A Finite Element Model for the Instability Analysis of Flexible Pipes Tensile Armor Wires

Gabriel Gonzalez, José Renato Mendes de Sousa and Luis Volnei Sudati Sagrilo

Civil Engineering Program, COPPE, Federal University of Rio de Janeiro, Rio de Janeiro 21941-596, Brazil

Abstract: The instability of the tensile armor wire of flexible pipes is a failure mode associated with deep and ultra-deep water applications. Real compressive forces acting on the pipe are necessary to trigger this process. The loss of stability may be divided into two distinct processes, according to the main direction of the wire's displacement: radial or lateral instability. This study aims at proposing a numerical tool for predicting lateral and radial critical buckling loads for the tensile armor wires of flexible pipes. A simple finite element model, based on springs and beams elements, was developed in *ABAQUS*® to deal with this problem in an efficient and reliable manner. A parametric study was conducted concerning the behavior of the critical load when the laying angle, the initial curvature and the total pipe length are varied. The results were consistent with previously published literature data and analytical expressions, proving its applicability to pipe engineering projects. It also has the advantage of approaching the problem three-dimensionally, which allows further modelling modifications, such as including friction effects.

Key words: Unbonded flexible pipes, instability, finite element modeling.

1. Introduction

Flexible pipes are multilayered composite structures in which the combination of metallic and polymeric layers is responsible for granting its characteristic behavior, i.e., a dichotomy between high axial stiffness and resistance and high compliance and deformability in bending. Metallic layers—such as the inner carcass, pressure armor and tensile armors, as shown in Fig. 1—are mainly responsible for the structural resistance of the pipe, whilst polymeric layers can be employed for sealing (plastic sheath), diminishing friction and wear between adjacent layers (anti-wear tape) or adding up further reinforcement to the pipe (high-strength tape). A typical cross-section of an unbonded flexible pipe is shown in Fig. 1 as well as its main layers.

Tensile armor layers are usually composed by several helically reinforcement wires with approximately rectangular cross-sections, presented in counter-wound pairs, laid at angles between 20° and 55° [1].

The main function of these wires is to resist axial forces and torsion. Another layer that has a great influence on the flexible pipe axisymmetric response is the high-strength tape (Fig. 2), commonly wrapped over the outer tensile armor layer and composed by high-strength polymeric fibers. This tape is frequently employed to restrain radial displacements of the tensile armor wires when the pipe is subjected to real axial compressive forces [2].

Fig. 1 Unbonded flexible pipe [2].

Corresponding author: G. M. Gonzalez, M.Sc., Ph.D. candidate, research field: offshore structural analysis.

Fig. 2 High strength tape [3].

Fig. 3 Instability modes of tensile armor [4].

Real compressive forces are more likely to occur in empty pipe conditions, in deep or ultra-deep water applications, and in large diameter pipes. These circumstances frequently are met during the installation of such pipes [3]. Real compression is responsible for some local instability mechanisms in flexible pipes, such as the radial and lateral buckling of their tensile armor wires, as respectively shown in Fig. 3.

Radial instability is commonly addressed as bird-caging, which is strictly linked to the pipe loss of capacity in restraining the radial expansion of the tensile armor wires when compressed. As stated in Refs. [3-5], the trigger of this phenomenon is mainly attributed to the failure of the high-strength tape. Nonetheless, Refs. [4] and [5] considered at least one extra path buckling mode in radial direction, called “wrinkled” mode. This happens when the high-strength tape remains intact, but it is not able to completely prevent radial displacements of the armor wires. The wrinkled mode is not addressed in this work.

Lateral instability occurs when, under high real compressive forces and dynamical cyclic bending, the radial displacement is prevented, forcing an adjacent equilibrium to be found on the laying surface of the wire [6]. Again, two distinct modes are mentioned in technical literature [4], regarding how the set of wires displaced after losing their stability, i.e., individually or as unique layer. It also should be highlighted the

extreme sensitivity of the critical load to the presence of friction [5].

Despite the efforts, lateral instability is not yet completely understood. For this reason, here, a simple finite element model based on beam and nonlinear spring elements is presented and its calculated critical loads are compared to literature consolidated results [6] and analytical expressions [5, 7, 8].

2. Finite Element Model

The model presented in this section was created using *ABAQUS v6.13*® and relies on an extension of the work done by Sousa [1]. This model is capable of estimating both lateral and radial critical loads. The FEM (finite element model), shown in Fig. 4, is developed based on beam and nonlinear spring elements only and, as its main objective is to predict the elastic buckling of a single wire, it considers geometrical nonlinearity and physical linearity.

