Theoretical Research on Stirling Reversible Heat Engine

Lingbin Kong

Luzhou Tianxu Tiancheng New Energy Co., Ltd., Longmattan District 646600, Luzhou City, Sichuan Province, China

Received: March 31, 2017 / Accepted: April 26, 2017 / Published: June 30, 2017.

Abstract: A mathematical model that describes the relationship between structural parameters and operating parameters was established based on the structure of Stirling reversible heat engine by Schmidt analyses. The calculation formulas of output power and average temperature of Stirling reversible heat engine were deduced, which disclosed the relationship between average temperature and ultimate pressure. A set of theoretical calculation formula of Stirling reversible heat engine was explored by combining definition of efficiency of heat engine. Research results provide theoretical references to design of parts and control system of Stirling reversible heat engine.

Key words: Stirling heat engine, structural parameters, operating parameters, formula.

1. Introduction

Stirling engine dated from 1816. However, it failed to achieve any substantial progress in several years later. The modern Stirling engine with practical values was developed until 1930s [1, 2]. However, it is strictly limited within special fields (e.g. submarine) [2] due to its complicated structure, large size and difficult sealing performances. The Stirling reversible heat engine applied China patent for invention on December 19th, 2007 [3], which eliminated all barriers against wide application of Stirling engine. The major part (regenerator) and core part of the control system (working condition controller) applied China patents for invention successively [4, 5]. With tight structure, small volume and wide power range, the Stirling reversible heat engine can completely replace modern internal combustion engine and turbine as power plants of vehicles, boats and power station. Moreover, it is a feasible technology that can replace latent heat pump and refrigerator and eliminate pollution caused by refrigerants.

Although the structural problem of Stirling reversible heat engine is solved, no theory that describes its operation law has been established. For

example, calculation of output power, calculation of average temperature and determination of working medium mass still remain unsolved. How does the number and connection mode of working chambers of Stirling reversible heat engine influence its performance? How to make quantitative analysis on such influence? How to search the optimal combination of structural parameters and operating parameters of Stirling reversible heat engine? These are important basic theoretical research topics of Stirling reversible heat engine.

Stirling reversible heat engine has at least three working chambers. For Stirling reversible heat engine with three working chambers, phase angle of all constant temperature processes is 120° and the phase angle of all constant volume processes is 60° . For Stirling reversible heat engine with four working chambers, phase angle of both constant temperature processes and constant volume processes is 90° . With the increase of working chambers, phase angle of constant temperature processes decreases, while the phase angle of constant volume processes increases accordingly. The Stirling reversible heat engine with six working chambers in dual ring connection is the simplest multi-ring connection Stirling reversible heat engine [6]. The phase angle of constant temperature processes is 60° and the phase angle of constant

Corresponding author: Lingbin Kong, engineer, research field: engineering thermophysics.

volume processes is 120° .

2. Calculation Formula of Output Power

Working chamber volume of Stirling reversible heat engine with four working chambers is composed of cylinder volume at the heat inlet (V_2), cylinder volume at the heat outlet (V_1) and through-flow volume (V_0). V_2 and V_1 change with the variation of output shaft angle. The variation law follows the variation law regulated by Schmidt analysis. The average temperature of working medium in V_0 is T_0 , the working medium temperature in cylinder at heat inlet is T_2 , and the working medium temperature in cylinder at heat outlet is T_1 . The working medium mass in working chamber is n , the working medium mass in cylinder at heat inlet is n_2 , the working medium mass in cylinder at heat outlet is n_1 , and the working medium mass in through-flow volume is n_0 . Then, there's an equation set I:

$$\begin{cases} n = n_1 + n_2 + n_0 \\ PV_1 = n_1 RT_1 \\ PV_2 = n_2 RT_2 \\ PV_0 = n_0 RT_0 \end{cases} \quad (\text{I})$$

