The Application of Continuous Spatial Scaling Model of NDVI in Validation and Its Sensibility Analysis to Various Image Characteristics

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Abstract: Based on the fractal theory, this study establishes a Continuous Spatial Scaling Model (CSSM) of the Normalized Difference Vegetation Index (NDVI) to address issues arising with spatial up-scaling in quantitative remote sensing. This model is able to quantitatively describe transformation relationships of the NDVI on continuous scales. Then the following experiments are accomplished: (1) the validation of ETM+ NDVI imagery is implemented based on the GEOEYE-1 image and its NDVI CSSM, and the following conclusion is obtained: because of bad stripes in the ETM+ image and the limited effect of destriping, the ETM+ NDVI image had a rather large error, and the error for the entire experimental imagery is about 25%, so the ETM+ NDVI product is not suitable for direct practical application; (2) Shatian Byland (Beihai City, China) is taken as the experimental area, and four images (two ETM+ images with wider and smaller coverage, respectively, a GEOEYE-1 image, and an HJ-1B CCD1 image) are studied. The most suitable scale levels are computed and compared for the four images, and a better understanding is obtained of the impact of various image characteristics (area of coverage, spatial resolution, and imaging quality) on determining the scale level for the NDVI CSSM.

Key words: Normalized Difference Vegetation Index (NDVI), scale transformation, similarity fractal dimension, validation, image characteristics.

1. Introduction

Spatial scale transformation is one of the most important basic issues arising in quantitative remote sensing [1]. In particular, spatial up-scaling has attracted much attention, since it can provide an effective way to solve a number of difficult problems, such as validation of quantitative remote sensing products [2-4]. There is no doubt that land surface reflectance presented scale effects [5, 6]. As the nonlinear combination of reflectance on red and near-infrared bands, the Normalized Difference Vegetation Index (NDVI) has more obvious and complex scale effects. Some studied it with statistic methods [7-11], and others utilized analytical methods such as Taylor Series Expansion (TSE) [12, 13].

However, as an important retrieval, the use of up-scaling for NDVI is confronted by these two issues in its implementation: (1) although the statistical methods were applied widely which could solve concrete problems well, the transformation formulas required lots of sample data and did not have clear physical meanings which were not applicable for different zones or sensors; (2) although the analytical methods had clear mathematical meanings, their kinds were limited and their precision was restricted because of some simplification and hypothesis conditions.

Based on the fractal theory, this study establishes a Continuous Spatial Scaling Model (CSSM) of the NDVI to address the above issues. This model is able to quantitatively describe the transformation relationships of the NDVI on continuous scales. Furthermore, the impact of different image characteristics (coverage

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area, spatial resolution, and imaging quality) on determining the most reasonable scale level for establishing the NDVI. CSSM is subjected to a thorough analysis.

2. Materials and Methods

2.1 Materials

The experimental area was located in the Shatian Byland of Eastern Hepu County, Beihai City in the Guangxi Zhuang Autonomous Region of China, which is located at 109°42'3" E to 109°45'36" E and 21°29'0" to 21°33'25". During the time at which the experimental images were acquired, from early to late October, the coverage in this zone comprised mainly eucalyptus, natural forest, mangrove, cassava, paddy fields, man-made constructions and water. This zone was selected for the experiment precisely because it is for such situations with different kinds of land object and with high spatial heterogeneity scaling of the NDVI using a fractal method which has potential advantages.

There were three original images: (a) a 2 m GEOEYE-1 image acquired on October 16, 2009, of $4,112 \times 3,036$ pixels; (b) a 30 m HJ-1B CCD1 image acquired on October 7, 2009; (c) a 30 m Landsat7 ETM+ image acquired on October 23, 2009. Preprocessing of these images was implemented as follows:

Atmospheric correction of the GEOEYE-1 image was done based on the ENVI FLAASH module, and its land surface radiance (Fig. 1) was obtained according to the transformation formula land surface radiance = land surface incident radiance $\times$ land surface hemispherical-directional reflectance [14] using 6S software.

The land surface radiance of the HJ-1B CCD1 image was obtained using the same method, and geometrical correction of the image was done with reference to the GEOEYE-1 image. Then, after preprocessing, the zone of the HJ-1B CCD1 image corresponding to the GEOEYE-1 image was taken as the experimental image (Fig. 2).

