Aggregate Consumption: An Analysis by Numeric Simulation

Ricardo Luis Chaves Feijó
University of São Paulo, Ribeirão Preto, Brazil

It is estimated that the aggregate consumption behavior of families by computational procedures in an abstract model contemplates the maximization of agent utility in each period (two dates), the attendance of budgetary restrictions and the conditions of general equilibrium (without production). It uses the technique of selection of candidate points in a simulation process with a portfolio of efficient assets and a hypothesis for the process of determining the returns of the securities: A GARCH process. By this technique, it compares the stochastic volatility patterns between the artificial series, obtained in the simulation, and the real series of household aggregate consumption in the US and Brazil.

Keywords: aggregate consumption, numeric simulation, general equilibrium, stochastic process

Introduction

Computational simulation exercises have been used instead of the purely analytic treatment, and the literature is well advanced. In this test, a very simple simulation procedure is offered. Here it does not try to determine prices, returns, and the choice of optimal portfolios simultaneously. The focus falls only on these last two classes of variables, and in the trajectory of consumption. Asset prices are here exogenous and simple processes, identified with a trivial random step. Nevertheless, the model is compatible with many of the hypotheses used in sophisticated theoretical treatment. The paper offers a programming exercise in the Matlab software language, applied to a simple but ingenious model that has as its scope a general equilibrium situation (without production) with a single goods and financial assets of the type that yields dividends, in which the returns follow a GARCH process. In such a model, the agents are utility maximizers in a static context, involving only two dates. However, the maximization exercise is repeated every couple of consecutive instants, in a process that walks over time. There is a stochastic path in the control and choice variables that carries a certain memory and, as such, conditions the choices of the agents. One thinks of an economy governed by a social planner who knows a range of possible states of nature, which are formed in each period, and chooses that, together with the choice of individual consumption, leads to maximum utility for the largest number of agents.1 Asset returns are exogenous, generated by sortition and in the structure of a GARCH process, however

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Ricardo Luis Chaves Feijó, associate professor, College of Economics, Business Administration and Accounting, University of São Paulo, Ribeirão Preto, Brazil.

Correspondence concerning this article should be addressed to Ricardo Luis Chaves Feijó, Rua Luís Correia de Melo, 250, ap. 1107, Vila Cruzeiro, São Paulo (SP), CEP 04626-220, Brazil.

1 The model could also describe a decentralized economy. However, in this case, in addition to being omniscient, the maximizing decisions of individual agents, operated also in counterfactual states, should take into account the maximizing choices of other agents in a coordination mechanism not offered by the markets.
it is up to the planner (the program itself) to choose, from counterfactuals, which, combined with the choice of consumption between two instants, leads to maximization on the question. That is, the problem of maximization also forces the omniscient planner to choose specific values of returns in the step, among infinite possibilities, in the stochastic trajectory of the return of each asset. The heroic hypothesis greatly simplifies the mathematical problem. The proposed simulation model can help explain the patterns that are observed in reality. In this case, the application is made in the study of the pattern of real curves of aggregate consumption of the countries. Computer draws are made here in the choice of the maximization point itself, therefore, without the need for the complex analytic solutions of the first order conditions. The solution offered is not intended to be accurate, but the approximations consider a margin of error stipulated by the program user. A specific test of the accuracy of the solution is not offered, only the application of it in the identification of patterns that are compared to those of the real economic variables.

The portfolio carries returns in the form of dividends, however, the agent does not live on them, nor from periodic withdrawing. He/she lives basically on labor income and only reinforces consumption, each period, spending the dividends received and using the possible positive results when operating in the portfolio in order to keep it always efficient over time. The income from the return on financial investments and the purchase and sale of assets is secondary to the payment of consumption. However, they affect the stochastic behavior of the consumption series, the focus of research. The agent wants the portfolio at each period to provide a desired return and, depending on this expectation, he/she forms a portfolio with a certain risk conditioned by its composition and the behavior of specific asset returns. Static choice also conditions the point in which the states of nature governing the returns of all assets, in the step in question, become compatible with individual maximization in the context of general equilibrium. Therefore, the individual static maximization also defines, jointly, the vector of returns of assets elected in the period.