The model adopts some hypotheses, such as: each tensile armor wire behaves independently of the others, no-friction between the wires and the adjacent layers is assumed, and the pipe bending effect can be simulated by altering the wire initial helical geometry. The two finite elements used are: three-dimensional beam elements-*B31*-formulated by Euler-Bernoulli theory and used to model a single tensile armor wire, and nonlinear springs- *Spring A*- used to represent the influence of the other layers of the flexible pipe into the wire.

2.1 Tensile Armor Curvature

Bending is quite important in triggering lateral instability, therefore its effects may not be neglected.

Fig. 4 Views of the FE model.

Initial imperfections, obtained from the geodesic, are hence introduced into its initial geometry. The geodesic path is chosen because it assumes that the less energetic consuming position is followed by the tensile armor wire when the pipe is bent. In other words, the wire is free to displace along its surface of settlement when the pipe is bent [9].

Eqs. (1)-(3) describe the geometry of a cylindrical helix, where θ is the angular coordinate, r , the mean radius of the tensile armor layer, L_{model} , the total length of the model and n_p , the number of pitches.

$$X_{hel} = r \cdot \cos \theta \quad (1)$$

$$Y_{hel} = \theta \cdot \frac{L_{model}}{2 \cdot n_p \cdot \pi} \quad (2)$$

$$Z_{hel} = r \cdot \sin \theta \quad (3)$$

The geodesic displacements, Eqs. (4) and (5), are separated in two directions: the tangential (u_t) and the binormal, or circumferential, (u_b). These displacements are given as functions of the angular coordinate and the pipe curvature (κ). Eqs. (4) and (5) were derived through the minimization of the arc-length of the bent helices [10].

$$u_t(\theta, \kappa) = -2 \cdot r^2 \cdot \frac{\kappa \cdot \sin \theta}{\tan \alpha_{hel}} \quad (4)$$

$$u_b(\theta, \kappa) = r^2 \cdot \frac{\kappa \cdot \sin \theta}{\tan^2 \alpha_{hel}} \quad (5)$$

Thereafter, these displacements must be merged into Eqs. (1)-(3). The tangential displacement shall be directly added to the Y_{hel} component, whilst the circumferential displacement needs to be transformed in an angular coordinate (φ) before it can be inserted into Eqs (1)-(3), which can be done by dividing the circumferential displacement by the minor radius of the torus. Thus, Eqs. (6)-(8) represent the bent armor wire.

$$X_{hel} = r \cdot \cos[\theta + \varphi(\theta, \kappa)] \quad (6)$$

$$Y_{hel} = \theta \cdot \frac{L_{model}}{2 \cdot n_p \cdot \pi} - u_t(\theta, \kappa) \quad (7)$$

$$Z_{hel} = r \cdot \sin[\theta + \varphi(\theta, \kappa)] \quad (8)$$

It is important to highlight that, when the curvature equals to zero, Eqs. (6)-(8) shall be reduced to Eqs. (1)-(3).

2.2 Nonlinear Spring Behavior

Finite element springs are responsible for relating forces to displacements, in the direction of its nodes. In order to calculate the characteristic curve (behavior) of the springs, the physical and geometrical characteristics of the high strength tape, inner carcass, pressure armor and plastic layers have to be considered. Some further simplifications also need to be considered:

- All materials are within their linear elastic limit,
- All remaining layers are considered as thin-walled cylinders,
- Radial pressure between all layers can be considered uniformly distributed,
- Boundary effects can be disregarded.

For a thin-walled cylinder of thickness h under internal pressure P_{int} , the radial displacement, u_n , is given by Eq. (9) [2].

$$u_n = \frac{P_{int} \cdot r^2}{E \cdot h} \quad (9)$$

In that case, the force exerted by the spring, F_{spr} , is given by the product of a stiffness, k_{spr} , and its normal, or radial, displacement:

$$F_{spr} = k_{spr} \cdot u_n = \frac{E \cdot h \cdot A_{cont}}{r^2} \cdot u_n \quad (10)$$

where A_{cont} stands for the contact area of each wire and its adjacent layer. Eq. (11) shows the contact area in terms of mean radius of the armor layer (r), number of wires (n_{wire}), number of divisions (n_{div}) and total length of the model (L_{model}).