Fig. 1 The GEOEYE-1 false-color composite image (R: 4, G: 3, B: 2) of the experimental area after preprocessing.

Fig. 2 The HJ-1B CCD1 false-color composite image (R: 4, G: 3, B: 2) of the experimental area after preprocessing.

First, the ETM+ image was destriped, and then its land surface radiance was obtained by the same method as for the other two images. Finally, a 400×300 pixel zone and a 272×200 pixel zone (the same area as the GEOEYE-1 image) were taken as experimental images, which will henceforth be referred to respectively as the ETM+ image with wider coverage and the ETM+ image with smaller coverage (Fig. 3).

The four post-processed images represent examples of images with different characteristics, namely, area of coverage (the ETM+ image with wider coverage versus the ETM+ image with smaller coverage), spatial resolution (the GEOEYE-1 image versus the ETM+

(a) The ETM+ image with wider coverage (400 × 300 pixels).

(b) The ETM+ image with smaller coverage (272 × 200 pixels).

Fig. 3 The ETM+ false-color composite images (R: 4, G: 3, B: 2) of the experimental area after preprocessing.

image with smaller coverage), and imaging quality (the HJ-1 CCD1 image versus the ETM+ image with smaller coverage). They can therefore be used to investigate the impact of these characteristics on determining the most reasonable scale level of the NDVI CSSM.

2.2 Methodology

The similarity fractal dimension was defined by

$$D = \log_1 u = \frac{\lg u}{\lg \frac{1}{s}} \quad (1)$$

and, using this, the scale and NDVI parameters were

transformed. According to Fig. 4, the CSSM of the NDVI was then established as follows. First, the land surface radiances (e.g., r_{11} , r_{12} , r_{21} and r_{22}) on the scale of the basic image (e.g., the 2 m scale of the GEOEYE-1 image) were summed by area to obtain the land surface radiance rad on larger scales, and the mean of NDVI ($NDVI$) on these larger scales was computed. Second, taking the basic image scale (e.g., the 2 m scale of the GEOEYE-1 image) as the base, the scale factor ($scale$) of the similarity fractal dimension was defined as the larger scale ($scale_up$) divided by the basic image scale (e.g., $scale_2$). Third, by computing the $\log_2$ arrays of $1/scale$ and $NDVI$, a linear regression was performed based on the arrays:

$$\log_2 NDVI = d \log_2 \frac{1}{scale} + b \quad (2)$$

where d is the slope of the fitting line and the constant b is its intercept. The NDVI CSSM was then established as

$$NDVI = 2^b \left(\frac{1}{scale} \right)^d \quad (3)$$

and its similarity fractal dimension D was calculated as

$$D = 2 - d \quad (4)$$

Because the land surface radiance satisfied the up-scaling rule of summing according to area [17], the NDVI model with land surface radiance was selected to

Fig. 4 Flowchart of the model for continuous scaling of the NDVI by a fractal method [15, 16].

establish a physically meaningful CSSM. A statistical estimation procedure was then applied to this model, with four relatively independent and related statistical parameters (r , p , rlo , rup) being chosen. Here, r is the correlation coefficient between the fitting line and the true curve in Eq. (2), valid for small p -values ($p < 0.05$), while rlo and rup are respectively the lower and upper limits of the confidence interval around $r = 0.95$, according to which the possibility is less than 0.05 that the true r is less than rlo .

Furthermore, on a different large scale, the difference between NDVI from Eq. (3) and the near-true value can be computed as [15]

$$Diff = NDVI_1 - NDVI_2 \quad (5)$$

where $NDVI_1$ is the mean NDVI on the larger scale from Eq. (3) and $NDVI_2$ is the near-true mean NDVI on the same scale (e.g., the mean of the NDVI image computed from the true land surface radiance image). Here, $Diff$ represents the error in $NDVI_1$. Thus, the ratio of $Diff$ to $NDVI_2$,

$$Error = Diff / NDVI_2 \quad (6)$$

represents the ratio of the error in $NDVI_1$ to $NDVI_2$. On this basis, $Max_of_abs(Error)$ can be defined as the largest absolute value of $Error$, which is the largest error in validating NDVI using the CSSM, and can therefore be chosen as an index of the applicability of the CSSM.

