Optimum Calculation of Partial Transmission Ratios of Mechanical Driven Systems Using a V-belt and a Two-Step Bevel Helical Gearbox

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Abstract: This paper introduces a new study on the optimum calculation of partial transmission ratios of mechanical drive system using a V-belt and two-step bevel helical gearbox for getting minimum size of the system. In the paper, based on moment equilibrium condition of a mechanic system including V-belt and a two-gear-unit of the gearbox, models for optimum calculation of the partial ratios of the V-belt and the gearbox were proposed. As the models are explicit, the partial ratios can be calculated accurately and simply.

Key words: Transmission ratio, gearbox design, optimum design, helical gearbox.

1. Introduction

In optimum gearbox design, the optimum determination of partial transmission ratios of the gearbox is the most important task. The reason for that is the partial ratios are main factors affecting the size, the dimension, the weight, and the cost of the gearbox [1]. Consequently, optimum determination of the partial ratios has been subjected to many researches.

Until now, there have been many studies on the prediction of the partial ratios gearboxes. The partial ratios were determined for different gearbox types, such as for helical gearboxes [1-7], for bevel-helical gearboxes [1, 5, 8, 9] and for worm-gearboxes [5, 10]. Beside, many methods have been used for finding optimum partial ratios. These methods are the graph method [1, 2, 4], the “practical method” [5] and modeling method [3, 6, 8, 9, 10].

From previous studies, it is clear that there have been many studies on the calculation of the partial ratios for

different types of gearboxes. However, until now, there have not been studies for mechanical driven systems using a V-belt and a two-step bevel helical gearbox. This paper introduces a result for optimal determination of partial ratios for mechanical systems using a V-belt and two-step bevel helical gearbox in order to get the minimum system cross-sectional dimension.

2. Calculation of Optimum Partial Transmission Ratios

For a two-step bevel helical gearbox (Fig. 1), the cross-sectional dimension is in minimum when [1]:

$$d_{e21} = d_{w22} \quad (1)$$

From above notice and from Fig. 1, it can be seen that, for a mechanical drive system using a V-belt and two-step bevel helical gearbox, the cross-sectional dimension of the system is minimum if:

$$d_2 = d_{e21} = d_{w22} \quad (2)$$

in which, d_2 is driven pulley diameter (mm); d_{e21} and d_{w22} are driven diameters of high speed and low speed step (mm).

For a two-step-bevel helical gearbox, based on the relations between gear ratios and driven diameters as

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Fig. 1 Calculation chema for optimum determination of partial transmission ratios.

well as based on condition Eq. (1), the optimum partial gear ratios u_1 and u_2 have been determined as Eq. (6):

$$u_2 = 1.1 \cdot u_h^{1/3} \quad (3)$$

After calculating u_2 , the high speed gear set u_1 can be found by $u_1 = u_h / u_2$.

From above analysis, for finding the optimum partial ratios of the systems in order to get the minimum system cross section, it is necessary to determine the diameters d_2 and d_{w21} .

2.1 Determining the Driver Pulley Diameter d_2

For a V-belt set, from tabulated data for determining allowable power [11], the following regression model for calculation of driver diameter d_1 (with the determination coefficient $R^2 = 0.9156$) was given:

$$d_1 = 269.7721 \cdot [p]^{0.7042} / v^{0.5067} \quad (4)$$

Theoretically, the peripheral velocity of the belt can be determined as Eq. (5):

$$v = \pi \cdot d_1 \cdot n_1 / 60000 \quad (5)$$

From Eq. (4) and Eq. (5), the diameter of the driver pulley can be determined by Eq. (6):

$$d_1 = 1130.2 \cdot [p_1]^{0.4674} / n_1^{0.3363} \quad (6)$$

Also, the diameter of driven pulley of a V-belt drive is calculated by [11]:

$$d_2 = u_d \cdot d_1 \cdot (1 - \varepsilon) \quad (7)$$

Substituting Eq. (6) into Eq. (7) gives

$$d_2 = 1130.2 \cdot u_d \cdot (1 - \varepsilon) \cdot [p_1]^{0.4674} / n_1^{0.3363} \quad (8)$$

In which, ε is slippage coefficient;

$\varepsilon = 0.01 \dots 0.02$ [11]; $[p_1]$ is the allowable power of the drive (kW); $[p_1]$ is calculated by Eq. (9):

$$[p]_1 = n_1 \cdot [T_1] / (9.55 \cdot 10^6) \quad (9)$$

Choosing $\varepsilon = 0.015$ and substituting it and Eqs. (9) into (8) gives:

$$d_2 = 0.6083 \cdot u_d \cdot n_1^{0.1311} \cdot [T_1]^{0.4674} \quad (10)$$

2.2 Determining the Driven Diameter d_{w21}

For pitting resistance of straight bevel gear unit, the Eq. (11) is needed [11]:

$$\sigma_{H1} = Z_{M1} Z_{H1} Z_{\varepsilon1} \sqrt{\frac{2T_{11} K_{H1} \sqrt{u_1^2 + 1}}{0.85 b d_{m1}^2 u_1}} \leq [\sigma_{H1}] \quad (11)$$

Also, permissible torque on the drive shaft is determined by

$$[T_{11}] = \frac{0.85 b d_{m1}^2 u_1}{2 \sqrt{u_1^2 + 1}} \frac{[\sigma_{H1}]^2}{K_{H1} (Z_{M1} Z_{H1} Z_{\varepsilon1})} \quad (12)$$

where

b - face width of bevel gear; b can be calculated from pitch con radius R_e by Eq. (13) [11]:

$$k_{be} = b / R_e \quad (13)$$

in which R_e is pitch cone radius and R_e is determined by [11]:

$$R_e = 0.5 m_{te} \sqrt{z_1^2 + z_2^2} \quad (14)$$

in which z_1 and z_2 are numbers of teeth of driving and driven bevel gears.