**Research Contest**

**Assumptions**

The strategy is to develop algorithms for computational use. In this context, the simulations consider financial assets traded in each period (at specific end-of-period dates) along a stochastic trajectory. There is only one goods. In such conditions, it is sought to find a stationary dynamic equilibrium characterized in a peculiar way. It supposes agents that maximize, at each period, the utility function. The originality of this paper is that it is proposing that the stochastic process of asset returns at each period leads to specific values drawn in the same random selection process that determines utility maximization consumption. The problem of maximizing families occurs at each period and assumes CRRA utility functions. It has a simple recursive dynamic model for the autonomous endowments of agents and a complex model, which follows a GARCH pattern, for the return of financial assets. It is assumed a finite set of assets. The difference among the agents focuses on their intertemporal preferences, which affect the choices between consuming on two different dates (middle and end of period), and the desired return rates of the asset portfolios. Focus is only on consumer utility maximization in a non-deterministic environment (but with non-stochastic utility functions). Equilibrium for this economy is a process of building portfolios and consumption decisions, given the formation of financial asset prices by a simple stochastic path, in which markets equilibrate.

Figure 1 shows the schematic representation of the cash flow in each period from the second one. The budget for agent \( h \) is formed by the end-of-period autonomous endowment \((d_{t+1}^h)\), plus the amount of the asset
portfolio return over that period and paid at the end of it ($R_{t+1}^h$), and the net return on the sale and purchase of assets in $t + 1$ ($r_{t+1}^h$). The agent can spend part of this total income in advance to use it as a source to pay consumption in $t$, before the formation of revenues in $t + 1$. In this case, he/she will have to borrow at the interest rate $i_t$, prefixed for the whole interval between $t$ and $t + 1$. This rate has stochastic behavior over the periods. Of course, the rates of return of the individual portfolios do not coincide with the interest rates of the loans (exogenous market data and the same for all agents). The fundamental maximization problem of the model seeks the maximum level of individual utility in choosing how much to consume at the middle and at the end of the period. That is, between $\lambda_t^h$ and $\lambda_{t+1}^h$. Such choice is made in an environment with a stochastic boundary, and therefore depends on the sortition of random variables that commands at the same time, together, the formation of all these variables. For the period in question, the decision is between consumption at $t$ and $t + 1$. Choice is always between $\lambda_t^h$ and $\lambda_{t+1}^h$, and decided given a projection of the budget constraint at $t + 1$, formed by $d_{t+1}^h + R_{t+1}^h + r_{t+1}^h$.

\[
\begin{array}{c}
0 \\
(1 + i_t)\lambda_t^h \\
t \\
\lambda_{t+1}^h \\
t + 1
\end{array}
\]

Figure 1. Cash flow for the period, values at $t + 1$, with income formation and consumption strategy in two instants.