$$A_{cont} = \frac{2 \cdot \pi \cdot r}{n_{wire}} \cdot \frac{L_{model}}{n_{div}} \quad (11)$$

Therefore, in order to account for the stiffness of the internal and external layers of the pipe, the spring stiffness can be written as:

$$k_{spr} = \begin{cases} A_{cont} \cdot \left(\frac{E_{kev} \cdot h_{kev}}{r_{kev}^2} + \frac{E_{pl} \cdot h_{pl}}{r_{pl}^2} \right) u_n < 0 \\ A_{cont} \cdot \left(\frac{E_{int} \cdot h_{int}}{r_{int}^2} + \frac{E_{pre} \cdot h_{pre}}{r_{pre}^2} + \frac{E_{pl} \cdot h_{pl}}{r_{pl}^2} \right) u_n \geq 0 \end{cases} \quad (12)$$

where, r_{kev} , r_{pl} , r_{int} and r_{pre} are the mean radius, E_{kev} , E_{pl} , E_{int} and E_{pre} are the Young modulus and h_{kev} , h_{pl} , h_{int} and h_{pre} are the thickness, regarding the high strength tape, the plastic sheaths, the inner carcass and the pressure armor layer, respectively. It is important to notice that inner carcass' and pressure armor's modulus of elasticity and thickness are calculated as shown in Refs. [1] and [3].

An additional rotational spring, placed in each node of the beams elements and connected to the ground, should be imposed to the wires in order to prevent the so called "fish-scaling" effects. The assumption followed here uses an iterative procedure to calibrate its stiffness. The calibration is based on a kinematical restraint in which the wire shall be considered as if it was sliding on a rigid supporting torus [7]. The wire axial rotation measured numerically may be stated as follows:

$$\theta_t = -\kappa_t \cdot u_b + \tau \cdot u_t \quad (13)$$

where κ_t is the Euler curvature, τ , the torsion, and u_n and u_b , the tangential and binormal displacements, respectively. The tolerance between axial rotation and the kinematical constrained used was equal to 1%. If the tolerance was not achieved, the stiffness is changed until the criterion is fulfilled.

Finally, in order to capture bird-caging phenomenon, it is necessary to input a yield limit to the radial springs, which represents the rupture of high-strength tape and outer plastic layers. This limit is determined in Ref. [5] and will be further discussed in the next item.

3. Analytical Expressions

This section aims to present two closed-form expressions for the estimation of the instability critical loads. The first expression, proposed in Ref. [5], is used for establishing the bird-caging trigger load. The

second expression, presented in Ref. [8], is related to the lateral buckling and represents a generalization of Ref. [7] for bent pipes.

3.1 Bird-Caging

Bird-caging is triggered by the loss of external support of tensile armor wire. This loss of support will occur when the high-strength tape fails, what would be followed by a sudden radial expansion of the armor wires [6]. The contact pressure suffered by this tape due to the wires which attempts to expand under real compression may be estimated by the hoop stress, giving:

$$\sigma_{tape} \approx \frac{n_{wire} \cdot \sigma_a \cdot A}{h_{kev}} \cdot \frac{\cos \alpha_{hel} \cdot \tan \alpha_{hel}}{2 \cdot \pi \cdot r} \quad (14)$$

where σ_a is the axial stress in the armor wire and A is the cross-section of a single armor wire. The compressive load that leads to the ultimate strength of the tape is considered as the radial buckling load.

3.2 Lateral Elastic Buckling

A closed-form solution for lateral buckling loads of a single tensile armor wire was derived from the differential equilibrium equations when the pipe is considered straight [7]. An expansion of this solution, in order to adapt this expression to its bent configuration, was presented in Ref. [8], and will be used here. In this expression, the curvature components of the wire-geodesic torsion (τ_g), normal curvature (κ_n) and geodesic curvature (κ_g), which are restricted to follow its geodesic path, are only considered in its critical radial position ($\theta = 0$). Furthermore, for pipe's radius of curvature greater or equal than 5 m ($R \geq 5$ m), their derivatives can be neglected ($\tau'_g = \kappa'_n = \kappa'_g \approx 0$).