Combining Eqs. (13) and (14) gives:

$$b = k_{be} R_e = 0.5 k_{be} m_{te} \sqrt{z_1^2 + z_2^2}$$

Or

$$b = 0.5 k_{be} m_{te} z_1 \sqrt{1 + u_1^2} \quad (15)$$

in which m_{te} is the average module of bevel gear set:

$$m_{te} = d_{e11} / z_1 \quad (16)$$

Substituting Eq. (16) into Eq. (15) results in

$$b = 0.5 k_{be} d_{e11} \sqrt{1 + u_1^2} \quad (17)$$

with d_{m1} — average diameter of driving bevel gear (mm); d_{m1} can be calculated from [11]:

$$d_{m1} = (1 - 0.5 k_{be}) d_{e11} \quad (18)$$

Substituting Eqs. (17) and (18) into Eq. (12) gives

$$d_{e21} = \left(\frac{[T_{11}] u_1^2}{0.2125 k_{be} (1 - 0.5 k_{be})^2 [K_{01}]} \right)^{1/3} \quad (19)$$

where

$$[K_{01}] = \frac{[\sigma_{H1}]^2}{K_{H1} (Z_{M1} Z_{H1} Z_{\varepsilon1})^2} \quad (20)$$

2.3. Determining the Partial Ratios

Combining Eqs. (2), (10) and (19) gives

$$0.6083 u_d n_1^{0.1311} [T_1]^{0.4674} = \left(\frac{[T_{11}] u_1^2}{0.2125 k_{be} (1 - 0.5 k_{be})^2 [K_{01}]} \right)^{1/3} \quad (21)$$

Based on the moment equilibrium condition of a mechanic system including V-belt and a two helical gear units the allowable torque on the drive shaft $[T_{11}]$ is determined by:

$$[T_{11}] = [T_1] \cdot u_d \cdot \eta_d \quad (22)$$

in which η_d is V-belt efficiency; η_d is from 0.956 to 0.96 [11].

Choosing $\eta_d = 0.995$ and substituting Eq. (22) into Eq. (21) gives

$$0.6083 u_d n_1^{0.1311} [T_1]^{0.4674} = \left(\frac{[T_1] u_d \eta_d u_1^2}{0.2125 k_{be} (1 - 0.5 k_{be})^2 [K_{01}]} \right)^{1/3} \quad (23)$$

From Eq. (3), the ratio speed of high speed unit u_1 can be calculated as

$$u_1 = u_h / u_2 = u_h / (1.1 \cdot u_h^{1/3}) \approx 0.91 \cdot u_h^{2/3} \quad (24)$$

Theoretically, the permissible torque on the drive shaft $[T_{11}]$ can be calculated from permissible torque on the output shaft $[T_r]$ by:

$$[T_{11}] = [T_r] / (u_\Sigma \cdot \eta_\Sigma) \quad (25)$$

where, η_Σ is the total efficiency of the system:

$$\eta_\Sigma = \eta_d \cdot \eta_{br}^2 \eta_o^3 \quad (26)$$

in which η_d is V-belt efficiency (η_d is from 0.956 to 0.96 [2]); η_{br} is helical gear transmission efficiency (η_{br} is from 0.96 to 0.98 [2]); η_o is transmission efficiency of a pair of rolling bearing (η_o is from 0.99 to 0.995 [2]). Choosing $k_e = 0.3$, $\eta_d = 0.995$, $\eta_{br} = 0.97$ and $\eta_o = 0.992$ [11], $[K_{01}] = 1.4608$ [12] and substituting Eqs. (24), (25) and (26) into Eq. (23) with the note that $u_h = u_\Sigma / u_d$ gives

$$u_d = \left(\frac{45.4283 \cdot u_\Sigma^{2.4022}}{[T_r]^{1.0689} \cdot n_1^{0.0023}} \right)^{3/10} \quad (27)$$

Eq. (27) is used to calculate the speed ratio of the V-belt driver. After having u_d , the partial speed ratios u_1 and u_2 of the high and low speed gear units can be found by Eqs. (24) and (3) respectively.

3. Conclusion

The minimum system cross-sectional dimension of a mechanical drive system using a V-belt and a two-step bevel helical gearbox can be obtained by optimally splitting the total transmission ratio of the system.

Models for calculated the partial ratios of the V-belt and the bevel helical gearbox were proposed for getting the minimum cross-sectional dimension of the system based on moment equilibrium condition of a mechanic system including a V-belt and a two-step bevel helical gearbox.

By using explicit models, the speed ratios of the V-belt driver and two-step bevel helical gear units can be determined accurately and simply.

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