The model generates vectors of relevant variables such as consumption, portfolio composition, and asset returns. It is considered a countable and infinite set $\Omega$ of states of nature ($\Omega = \{1, 2, \ldots\}$, generated from successive draws. $\Omega$ is defined for each period. The individual consumption plan $\lambda^h$ specifies consumption at date $t$ and at date $t + 1$, $\lambda^h = (\lambda_t^h, \lambda_{t+1}^h)$. The consumption at date $t$ is represented by $\lambda_t^h \in R$ and the consumption plan at date $t + 1$ by $\lambda_{t+1}^h$. The consumption pair on two dates is defined by the vector $\lambda^h : \Omega \rightarrow R^2$. $r_t(\omega)$ is the return of asset $i$ in the state $\omega$. For $N$ assets $(a_1, \ldots, a_N)$, it has an individual portfolio represented by $\theta^h = (\theta_1^h, \ldots, \theta_N^h) \in R^N$. The return of the portfolio is $r_h = \sum_{i=1}^{N} \theta_i^h r_i(\omega)$. The vector of the asset prices is represented by $q \in R^N$. The only consumer good in question is always unit price, and asset prices are quoted on this single goods (the summary). $\theta_i^h$ represents the relative participation of asset $i$ in total assets of all types involved in the portfolio. Of course, $\sum_{i=1}^{N} \theta_i^h = 1$. Let $Q$ be the total of assets, the absolute quantity of each asset $i$ allocated in the portfolio is therefore $\theta_i^h Q$. Thus, the value of the portfolio, at prices $q$, is $q \theta^h Q = Q \sum_{i=1}^{N} q_i \theta_i^h$.

Given the period in question, a combination of consumption between two dates is imposed by the program (which functions as a central planner) which indicates, among the drawn points, which obey the total endowment, and maximize the utility function for the largest number of agents.

The Models

Below is a general logarithmic utility function of the type:
\[(1) \quad U^h(x) = U^h(x_t^h, x_{t+1}^h) = \ln(x_t^{h^*} \left( p_t^h x_t^h \right)^{\beta}) \text{. For } \alpha, \beta > 0.\]  

The individual autonomous endowment \(d_t^h \) is a component of the total endowment that also aggregates, to each period, in addition, the return on the portfolio of financial assets and the gains from the purchase and sale of assets in the financial market. Such autonomous endowment depends on rents exogenous to the model, such as labor income, and is generated here simply by a simple random process. It follows a recursive path given by the expression:

\[(2) \quad d_t^h = d_t^h (1 + z^h + \tau^h \eta^h) \]  

where \(z^h \) represents the deterministic term of the growth trend of the individual endowment and \(\tau^h \) is the standard deviation of the random term of the endowment growth. \(\eta^h \) is a random shock of normal distribution whose specific value, in each period and for each agent, is associated to a certain state of nature.

Given the budget constraint \(B_{t+1}^h \) of the consumer with a certain total endowment (which includes the autonomous endowment as well as the periodic return of the assets and the gain of the operations with them), the individual maximizing equilibrium, at each period, makes the asset portfolio efficient, and is the set of triples \((x^h, \psi^h, \psi^h)\) where \(x^h \) is the feasible consumption vector \((x_t^h, x_{t+1}^h)\). \(\psi^h \) is a vector of purchases and sales of financial assets, at the end date of the period in question, compatible with portfolio maintenance in the combination of efficiency (efficient portfolio) \((rl_t^h = \psi_t^h(\omega) - \phi_t^h(\omega))\). The portfolio of assets in the period, specific to each agent \(h\), presents an end-of-period return \(r_{t+1}^h(\omega)\). The budget constraint establishes the condition:

\[(3) \quad (1 + i_t)x_t^h + x_{t+1}^h \leq B_{t+1}^h(\omega) \]

Assuming unit prices for the asset in question and given the prefixed exogenous interest rate \(i_t\), the budget constraint that obeys the individual choice of consumption is formed, for each \(\omega \in \mu^h\), by the equation:

\[(4) \quad d_{t+1}^h(\omega) + r_{t+1}^h(\omega), \eta_{t+1}^h(\omega). \theta_{t+1}^h(\omega) + q_{t+1}(\omega) (\psi_{t+1}^h(\omega) - \phi_{t+1}^h(\omega))^T = B_{t+1}^h(\omega) \]