Taking the four statistical indices (r , p , rlo and rup) together with the index $Max_of_abs(Error)$, the CSSM can be comprehensively estimated on the basis of statistics and applicability. These indices are mutually dependent and form a five-index estimation system for the NDVI CSSM. According to the flowchart shown in Fig. 4, different large scales can be obtained from the basic image scale (e.g., the 2 m of the GEOEYE-1 image), with the largest of these scales corresponding to the final scale level. For a basic image (e.g., GEOEYE-1), different final scale levels can be set, and different NDVI CSSMs can be established for each of

these. From these models, the most appropriate NDVI CSSM can be chosen once the parameter values for the estimation system have been set.

In this study, the appropriate scale levels for the experimental images were defined as follows. First, the NDVI CSSM of an image was established at each level. Second, the NDVI CSSMs at each level were analyzed based on the standard criterion ($r \geq 0.8$, $p < 0.05$ and $rlo \geq r \leq rup$), and the levels matching the criterion were selected. Third, with the aim of determining the NDVI CSSM with the widest range of scales, based on the other criterion $Max_of_abs(Error) \leq 0.05$, the most appropriate scale level could be defined from the available scale levels computed above.

To quantitatively analyze the reason for the NDVI scale effect, a spatial heterogeneity index (SHI) was defined to estimate the spatial heterogeneity of the images [18]. The SHI of an image pixel $f(i, j)$ is defined as the sum of the absolute differences between a pixel and its eight neighbors:

$$SHI_{ij} = \sum_{a=-1}^1 \sum_{b=-1}^1 |f(i, j) - f(i+a, j+b)| \quad (7)$$

Thus, $Mean\ SHI$ for an image of size $m \times n$ pixels can be computed as

$$Mean\ SHI = \frac{1}{m \cdot n} \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} SHI_{ij} \quad (8)$$

In general, the greater the value of $Mean\ SHI$ for an NDVI image is, the more obvious the scale effect of the image is.

3. Results and Discussion

3.1 Experiments and Results

The following experimental results were obtained.

(a) Validation of ETM+ NDVI imagery based on the GEOEYE-1 image and its NDVI CSSM

In the experiment, the NDVI CSSM of the GEOEYE-1 image (Fig. 1) was established on the basis of fractal theory and a five-index estimation system ($r \geq 0.8$, $p < 0.05$, $rlo \geq r \leq rup$ and $Max_of_abs(Error) \leq 0.05$), and, by integrating the spectral normalization, the ETM+ NDVI image (Fig. 3b) was validated. From

each image, 22 sample images of same areas were selected, which had four kinds of ranges (150 pixel $\times$ 150 pixel, 600 pixel $\times$ 600 pixel, 1,500 pixel $\times$ 1,500 pixel and 4,112 pixel $\times$ 3,036 pixel for GEOEYE-1 image, which were respectively corresponded to 10 pixel $\times$ 10 pixel, 40 pixel $\times$ 40 pixel, 100 pixel $\times$ 100 pixel and 272 pixel $\times$ 200 pixel of ETM+ image) and contained different kinds of ground objects described in “2.1 Materials”. Then, on a 30 m scale, the NDVI values from the model and the ETM+ sample images were computed and compared, as shown in Fig. 5a. Fig. 5b shows the NDVI errors for the ETM+ sample images computed from Eq. (6) and compared with the upper and lower limits of acceptable error (± 0.05), which were defined according to the estimation criterion $Max_of_abs(Error) \leq 0.05$. The experimental results were analyzed, with the following conclusion: because of the influence of the bad stripes in the ETM+ image and the limited effect of destriping, the ETM+ NDVI image had a rather large error, and the error for the entire experimental imagery was about 25%, so the ETM+ NDVI product was not suitable for direct practical application. Meanwhile, the potential value of NDVI CSSM for validation was demonstrated.

For the ETM+ images with wider and smaller coverage, the NDVI CSSMs were respectively established at the most suitable scale levels (*Levels* 267 and 181) (Figs. 6 and 9), their *Errors* were computed (Figs. 7 and 10), and the *SHIs* of the two NDVI images were computed (Figs. 8 and 11).

(b) Comparison of NDVI CSSMs based on the ETM+ images with wider and smaller coverage

In Figs. 7 and 10, the absolute values of *Errors* are generally non-zero: for the ETM+ image with wider coverage, the minimum absolute value is 3.6308×10^{-5} (at *up-scale* = 5,370 m) and the maximum is 0.0497 (at *up-scale* = 30 m), and for the ETM+ image with smaller coverage, the minimum is 1.3217×10^{-5} (at *up-scale* = 630 m) and the maximum is 0.0237 (at *up-scale* = 3,000 m). For the most suitable NDVI CSSMs, the maximum absolute values of *Errors* are

(a) Comparison of NDVI values according to the model and ETM+ sample images.