The output power of the working chamber (W) includes output powers caused by volume changes of cylinders at heat inlet and heat outlet. If the output powers of cylinders at heat inlet and heat outlet are W_2 and W_1 , the piston sectional areas at heat inlet and heat outlet are A_2 and A_1 , and the stroke length is L . The starting point of constant-temperature expansion process is used as the baseline. The relative output shaft angle of piston displacement is α and there's a sine functional relationship between piston displacement and the output shaft angle. The output power caused by volume expansion is regulated positive, then

$$V_2 = \frac{1}{2} A_2 L (1 + \sin \alpha) \quad (1)$$

$$V_1 = \frac{1}{2} A_1 L (1 - \cos \alpha) \quad (2)$$

$$dW_2 = PdV_2 = \frac{n_2 RT_2 dV_2}{V_2} \quad (3)$$

Substitute n_2 calculated from the equation set I into Eq. (3)

$$dW_2 = \frac{nRT_1 T_2 T_0 A_2 L \cos \alpha d\alpha}{A_1 L T_2 T_0 (1 - \cos \alpha) + A_2 L T_1 T_2 (1 + \sin \alpha) + 2V_0 T_1 T_2} \quad (4)$$

Similarly,

$$dW_1 = \frac{nRT_1 T_2 T_0 A_1 L \sin \alpha d\alpha}{A_1 L T_2 T_0 (1 - \cos \alpha) + A_2 L T_1 T_2 (1 + \sin \alpha) + 2V_0 T_1 T_2} \quad (5)$$

W is the sum of W_2 and W_1 .

$$dW = dW_1 + dW_2 \quad (6)$$

$$dW = \frac{nRT_1 T_2 T_0 L (A_1 \sin \alpha + A_2 \cos \alpha) d\alpha}{A_1 L T_2 T_0 (1 - \cos \alpha) + A_2 L T_1 T_2 (1 + \sin \alpha) + 2V_0 T_1 T_2} \quad (7)$$

The output power of one Stirling circle (output shaft rotates for one circle) is:

$$W = \int_0^{2\pi} \frac{nRT_1 T_2 T_0 L (A_1 \sin \alpha + A_2 \cos \alpha) d\alpha}{A_1 L T_2 T_0 (1 - \cos \alpha) + A_2 L T_1 T_2 (1 + \sin \alpha) + 2V_0 T_1 T_2} \quad (8)$$

Eq. (8) is the output power of one Stirling circle of one working chamber. The output power of a Stirling engine with four working chambers in one Stirling circle is four times of W . The output power of a Stirling reversible heat engine with six working chambers in dual ring connection in one Stirling circle is:

$$W = \int_0^{2\pi} \frac{nRT_1 T_2 T_0 L \left[A_1 \sin \alpha + A_2 \cos \left(\alpha + \frac{\pi}{6} \right) \right] d\alpha}{A_1 L T_2 T_0 (1 - \cos \alpha) + A_2 L T_1 T_2 \left[1 + \sin \left(\alpha + \frac{\pi}{6} \right) \right] + 2V_0 T_1 T_2} \quad (9)$$

Eq. (9) reflects that the Stirling reversible heat engine with six working chambers in dual ring connection can increase enhancement degree significantly and is applicable to high-power Stirling engine.

3. Calculation Formula of Average Temperature

Average temperature (T_0) and working medium mass in working chamber (n) involved in the calculation of output power involves could not be tested directly, but only could be inferred theoretically through related parameters.

T_0 is related with size and structure of through-flow volume. The through-flow volume (V_0) consists of heater volume (V_H), regenerator volume (V_{RD}) and cooler volume (V_C).

$$V_0 = V_H + V_{RD} + V_C \quad (10)$$

The heater temperature is temperature at heat inlet (T_2), the average temperature of regenerator is T_{RD} , and the cooler temperature is the temperature at heat outlet (T_1). At any operating point of the Stirling heat engine when the pressure is P , there's:

$$\frac{PV_0}{RT_0} = \frac{PV_H}{RT_2} + \frac{PV_{RD}}{RT_{RD}} + \frac{PV_C}{RT_1} \quad (11)$$