In this equation, the term \(r_{t+1}^h(\omega), q_{t+1}(\omega)\) represents the amount of interest periodically paid by portfolio \(= B_{t+1}^h\). It is the product of the rate of return times the value of the portfolio. Note that it is assuming that the total number of assets of all types is equal to one, and the individual quantities of each asset type are fractions. This does not affect simulation results and simplifies calculations and interpretation of results. In this context, \(\theta_{t+1}^h\) can be read as the quantity of assets \(i\) in the portfolio (negative in the case of short sale). If it considers the value of the portfolio as the individual’s financial wealth, it is, for each period, \(q_{t+1}(\omega)\), \(\theta_{t+1}^h(\omega)\), or simply a weighted average of the stochastic prices \(q_{t+1}(\omega)\). The triple representing the individual equilibrium at each end-of-period date, and for an asset price vector \(q_t\), is formed in such a way that the budget condition (4) is satisfied and, in addition, the function \(U^h(x^h)\) reaches a maximum value at the considered point. That is, individual plans are financially viable (for each \(h\)) and consumers optimize in their budget sets:

\[(5) \quad \text{If } (1 + i_t)y_t^h + y_{t+1}^h \leq B_{t+1}^h(\omega) \text{ then } U^h(x^h) \geq U^h(y^h) \]

Under conditions of general equilibrium, it must impose that the chosen point \(x^h\) simultaneously maximizes utility for all individuals considered in the simulation model. Or, when it is not possible to find this point, among the states of nature drawn, the program selects one of them that maximizes the utility for the maximum of agents, among the required minimum maximizer agents \((h_m)\), informed by the user. The condition

\(^2\) Also, if any of the two, \(x_t^h\) and \(x_{t+1}^h\), are null the utility function assumes the value zero.
of utility maximizing equilibrium, considering the maximum of agents, would be formalized as follows:

\[ U^h(x^h) \geq U^h(y^h), \ (1 + i_{\ell}) y^h + y^h_{t+1} \leq B^h_{t+1}(\omega) \]

Another condition is imposed in the simulation model, of the market general equilibrium: there is a requirement that, for each period, the quantity available of the single goods in question, in order to attend the consumption of all agents on the two middle and end-of-period dates, is fixed. The quantity is offered in an amount such that the value of the good stock (the quantity of it, since its price is always unitary) is equal to the sum of the individual endowment of the period \( \sum_{h=1}^{H} x^h_t + \sum_{h=1}^{H} x^h_{t+1} = \sum_{h=1}^{H} B^h_{t+1} \) or, rather, in the stochastic context, that both are equivalent to less than an arbitrarily small margin \( \epsilon_1 (\sum_{h=1}^{H} (x^h_t + x^h_{t+1} - B^h_{t+1}) \leq \epsilon_1) \). Thus, on each date considered, the quantity offered of the goods (dictated by the aggregate endowment) is always equal to the total demanded. Once this condition is satisfied, the market for consumer goods is considered in general equilibrium. It also imposes, for this equilibrium, that, for each period, the aggregation for all assets of the sum of the excess purchased from each specific asset, considering all the agents, is null or sufficiently small, within the arbitrary margin \( \epsilon_2 (|\sum_{h=1}^{H} (\psi^h_{t+1 i} - \psi^h_{0 i})| \leq \epsilon_2) \). In the sum of the results of all agents, therefore, the volume of asset \( i \) sold approximates the total volume purchased from it when all assets are considered. With the presence of short sale, a third inequality must be observed, because it is necessary that short sale operations have an upper limit, thus avoiding that the agents jointly uncontrolled to run into debt. In this sense, there must be a limit on the differences between the values sold (which may also indicate short sale) between the two start and end times, in the sum of all the agents, or that they are equal to or less than \( \epsilon_3 \) as small as one wants \( |\sum_{h=1}^{H} \sum_{i=1}^{N} (\psi^h_{t+1 i} - \psi^0_{0 i})| \leq \epsilon_3 \). So for the general equilibrium of markets, one must observe:

\[ |\sum_{h=1}^{H} (x^h_t + x^h_{t+1} - B^h_{t+1})| + |\sum_{i=1}^{N} \sum_{h=1}^{H} (\psi^h_{t+1 i} - \psi^h_{0 i})| + |\sum_{h=1}^{H} \sum_{i=1}^{N} (\psi^h_{t+1 i} - \psi^0_{0 i})| \leq \epsilon \]