$$\kappa_n = -\frac{1}{r} \sin^2 \alpha_{hel} - \frac{\kappa}{1 + r\kappa} \cos^2 \alpha_{hel} \quad (15)$$

$$\tau_g = \left(\frac{\kappa}{1 + r\kappa} - \frac{1}{r} \right) \cdot \sin \alpha_{hel} \cos \alpha_{hel} \quad (16)$$

$$\kappa_g = \frac{d^2 u_b}{ds^2} \quad (17)$$

The closed form solution is shown in Eq. (18). Its

step-by-step deduction and hypothesis can be found in Ref. [9].

$$P_{cr} = n_{wire} \cdot \cos \alpha_{hel} \left[GJ \cdot \left(\kappa_n^2 - \frac{\cos^2 \alpha_{hel}}{r} \cdot \kappa_n \right) + 4 \cdot \tau_g^2 \cdot EI_b + EI_n \left(2 \cdot \tau_g^2 - \frac{\cos^2 \alpha_{hel}}{r} \cdot \kappa_n \right) \right] \quad (18)$$

4. Case Study

The following section presents the case study of a 6" nominal diameter flexible pipe. The main geometric and physical characteristics of this pipe are presented by Østergaard [6] and also shown in Table 1.

The model utilized for bird-caging studies had its total length equals to four pitches of the external armor layer. The boundary conditions used in all analyses were applied to the centerline of the pipe. In one end of the model all degrees of freedom are restrained, while in the opposite end only the axial displacement is set free. The results of bird-caging critical loads for different ultimate strength of the high-strength tape are presented in Table 2.

In Table 3, the lateral buckling critical load, calculated with analytical expression, FEM and Østergaard's model [6], is shown for three different radius of curvature. It is also determined the lateral buckling critical load experimentally. The experimental results showed two very large ranges of critical load, but only

Table 1 6" flexible pipe physical and geometric properties.

Layers (material)	Properties
Inner tensile armor (Carbon Steel)	External diameter = 201.0 mm, Thickness = 3.0 mm, Number of wires = 52, Laying Angle = + 26.2°, Cross-Sectional Area = 30.0 mm ² , Moment of Inertia = 22.5 mm ⁴ , Young Modulus = 210 GPa, Poisson Ratio = 0.3
Outer tensile armor (Carbon Steel)	Thickness = 3.0 mm, Number of wires = 54, Laying Angle = - 26.2°, Cross-Sectional Area = 30.0 mm ² , Moment of Inertia = 22.5 mm ⁴ , Young Modulus = 210 GPa, Poisson Ratio = 0.3
High-strength tape (Kevlar)	Thickness = 1.5 mm, Young Modulus = 27 GPa; Poisson Ratio = 0.4
Plastic Sheath (Polyamide11)	Thickness = 6.0 mm; Young Modulus = 400 MPa; Poisson Ratio = 0.4

Table 2 Comparison of bird-caging critical loads for different ultimate strength of the high-strength tape.

Analyzed Conditions	Analytical, Eq. (14)	FEM
$\sigma_u = 250$ MPa	1.37 MN	1.48 MN
$\sigma_u = 500$ MPa	2.26 MN	2.42 MN
$\sigma_u = 1,000$ MPa	4.04 MN	4.33 MN

Table 3 Comparison of lateral buckling critical loads for different initial curvature of the pipe.

Analyzed Conditions	Østergaard's [6]	Analytical, Eq. (18)	FEM
$\kappa = (11 \text{ m})^{-1}$	110.0 kN	115.6 kN	121.8 kN
$\kappa = (20 \text{ m})^{-1}$	110.0 kN	114.3 kN	116.2 kN
$\kappa = (0 \text{ m})^{-1}$	110.0 kN	112.6 kN	110.3 kN

Table 4 Comparison of lateral buckling critical loads for different laying angles.

Analyzed Conditions	Østergaard's [6]	Analytical, Eq. (18)	FEM
$\alpha_{hel} = 32^\circ$	138.5 kN	147.1 kN	142.8 kN
$\alpha_{hel} = 26.2^\circ$	110.0 kN	114.6 kN	117.7 kN
$\alpha_{hel} = 22^\circ$	91.8 kN	94.6 kN	95.2 kN

Table 5 Comparison of lateral buckling critical loads for different numbers of pitches.

Analyzed Conditions	Østergaard's [6]	Analytical, Eq. (18)	FEM
$n_p = 1$	262.9 kN	143.6 kN	329.9 kN
$n_p = 2$	122.1 kN	122.1 kN	142.8 kN
$n_p = 3$	116.9 kN	118.1 kN	128.4 kN

the more conservative will be presented. The experimental critical loads were within the range of 80-160 kN [6].

