 450, Fig. 8 depicts the streamlines (top row) and isotherms (bottom row) for the square as well as T-cavities specifically at Ra = 10^5. In the square cavity, two counter-rotating cells are present
starting at \( Ra = 5 \times 10^4 \) with one of the cells becoming dominant at higher \( Ra \) as visible in Fig. 8 at \( Ra = 10^5 \). In T-cavities, results at \( Ra = 5 \times 10^4 \) and \( Ra \geq 3 \times 10^5 \) are similar to those of \( Pr = 6 \), however at \( Ra = 10^5 \) four compressed counter-rotating cells are formed throughout the entire domain in T-A in contrast to two cells in the upper region and one or two weaker ones in the lower region of T-B. Fig. 9 compares the time evolution of \( Nu_h \) in three cavities at various \( Ra \). Increasing \( Ra \) to \( 3 \times 10^5 \) leads to regular periodic behavior in both T-cavities (after, for example, \( t = 0.45 \) in T-A), as opposed to steadiness persisting in the square cavity. At \( Ra = 5 \times 10^5 \), \( Nu_h \) is steady in the square enclosure, while exhibiting chaotic variation
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Fig. 9  Temporal variation of $\text{Nu}_H$ for $Pr = 450$ at various $Ra$.

Fig. 10  $\bar{Nu}_H$ versus $Ra$ for $Pr = 0.024, 0.71, 6,$ and $450$.

in T-A and regular periodic behavior in T-B. Further increasing $Ra$ to $7.5 \times 10^5$ yields regular periodic $\text{Nu}_H$ behavior in the square and T-B cavities, in contrast to the chaotic behavior in T-A. Finally at $Ra = 10^6$, $\text{Nu}_H$ shows sinusoidal oscillations in square enclosure and T-B, and regular non-sinusoidal pattern in T-A.

A summary is provided in Fig. 10 in terms of $\bar{Nu}_H$ (mean $\text{Nu}_H$) which is selected here as the arithmetic average of the last 10% of the time-dependent variations examined thus far for each $Pr$ and $Ra$ in T-A and T-B. $\bar{Nu}_H$ tends to increase with both $Pr$ and $Ra$ in both cavities, while it is considerably smaller for T-B, as expected. In T-B, the difference between results for $Pr = 6$ and 450 nearly diminishes at higher $Ra$. An alternative measure of the degree of fluctuations is the standard deviation in $\text{Nu}_H$ as depicted in Fig. 11. Standard deviations for $Pr = 0.71$ are zero in all three configurations since the results are stable. In the square cavity, $Pr = 0.024$ is the most unstable at all $Ra$ except $Ra = 7.5 \times 10^5$, and $Pr = 450$ is the least unstable compared to $Pr = 0.024$ and 6. Conversely, in T-A, the results for $Pr = 450$ are most unstable at $Ra \leq 5 \times 10^5$, while those for $Pr = 6$ are the most unstable at $Ra \geq 7.5 \times 10^5$. In T-B, $Pr = 6$ manifests the most unstable behavior at $Ra \geq 3 \times 10^5$. 
6. Conclusion

The effect of cavity shape (T-shaped versus square) on flow and temperature fields is examined over a range of $Ra = 5 \times 10^4$-$10^6$ with the walls subject to different thermal boundary conditions. The cavity volume is filled with fluids of four different $Pr$. The predictions have shown that $\overline{Nu}_H$ in T-A and T-B increases with both $Pr$ and $Ra$ considered here. This observation is also true for $Pr = 0.024$, 6, and 450 in the square cavity, but not for $Pr = 0.71$ which exhibits a transition to a different cell structure. Furthermore, the influence of $Pr = 0.024$ and 6 on $\overline{Nu}_H$ in the square cavity is less distinct compared to T-A and T-B. The lowest $\overline{Nu}_H$ is obtained for $Pr = 0.024$, and the highest for $Pr = 450$ in T-A and T-B, whereas $Pr = 0.71$ gives lowest $\overline{Nu}_H$ at $Ra = 5 \times 10^5$ and $10^6$ in the square cavity. In T-B, $\overline{Nu}_H$ turns out to increase weakly with $Pr$ from 6 to 450 for $Ra > 5 \times 10^5$.

References


