Mathematical Model of Thermoelectric Peltier Module

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Abstract: TEM (thermoelectric Peltier modules) are devices that convert electrical power in a temperature gradient. Its action is based on Peltier effect. The devices are relatively simple and with small dimensions, they possess a good construction reliability and long service life (over 200,000 hours). They do not have any moving parts or environmentally harmful refrigerants. Modern TEM are highly effective—with power up to several hundreds of watts. In recent years, thermoelectric cooling systems have been widely used. They have been applied in modern vehicles for cooling and preserving products during their transportation, in portable cooling bags, computer, military and medical equipment. Usually in TEM catalogue data present some transducer characteristics and maximum parameters, but they are insufficient to create highly efficient thermoelectric systems. The purpose of this article is to offer a relatively easy method for modeling of TEM, and the results are presented in tabular and graphic form by Matlab.

Key words: Thermoelectric Peltier modules, thermoelectric cooling systems, modeling, TEC, TEG.

1. Introduction

TEM (thermoelectric Peltier modules) are devices which convert electrical power in temperature gradient. In their work they use the effect of Peltier, consisting of simultaneous heating and cooling of the two opposite sides of TEM [1, 2].

TEM have increasing interest due to simultaneous improving of their economic and technical parameters as well as and wide application which they get.

As a result, the producer of TEM supplied to the market with the wide assortment of modules with different thermoelectrically parameters, shape and sites [3].

Usually in the data sheets for given TEM some converting characteristics and maximum permissible parameters are shown: maximum temperature difference between the sides of the TEM— ΔT_{max} , maximum current— I_{max} , maximum supplying voltage— U_{max} , and maximum absorbed from the cool side of TEM power— $Q_{c_{max}}$ [4, 5].

For creating one high effective TES

(thermoelectrically system), except these data it is necessary for the optimal parameters of the real module to be known, as well as the base thermoelectrically parameters used for modules materials—coefficient of Zeebek α [V/K], specific resistance of the materials ρ [$\Omega \cdot \text{cm}$] and the coefficient of thermal conductivity k [W/cm.K].

Unfortunately the producer does not show that information in the data sheets and that is why it is necessary to have a method for calculation of these parameters.

The goal of this paper is the easy and useful method for calculation of the thermoelectrical parameters of TEM.

These are α_m , ρ_m , k_m , coefficient of conversion η , quality factor Z_0 and the parameters of the materials α , ρ and k , based on the information from the producer for the limited parameters of TEM.

The results observed are presented in tabular and graphical mode with the help of graph editor MATLAB.

2. Mathematical Analysis

2.1 Expression of Base Dependences for Cooling TEM

Next Eqs. (1-4) are fundamental and they are

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described in books and papers [6-8]:

$$Q_c = 2N \left[\alpha I T_c - \frac{1}{2} I^2 \frac{\rho}{G} - k G \Delta T \right], \quad (1)$$

where,

- N – number of thermocouples in TEM;
- G – factor of geometry expressing the relation between the surface and height of the semiconductor element;
- I – electrical current.

The voltage U is given with:

$$U = 2N \left[I \frac{\rho}{G} + \alpha \Delta T \right] \quad (2)$$

And the consummated power from TEM W is:

$$W = U \cdot I \quad (3)$$

Quality factor Z_0 is the parameter, directly connected with the possibility of TEM to pump thermal power:

$$Z_0 = \frac{\alpha^2}{\rho \cdot k} \quad (4)$$

Definition of the parameters α_m , ρ_m and k_m :

$$\alpha_m = 2 \cdot \alpha \cdot N \quad (5)$$

$$\rho_m = \frac{2 \cdot \rho \cdot N}{G} \quad (6)$$

$$k_m = 2 \cdot N \cdot k \cdot G \quad (7)$$

Using Eqs. (5)-(7), the Eqs. (1), (2) and (4) can be presented as:

$$Q_c = \alpha_m I T_c - 0,5 I^2 \rho_m - k_m \Delta T \quad (8)$$

$$U = \alpha_m \Delta T + I \rho_m \quad (9)$$

$$Z_0 = \frac{\alpha_m^2}{\rho_m \cdot k_m}, \quad (10)$$

2.2 Calculating Thermal Electrical Parameters of TEM

After reading the parameters from the producer data

sheet: ΔT_{max} , I_{max} , U_{max} and $Q_{c_{max}}$, the thermoelectrical parameters of TEM— Z_0 , α_m , ρ_m and k_m could be calculated.