In the model of simulation, in each period, a state of nature is revealed, among a finite set of possible states. For each state of nature \( \omega \), it can associate a corresponding endogenous variable \( x(\omega) \). Starting from the maximization exercise at each period, an intertemporal model is constructed by chaining the problem in a recursive process. As for the returns of assets, it works with the hypothesis that they follow a GARCH process. At each period, the growth process of the individual autonomous endowment and the evolution of the returns of each asset \( i \) depend fundamentally on the respective random numbers \( \eta^h \) and \( \xi_i \) (the latter also of normal distribution and used in the calculation of returns). The idea of the numeric simulation is to associate a certain state of nature with each successive step. By restricting the processing load, the simulation model works with nine types of assets and 10 agents. \( \tau^h \) and \( \varphi^h \) are generated by random draw, once in the experiment, and kept fixed. The same for the value of \( \rho^h \), in all three cases based on a uniform distribution. The series of interest rate \( i_{\ell} \) is also generated only once in the experiment, always by a random process of normal distribution, so that, following periods, it traverses a simple stochastic path.

In a given experiment, estimates of the utility associated with the consumption of a given individual are made at each period and the subsidiary conditions of the problem are met. At the same time, it seeks to satisfy the subjective maximization condition in each period. The equilibrium point must meet the conditions that ensure that both the consumer goods market and the asset market are in equilibrium, alongside the individual utility maximization. The vector describing the state of the random variables \( (\eta^h, \xi_i) \) depends on the step in question. There is dependence on past periods (within a pre-established time window) in relation to the returns of assets by
a GARCH process. It chooses to let the computer examine the different points drawn \((x^h_t, x^h_{t+1})\), for each period, and calculate the numeric value of the associated utility in each one. An unlimited sequence of draws is stopped when an individual maximizing point is found which is also the maximum utility for all agents considered (or for a previously specified minimum number of agents). In this way, it is possible to describe the system state changes under a particular set of occurrences (the experiment in question), greatly simplifying the explosive possibilities of paths that could be followed by the stochastic process of the control variable, given the infinite combination of the values drawn from the random variables \((\eta^h, \xi_i)\) over the periods.

The MATLAB program, in each period, performs several rounds of drawings that make up, each one, exits of the programming loop. In the model, the maximization occurs given the possible occurrences of an autonomous random process in the exogenous variables that define the structure of the stochastic process itself. The central planner acts as the computational program and knows all the states of nature drawn in each round of programming (combinations of \(\eta^h\) and \(\xi_i\)), and the range of states of nature previously drawn in other rounds of the same program loop. The planner (and the program) chooses the state of nature that maximizes individual utility to the maximum of agents (general subjective equilibrium) and thereby selects not only the individual consumption in each period but the combination \((\eta^h, \xi_i)\) that conditions the trajectory of the endowment and the return of assets (in this case, affecting the result of a GARCH process of the returns).

Thus, the optimum stochastic path control is not done by estimating the stochastic consumption function, but by sequentially solving, through the selection of lottery results, a static optimization problem applied to each period. Equation (3) governs the potential consumption of each agent in the following periods, which is endogenously estimated by means of a structural equation of budget equilibrium in which the amount consumed equals the sum of the endowment at the end of the period in question plus the financial return of the portfolio and net income from the purchase and sale of assets on this date. \(x^h_t\) and \(x^h_{t+1}\) arise with the draw of numbers assigned to random variables. The program attempts to reach the maximization point by comparing the utility values estimated for all the points reached. To do so, the program allows changes in both \(x^h_t\) and \(x^h_{t+1}\). The process of generating \(x^h_t\) occurs at the same time as estimating \(x^h_{t+1}\). The program generates both in the same random process, in the two consumptions applying the condition that the general market equilibrium in \(t + 1\) is met. Then, the additional condition of utility maximization is imposed which leads to the selection of a single pair \(\left(x^h_t, x^h_{t+1}\right)\) as a candidate point, that is, maximizing equilibrium when compared with other points related to the statistically equivalent budget equation.