The regenerator volume is the middle part of through-flow volume. They have similar structures. T_{RD} is very close to numerical values of T_0 , which have no strict difference in practical application. Replacing T_{RD} with T_0 has small errors. With respect to numerical values, it can completely believe that:

$$T_0 = T_{RD} \quad (12)$$

Combine Eqs. (10)-(12), it can get:

$$T_0 = \frac{V_H + V_C}{V_H/T_2 + V_C/T_1} \quad (13)$$

Eq. (13) shows that the average temperature is determined by heater volume, cooler volume,

regenerator volume and their proportions as well as temperature at heat inlet and heat outlet.

4. Relationship between Average Temperature and Ultimate Pressure

During the operation of Stirling reversible heat engine, there's a determined relationship between working medium mass and ultimate pressure. Each working circle has only one pressure minimum and one pressure maximum.

The pressure minimum of Stirling reversible heat engine with four working chambers occurs at the intersection between constant-volume exothermic process and constant-temperature compression process. The maximum pressure occurs at the intersection between the constant-volume heating process and constant-temperature expansion process.

At the minimum pressure point, volume at heat inlet is half of cylinder volume and the volume at heat outlet is equal to the whole cylinder volume. The relationship among minimum pressure ($P_{\min}$), n , A_2 , A_1 , L , V_0 , T_0 , T_2 , T_1 and the ideal gas constant (R) is:

$$n = \frac{P_{\min} A_2 L}{2RT_2} + \frac{P_{\min} V_0}{RT_0} + \frac{P_{\min} A_1 L}{RT_1} \quad (14)$$

At the maximum pressure point, volume at heat inlet is half of cylinder volume and the volume at heat outlet is zero. The relationship between $P_{\max}$ and corresponding parameters is:

$$n = \frac{P_{\max} A_2 L}{2RT_2} + \frac{P_{\max} V_0}{RT_0} \quad (15)$$

By solving Eqs. (14) and (15), it gets:

$$T_0 = \frac{2V_0 T_1 T_2 (P_{\max} - P_{\min})}{2T_2 A_1 L P_{\min} - T_1 A_2 L (P_{\max} - P_{\min})} \quad (16)$$

Substitute Eq. (16) into the Eq. (15), then,

$$n = \frac{P_{\max} P_{\min} A_1 L}{RT_1 (P_{\max} - P_{\min})} \quad (17)$$

Ultimate pressure values determine the working

medium mass. Working medium mass could be calculated in practice by measuring the ultimate pressures. Ultimate pressures are basis for the control system to determine the control range.

The direct relationship between ultimate pressures and average temperature could be concluded from Eqs. (14) and (15):

$$\frac{P_{\max}}{P_{\min}} = \frac{A_1 L / T_1}{A_2 L / 2T_2 + V_0 / T_0} + 1 \quad (18)$$

Obviously, given fixed structural parameters, the ultimate pressure ratio is determined by temperature and through-flow volume.

The relationship between average temperature of Stirling reversible heat engine with six working chambers and ultimate pressures is very complicated. However, it still has only one minimum pressure and one maximum pressure in one working circle.

5. Theoretical Calculation for Stirling Reversible Heat Engine Design

The Stirling reversible heat engine with four working chambers is used as engine. The theoretical calculation is to determine heat dissipating capacity (Q_1), heat absorption capacity (Q_2) and regeneration capacity (Q_3) according to definition of efficiency of heat engine by following the law of conversation of energy and combining function transformation characteristics during thermodynamic processes. The relationship of these parameters is:

$$\eta = \frac{Q_1 + Q_2}{Q_2} \quad (19)$$

The efficiency of heat engine (η) is about 0.3.