Table 4 and Table 5 compare the results obtained by the same models previously stated for Table 3 when different laying angles and total number of pitches, respectively, were considered. Standard radius of curvature, laying angle and number of pitches for the analysis are equal $\kappa = (20 \text{ m})^{-1}$, $\alpha_{hel} = 26.2^\circ$ and $n_p = 3$, respectively. When not specified otherwise, the standard parameters were applied.

5. Conclusions

This work proposed a study on the instability of the tensile armor wires of flexible pipes. A numerical tool devoted to the calculation of the critical load due to bird-caging and lateral elastic buckling of the wires was presented. The FE tool was capable of estimating the instability loads within some engineering

acceptable deviation (3-20%) with respect to previously published studies.

Some tendencies and observations deserve to be highlighted. Firstly, one could have observed that, in Ref. [6], the results do not vary when the radius of curvature changes. This behavior is attributed to the capability of Østergaard's model [6] to represent the bend of the pipe with simplification. This simplification can be justified by the prescription of a fixed displacement path for the bent helix, the geodesic, which Østergaard's model does not [6]. This addresses Østergaard's model as less constrained than the FEM model.

Next, it should be underlined that, for greater curvatures, both analytical and FEM showed higher critical loads. This same tendency is obtained when the lay angle is considered, but the opposite is observed for the number of pitches. Regarding the lay angle parameter, the behavior observed was as expected, once for higher angles (closer to 90°) the wire tends to respond more to radial loads, while for lower angles the wire is responsible for resisting mainly to axial loads. Apropos the number of pitches, it was noticed that the increase of it diminishes the critical load, which stabilizes when the number of pitches is equal to four. Finally, the divergence of the values acquired for 1 pitch is attributed to the influence of boundary conditions, which are quite distinct in each model.

References

- [1] De Sousa, J. R. 2005. "Finite Element Local Analysis of Flexible Risers." Rio de Janeiro: Doctoral Thesis, Federal University of Rio de Janeiro-COPPE.
- [2] Brack, M., Troina, L. M., and de Sousa, J. R. 2005. "Flexible Riser Resistance against Combined Axial Compression, Bending, and Torsion in Ultra-deep Water Depths." In *Proceedings of the International Conference on Offshore Mechanics and Arctic Engineering-OMAE*, Halkidiki, Greece.
- [3] De Sousa, J. R., Viero, P. F., Magluta, C., and Roitman, N. 2010. "An Experimental and Numerical Study on the Axial Compression Response of Flexible Pipes." In *Proceedings of the International Conference on Offshore Mechanics and Arctic Engineering-OMAE*, Shanghai, China.
- [4] Vaz, M. A., and Rizzo, N. A. S. 2011. "A Finite Element Model for Flexible Pipe Armor Wire Instability." *Marine Structures* 24 (3): 275-91.
- [5] Sævik, S., and Thorsen, M. 2012. "Techniques for Predicting Tensile Armour Buckling and Fatigue in Deep Water Flexible Risers." In *Proceedings of the International Conference on Ocean, Offshore and Arctic Engineering-OMAE*, Rio de Janeiro, Brazil.
- [6] Østergaard, N. H. 2012. "On Lateral Buckling of Armouring Wires in Flexible Pipes." Aalborg: Doctoral Thesis, Aalborg University.
- [7] Sævik, S., and Ji, G. 2014. "Differential Equation for Evaluating Transverse Buckling Behavior of Tensile Armour Wires." In *Proceedings of the International Conference on Ocean, Offshore and Arctic Engineering-OMAE*, 1-6.
- [8] Gonzalez, M. G., de Sousa, J. R. M., and Sagrilo, L. V. S. 2016. "Analytical Expression for the Prediction of Lateral Buckling of Flexible Pipe's Tensile Armour." *XXXVII Jornadas Sudamericanas de Ingeniería Estructural*, Assunción, Paraguay.
- [9] Leroy, J. M., and Estrier, P. 2001. "Calculation of Stresses and Slips on Helical Layers of Dynamically Bent Flexible Pipes." *Oil & Gas Science and Technology* 56 (6): 545-54.
- [10] Féret, J. J., and Bournazel, C. L. 1987. "Calculation of Stresses and Slips in Structural Layers of Unbonded Flexible Pipes." *Journal of Offshore Mechanics and Arctic Engineering* 109: 263-9.