For the utility maximization in two dates of the same period exercise, the relevant budget line obeys Equation (4). Defining the amount of the yield at \(t + 1\) for agent \(h\) \((r^{h+1}_{t+1}(\omega), q_{t+1}(\omega), \theta^h_{t+1} T(\omega))\) as being \(R^h_{t+1}\), and the net return on asset transactions at \(t + 1\) as \(r l_{t+1}\), it has the equation:

\[
(8) \quad (1 + i_t)x_t + x_{t+1} = d_{t+1} + R_{t+1} + r l_{t+1}
\]

The second member of the equation can be thought of as the individual budget constraint for the problem of the choice of consumption between two dates. The budget constraint to be considered shall be the sum of the autonomous endowment at the final date with the amount of the return on the portfolio, plus the net return on asset purchase and sale transactions at the end of the period. The choice of the consumption pair is then subject to the common constraint (8). Candidate point must be thought of as a pair that satisfies it and is a utility maximizer.
However, note that the equation allows us to know only the sum \((1 + i_t)x_t + x_{t+1}\) that satisfies the budgetary conditions imposed for the choice between two dates. Nevertheless, it is necessary to examine points distributed by the stochastic frontier in question. Then, the maximum value of the utility is searched by applying the utility function to these points. For a certain agent, when the utility function assumes the greatest value at the point \((x_t^i, x_{t+1}^j)\), that point configures the values of \(x_t\) and \(x_{t+1}\) chosen in the round. This is the first coordinate pair chosen in the selection round. Of course, the process repeats itself for the other agents in the same program loop. In each round, it selects the pair \((x_t, x_{t+1})\) that, by construction, satisfies the maximum utility condition of the agent in question among the points surveyed (defines the candidate point) for all other agents. Next, it tries to compare the candidate points to see which is the dominant candidate, the point that generates the maximum utility for the largest number of agents. The process is repeated every time. In each, there is a choice of \((x_t^h, x_{t+1}^h)\) for each agent \(h\). Thus, for an agent \(h\) in question, one arrives at the stochastic series of consumption. Therefore, a time series is generated for the maximizing consumption in each agent \(h\).

Subjective general equilibrium models often require that, in maximizing equilibrium, a single level of consumption can maximize the utility of all agents involved. The \(x_t^h\)s, estimated by the computational process described, refer to the same candidate point, and this does not necessarily imply that it is the same pair of consumption coordinates \((x_t^h, x_{t+1}^h)\). Nevertheless, one can work with the idea of representative individual consumption, obtained simply by adjusting the vector of the individual consumption \((x_t, x_t^1, \ldots, x_t^{10})\), at each \(t\) by a normal distribution. The mode of the distribution is taken as representative consumption at time \(t\). As the market is efficient, let’s look at the formal conditions for choosing the optimal portfolio. Taking into account the well-known theory of portfolio construction, based on the mean and the variance, proposed by Markowitz, let’s look at the formal conditions for efficient portfolio. The one maximizes the expected return and minimizes risk, considering a portfolio of active \(N'\) assets. If the wealth, or available money, is one (for simplicity), the fraction of a unit of wealth invested in asset \(i\) it already calls \(\theta_i\) \((i = 1, \ldots, N)\). The return rates of \(N\) assets are expressed in the vector of random components \(r = (r_1, \ldots, r_N)^T\). The expected return is \(\bar{r} = E r = (\bar{r}_1, \ldots, \bar{r}_N)^T\). The matrix of the returns of the assets’ covariance is represented by \(V = (\sigma_{i,j})\), where \(\sigma_{i,j} = cov(r_i, r_j)\), \(i, j = 1, \ldots, N\). The standard deviation of the asset is considered a measure of risk. A certain combination of assets defines a specific portfolio. The expected return of this portfolio is given by \(r_p = \bar{r}^T \theta\). Also, defined for the portfolio is a variance, understood as the variance of its return. Such a variance can be expressed as \(\sigma_p^2 = \theta^T V \theta\). Portfolio risk is the standard deviation corresponding to this variance.