$$Q_2 + Q_3 =$$

$$\int_{-0.5\pi}^{0.5\pi} \frac{nRT_1 T_2 T_0 L (A_1 \sin \alpha + A_2 \cos \alpha) d\alpha}{A_1 L T_2 T_0 (1 - \cos \alpha) + A_2 L T_1 T_0 (1 + \sin \alpha) + 2V_0 T_1 T_2} \quad (20)$$

Eq. (20) is the heat absorbed by working medium

from heat inlet and by the regenerator during constant volume heating processes and constant temperature expansion processes. Let $Q_{23} = Q_2 + Q_3$.

$$Q_1 - Q_3 =$$

$$\int_{0.5\pi}^{1.5\pi} \frac{nRT_1 T_2 T_0 L (A_1 \sin \alpha + A_2 \cos \alpha) d\alpha}{A_1 L T_2 T_0 (1 - \cos \alpha) + A_2 L T_1 T_0 (1 + \sin \alpha) + 2V_0 T_1 T_2} \quad (21)$$

Eq. (21) is the heat released by working medium from the heat outlet and by the regenerator during constant volume heating processes and constant temperature expansion processes. Let $Q_{13} = Q_1 - Q_3$.

Then,

$$Q_1 = \frac{\eta - 1}{\eta} (Q_{23} + Q_{13}) \quad (22)$$

$$Q_2 = \frac{Q_{23} + Q_{13}}{\eta} \quad (23)$$

$$Q_3 = Q_{23} - \frac{Q_{23} + Q_{13}}{\eta} \quad (24)$$

In theoretical calculation of using Stirling reversible heat engine with four working chambers as engines, efficiency of heat engine is determined as 0.3 and parameters (e.g. $P_{\max}$, V_0 , V_H , V_C , cylinder radius r_2 , cylinder radius at heat outlet r_1 , T_2 and T_1) are determined. Q_1 , Q_2 and Q_3 are calculated by combining Eqs. (8), (13), (15), (17), (20)-(24) as well as the calculation formula of piston sectional area. Structural parameters are corrected continuously to search the optimal combination of structural parameters and operating parameters. The output power of one working chamber could be calculated by introducing the engine speed. The total output power is the sum of output powers of four working chambers. The theoretical calculation of Stirling reversible heat engine with six working chambers in dual ring connection just has to replace Eq. (8) by Eq. (9). The total output power is the sum of output powers of six working chambers. This calculation formula is used in theoretical

calculation of high-power Stirling engine design for the use of high temperature gas cooled reactor-based nuclear power generation [7].

The theoretical calculation of using Stirling reversible heat engine as heat pump or refrigerator is same with that of using it as an engine except for the opposite output powers and lower temperature at heat inlet than that at heat outlet.

6. Conclusions

The mathematical model established by Schmidt analysis illustrates reversibility of Stirling reversible heat engine theoretically and deduces the calculation formula of output power. The calculation formula of average temperature of Stirling heat engine is inferred by studying its through-flow volume structure and functions, which ends the history of empirical computation of average temperature of Stirling heat engine. Besides, a set of theoretical calculation

formulas of Stirling reversible heat engine are formed by combining definition of efficiency of heat engine. They provide some theoretical bases for designing the parts and control system of Stirling reversible heat engine.

References

- [1] Qian, G. Z. 1982. *Thermomotor*. Beijing: National Defence Industry Press.
- [2] Jin, D. H. 2010. *Stirling Engine Technology*. Harbin: Harbin Engineering University Press.
- [3] Kong, L. B. 2009. Stirling reversible heat engine. Chinese Patent Gazettes.
- [4] Kong, L. B. 2012. A heat-exchanger type regenerator for Stirling heat engine. Chinese Patent Gazettes.
- [5] Kong, L. B. 2011. A working condition controller for Stirling heat engine. Chinese Patent Gazettes.
- [6] Kong, L. B. 2012. Multi-ring connection Stirling reversible heat engine. Chinese Patent Gazettes.
- [7] Kong, L. B. 2014. A high temperature gas cooled reactor-based nuclear power generation and sea water desalting plant. Chinese Patent Gazettes.