This method uses three of the limit parameters— ΔT_{max} , I_{max} and U_{max} .

$$Z_0 = \frac{2 \Delta T_{max}}{(T_h - \Delta T_{max})^2} \quad (11)$$

$$\alpha_m = \frac{U_{max}}{T_h} \quad (12)$$

$$k_m = \frac{(T_h - \Delta T_{max}) U_{max} I_{max}}{2 T_h \Delta T_{max}} \quad (13)$$

$$\rho_m = \frac{(T_h - \Delta T_{max}) U_{max}}{T_h I_{max}}, \quad (14)$$

where,

- T_h is the temperature of the hot side of TEM.

After the calculation of thermoelectrical parameters of the module, the thermophysical parameters of the semiconductors are calculated, from which the thermocouples are created—coefficient of Zeebec α , specific resistivity of the materials ρ and coefficient of the thermal conductivity k .

It is done with the help of Eqs. (5-7), but only when the number of thermocouples N and geometry factor G are known.

For calculation of the converting coefficient η and thermal resistivity of the hot radiator R_h next equations are used:

$$\eta = \frac{Q_c}{W} \quad (15)$$

$$R_h = \frac{T_h - T_a}{Q_c + W} \quad (16)$$

3. Discussion and Results

The algorithm which is used for calculation of thermoelectrical parameters is shown in Fig. 1.

After the initial definition of the conditions and checking for correct their import the next calculations are done:

Fig. 1 The algorithm of modeling.

- Physical characteristics of the chosen thermoelectric module—quality factor Z (K^{-1}); coefficient of Zeebec α_m (V/K); the resistance of the module ρ_m (Ω) and coefficient of module resistivity k_m (W/K);

- The basic physical characteristics of the used for TEM thermoelectrical elements: coefficient of Zeebec α_m (V/K); specific resistance ρ ($\Omega \cdot \text{cm}$) and coefficient of thermal conductivity k (W/cm.K).

The visualization of the results from the modeling of the thermal electrical module is performed with the

help of the program product realized on the base the graphical editor MATLAB.

The program gives wide possibilities for the user, who can import high number parameters: limited parameters of TEM, shown in the producer data catalogue — ΔT_{max} , I_{max} , U_{max} , and well the working condition in which thermoelectrical module will be used.

They are:

Input current I (in determined limits);

Temperatures of the hot and cool side of the module— T_h and T_c . The hot T_h is firmly determined, but T_c is determined limits;

The ambient temperature T_a ;

The number of the semiconductors thermocouples N ;

Geometry factor G .

Practically it can be simulated the work of every one arbitrary chosen TEM, if for all input data.

On Fig. 2 the table the input/exit part of the working interface is shown.

The results, except for those in the table, are presented in graphical type.

The program proposes possibility four types of dependence to look at and analyze.

Coefficient of performance η as a function of the input current I : $\eta = f(I)$ – Fig. 3;

Coefficient of the performance η as a function of the voltage supply U : $\eta = f(U)$ – Fig. 4;

The absorbed thermal power Q_c from the cool side of TEM as a function of input current I : $Q_c = f(I)$ – Fig. 5;

The absorbed thermal power Q_c from the cool side of TEM as a function of voltage supply U : $Q_c = f(U)$ – Fig. 6;

From Figs. 3 and 4 the efficiency of the given TEM can be estimated in dependence on DC mode of work.

The graphics from Figs. 5 and 6 show optimal values of input current and the maximum values of voltage absorbed thermal power from the cool side of TEM could be reached.

Input the Performance Specifications:

Th= 300 K
dTmax= 60 K
Imax= 10 A
Umax= 12 V

Calculate the Physical Characteristics:

Z= 0.00208333 1/K
Sm= 0.04 V/K
Rm= 0.96 Ω
Km= 0.8 W/K

Input the Operating Conditions:

I= 0:10 A
Tc= 300:-20:240 K
Ta= 298 K
N= 127
G= 0.072 cm

Calculate the Basic Physical Properties:

p= 0.000272126 Ω cm
s= 0.00015748 V/K
k= 0.0437445 W/(cm K)

Calc **Clear**

Fig. 2 Input and calculated part of the program interface.

Fig. 3 Dependence of the performance coefficient η on the input current I: $\eta = f(I)$. Fig. 4 Dependence of the performance coefficient η on the voltage supply U: $\eta = f(U)$. Fig. 5 Dependence of the absorbed thermal power Q_c on the input current. Fig. 6 Dependence of the absorbed thermal power Q_c on the voltage supply U: $Q_c = f(U)$.

4. Conclusion

The realized mathematical model of thermoelectrically cooling module is a method with

which the user easily can calculate the base thermal physical parameters of TEM and for the semiconductors thermocouples, and in graphical way to report the absorbed thermal power Q_c in dependence on input current and voltage using a catalog information.

On the base of the received results it can select suitable cooling TEM in the design of thermoelectrically cooling -heating system.

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