(b) NDVI errors for the ETM+ sample images.

Fig. 5 Validation of ETM+ NDVI.

less than 0.05, which is consistent with the criterion $Max_of_abs(Error) \leq 0.05$, and therefore the CSSMs can be applied for validation of the NDVI. In Figs. 8 and 11, *Mean SHI* at different *up-scales* would be the same if the ETM+ NDVI images were spatially uniform and not heterogeneous. In both of these figures, however, *Mean SHI* varies clearly on different scales, indicating that the ETM+ NDVI images are of high spatial heterogeneity, which is the reason for the NDVI scale effect and the *Errors* generated.

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Fig. 6 Establishing the NDVI CSSM at the most suitable scale level (Level = 267) for the ETM+ image with wider coverage.

Fig. 9 Establishing the NDVI CSSM at the most suitable scale level (Level = 181) for the ETM+ image with smaller coverage.

Fig. 7 Difference between the NDVI results from the two methods (Level = 267) for the ETM+ image with wider coverage.

Fig. 10 Difference between the NDVI results from the two methods (Level = 181) for the ETM+ image with smaller coverage.

Fig. 8 The scale versus SHI curve of the image (Level = 267) for the ETM+ image with wider coverage.

Fig. 11 The scale versus SHI curve of the image (Level = 181) for the ETM+ image with smaller coverage.

Table 1 Comparison of the NDVI CSSMs based on the ETM+ Images with wider and smaller coverage.

Image	Mean <i>SHI</i>	Most suitable model	Fractal dimension	Scale range	Error range
ETM+ image with wider coverage	0.4200	$\log_2 NDVI = -0.0347 \log_2 \frac{1}{scale} - 1.1296$	2.0347	30 m to 8,010 m	3.6308×10^{-5} to 0.0497
ETM+ image with smaller coverage	0.3823	$\log_2 NDVI = -0.0211 \log_2 \frac{1}{scale} - 0.8675$	2.0211	30 m to 5,430 m	1.3217×10^{-5} to 0.0237

On this basis, and taking the mean *SHI*, the most suitable model, the fractal dimension, the scale range, and the error range as indices, the NDVI CSSMs of the ETM+ images with wider and smaller coverage are compared in Table 1.

It can be concluded from Table 1 that, for images from the same sensor with similar land cover type, the wider the area of coverage is and the greater the spatial heterogeneity of the image is; the higher the scale level of the model is and the wider its scale range is.

(c) Comparison of NDVI CSSMs based on the GEOEYE-1 image and ETM+ image with smaller coverage

For the GEOEYE-1 image and the ETM+ image with smaller coverage, the NDVI CSSMs were respectively established at the most suitable scale levels (Levels 495 and 181) (Figs. 9 and 12). Their *Errors* were computed (Figs. 10 and 13), and the *SHIs* of the two NDVI images were computed (Figs. 11 and 14).

In Figs. 13 and 10, the absolute values of *Errors* are generally non-zero: for the GEOEYE-1 image, the minimum absolute value is 1.0697×10^{-5} (at *up-scale* = 448 m) and the maximum is 0.0127 (at *up-scale* = 868 m), and for the ETM+ image with smaller coverage, the minimum is 1.3217×10^{-5} (at *up-scale* = 630 m) and the maximum is 0.0237 (at *up-scale* = 3,000 m). For the most suitable NDVI CSSMs, the maximum absolute values of *Errors* are less than 0.05, which is consistent with the criterion $Max_of_abs(Error) \leq 0.05$, and therefore the CSSMs can be applied to validation of the NDVI. In Figs. 11 and 14, the *Mean SHI* at different *up-scales* would be the same if the GEOEYE-1 and ETM+ NDVI images were spatially uniform and not heterogeneous. In both of these figures, however, *Mean*

SHI varies clearly on different scales, indicating that the two NDVI images are of high spatial heterogeneity, which is the reason for the NDVI scale effect and the *Errors* generated.

On this basis, taking the mean *SHI*, the most suitable model, the fractal dimension, the scale range, and the

Fig. 12 Establishing the NDVI CSSM at the most suitable scale level (Level = 495) for the GEOEYE-1 image.