Instead of generating purely random portfolio choices, it works with efficient portfolios. Thus, the quantities purchased (\(\varphi^h\)) by agent \(h\) at each step of the stochastic process are such that, at the end, the individual will have an efficient portfolio \(\theta^*\) in the period in question. Let’s see how the agent builds efficient portfolio. It looks only at the agent expected returns of the portfolio and at the covariance structure of the individual returns, of each asset with each other. The problem of the individual choice of the efficient portfolio can be formalized as follows: the agent seeks to achieve a certain level of desired (targeted) return \(r^{h*}\) with minimal risk. The corresponding portfolio is called the portfolio of minimum variance, that is, it has the lowest risk. Formalizing:

\[\text{(9)} \min \frac{1}{2} \theta^T V \theta \text{ subject to } 1^T \theta = 1 \text{ and } \bar{r}^T \theta = r^{h*}, \text{ for given desired return } r^{h*}.\]

\(^3\text{Note that } \theta_i < 0 \text{ also has economic significance. It translates the situation of short sale.}\)
The most common occurrence is when $1^TV^{-1}r^e \neq 0$. With each draw of the row of the matrix, the temporal trajectory of the returns of the assets is changed and the variance and covariance relations are changed (since that depends on the draw history). Memory of the returns and the new return, in the step in question, generated by the lottery that occurs in it, are considered. Thus the sequence of fluctuations is fixed for the window (successive past stages, each one with a predetermined size), except for the last entry of it. A window of observation of initial size of 10 stages is constructed, within which the drawings, which characterize the temporal trajectory of the return of the assets, have already been established. It advances with the window every step of the process, and of the analysis, making it grow in size (the initial size of the window is fixed (virtual window) and new periods are incorporated). In the first step of the stochastic process being examined, it adds a period (the present one), whose values of the random variables are being drawn, and therefore, by lot, it determines the matrix $V$. The procedure will be repeated in the following steps, adding new draws whose values are incorporated in the structured model. Several draws are made at the stage in question because different values for the inputs of $V$ are tested until the consequent value of $\theta^h$, per individual, satisfies the general equilibrium conditions being tested in the program loop. The method makes the matrix $V$ random at every step and with each program turn on it. Changing the last entry in the time window of the returns can also affect the expected return of each asset, the averages of the calculated values within the window. Thus, also the variable $r^e_i$ is subject to repetitive draws at each step. In the period in question, the quantity demanded of each asset is the one that, by construction, keeps the portfolio efficient. The agents are also different regarding the respective target return. In the portfolio risk minimization problem, each agent presents its portfolio target return, expressed in the variable $r^{h^*}$ (which in the simulation is attributed by a certain criterion to be commented). The choice of the optimal portfolio ($\theta^*$) depends on $r^{h^*}$. Then each agent $h$ is characterized by an own $r^{h^*}$, and the individual efficient portfolio depends fundamentally on this.

The Experiments

Summing up the average individual maximizer consumptions, at each instant of time, it arrives at the estimation of the utility-maximized aggregate consumption trajectory for the model with 10 agents. The maximizing aggregate consumption curve depends on a sequence of random occurrences. It is intended to generate 10 of these aggregate consumption curves and to analyze their aggregate behavior. The focus is on the volatility of the curve at each point of the stochastic process at each of the time points considered. The thesis in question, corroborated by the experiments, is that the average aggregate consumption curve (average of the distribution estimated at each stage, based on a normal curve), considering all the draws made and that explain the volatility of it, presents a variable standard deviation.

The program seeks to find the efficient portfolio in every period. Thus, it is possible to calculate the quantities bought and sold by the agents at each period end date. It is estimated, therefore, the returns of assets and of efficient portfolio at each end date of period. With this, the budget constraint is constructed. Following is the program for the identification of candidates. The program follows the tests for utility maximization. Then, after completing the programming loop for candidate selection, the general equilibrium condition is tested. Finally, it is estimated the average individual maximizing consumption, the aggregate consumption per rounds and the average of it in all the rounds of the experiment in question. It is very interesting to examine the

volatility of average aggregate consumption on each of the dates considered. The standard deviation of this consumption is taken at each stage in 10 complete rounds of sortition, such as the volatility indicator. It is noted in the experiment that the standard deviation in each of the 18 effective points was considered different from one another. The variation of the standard deviation of the aggregate consumption trajectory is a fact of great empirical appeal, since the simulations lead to a description of the aggregate consumption compatible with the observed behavior of the variation of point-to-point volatility in the series of actual consumption of the empirical basis. The indicator of volatility is the relation between the standard deviation (sigma) and the average. The sigma/average ratio of aggregate consumption is only slightly less volatile than the sigma series itself, but it also varies greatly throughout the stochastic process analysis window in question. Figure 2 shows the bundle of 10 distinct trajectories of aggregate consumption associated with 10 draw sequences. In the experiment, it has the pattern of volatility observed in the 10 draws at each point in time. Considering all the trajectories, there is a great variation of the standard deviation between the points.

Figure 2. Aggregate consumption trajectories in 10 draw sequences. Given experiment.

It is examined the volatility pattern of those real series. The volatility of the curve as a whole is estimated by observing the behavior of the series at constant intervals, in order to infer the volatility that can be associated with different segments of the trajectory in question. Thus, annual intervals (four quarters) are taken. At intervals of this size, it can analyze 17 segments of the trajectory, of aggregate consumption. It obtains averages and standard deviations (sigmas) for successive intervals extracted from the trajectory of this consumption for American and Brazilian families. In Figure 3, it examined data on the series of aggregate consumer consumption in the USA and Brazil. It is soon seen that, in both cases, the sigmas vary throughout the series. Figure 4 allows a quick inspection in the results of the calculations of the sigma/average relation in the real series and in the other one obtained in the simulation. It can now compare the results of empirical observation with the outputs of computational exercises. It is first noted that this coefficient is much lower in the real series than in the simulations.

Figure 4. Sigma/average relation trajectory for consecutive instants of the time.

Conclusion

The computational simulation is not intended to provide a theoretical explanation for the pattern of volatility observed in the real series. The real sigma/average coefficient, in the American case, ranged from 0.0028 to 0.019, a variation range, therefore, of 0.016, or more than five times the coefficient calculated for the beginning of the series. For the Brazilian case, the numbers are 0.003 and 0.020, variation of 0.017. The observed ranges of volatility could not be explained only by a problem of statistical fluctuation. For this question, these do not explain the variable pattern of volatility. If it were the decisions of millions of agents and hundreds of factors that shift aggregate consumption up or down the trend line, there would be no reason for the observed change in the pattern of volatility over time. The question of volatility variation deserves some theoretical explanation. Simulation replicates such a pattern of change in volatility. Clearly the simulation result suggests a conjecture for the volatility pattern of the real series. There are important advances in relation to the
literature, such as the incorporation of an omniscient planner choice model into a meticulous Matlab program, the idea of efficient portfolio formation over periods, and the target return criterion as opposed to traditional optimal control in the intertemporal maximization of the discounted utilities. The incorporation of asset returns that follow a GARCH process has also been little explored in the literature.

**References**