Fig. 13 Difference between the NDVI results from the two methods (Level = 495) for the GEOEYE-1 image.

Fig. 14 The scale versus SHI curve of the image (Level = 495) for the GEOEYE-1 image.

error range as indices, the NDVI CSSMs of the GEOEYE-1 image and the ETM+ image with smaller coverage are compared in Table 2.

It can be concluded from Table 2 that, for images from different sensors of the same area of coverage and the same land cover type, the higher the spatial resolution is and the lower the spatial heterogeneity of the image is; the finer the scale of the model is but the smaller its scale range is, starting from a smaller scale corresponding to the source image's resolution and containing scales corresponding to integer multiples of the source image resolution. Conversely, the lower the spatial resolution is and the higher the spatial heterogeneity of the image is; the coarser the scale of the model is but the wider its scale range is, starting from a larger scale corresponding to the source image resolution and containing scales corresponding to integer multiples of the source image resolution.

Comparison of NDVI CSSMs based on the HJ-1 CCD1 image and ETM+ image with smaller coverage

For the HJ-1 CCD1 image and the ETM+ image with smaller coverage, the NDVI CSSMs were respectively established at the most suitable scale levels (*Levels* 24 and 181) (Figs. 9 and 15). Their *Errors* were computed (Figs. 10 and 16), and the *SHIs* of the two NDVI images were computed (Figs. 11 and 17).

In Figs. 10 and 16, the absolute values of *Errors* are generally non-zero: for the HJ-1 CCD1 image, the minimum absolute value is 2.3396×10^{-4} (at *up-scale* = 180 m) and the maximum is 0.0068 (at *up-scale* = 600 m), and for the ETM+ image with smaller coverage, the minimum is 1.3217×10^{-5} (at *up-scale* = 630 m) and the maximum is 0.0237 (at *up-scale* = 3,000 m). For the most suitable NDVI CSSMs, the maximum absolute values of *Errors* are less than 0.05, which is consistent with the criterion $Max_of_abs(Error) \leq 0.05$, and therefore the CSSMs can be applied to validation of the NDVI. In Figs. 11 and 17, the *Mean SHI* at different *up-scales* would be the same if the HJ-1 CCD1 and ETM+ NDVI images were spatially uniform and not heterogeneous. In both of these figures, however, *Mean SHI* varies clearly on different scales, indicating that the two NDVI images are of high spatial heterogeneity, which is the reason for the NDVI scale effect and the *Errors* generated.

On this basis, taking the mean *SHI*, the most suitable model, the fractal dimension, the scale range, and the error range as indices, the NDVI CSSMs of the HJ-1 CCD1 image and the ETM+ image with smaller coverage are compared in Table 3.

It can be concluded from Table 3 that, for images from different sensors of the same area of coverage and land cover type, the better the imaging quality is and the better the capability to describe complex structure

Table 2 Comparison of NDVI CSSMs based on the GEOEYE-1 image and the ETM+ image with smaller coverage.

Image	Mean <i>SHI</i>	Most suitable model	Fractal dimension	Scale range	Error range
GEOEYE-1 image	0.3746	$\log_2 NDVI = -0.0120 \log_2 \frac{1}{scale} - 1.1122$	2.0120	2 m to 990 m	1.0697×10^{-5} to 0.0127
ETM+ image with smaller coverage	0.3823	$\log_2 NDVI = -0.0211 \log_2 \frac{1}{scale} - 0.8675$	2.0211	30 m to 5,430 m	1.3217×10^{-5} to 0.0237

Fig. 15 Establishing the NDVI CSSM at the most suitable scale level (Level = 24) for the HJ-1 CCD1 image.

Fig. 16 Difference between the NDVI results from the two methods (Level = 24) for the HJ-1 CCD1 image.

Fig. 17 The scale versus SHI curve of the image (Level = 24) for the HJ-1 CCD1 image.

and the spatial heterogeneity of the coverage is; the higher the scale level is and the wider the scale range of the model is.

3.2 Discussion

On one hand, from formula 3, the NDVI values by the model are the entire statistical feature of images on each scale, and the CSSM describes the relation between mean NDVIs of images on each scale, not of corresponding pixels. Therefore, it is a whole-image validation method based on NDVI CSSM. Although it is reasonable and reliable [17], which is still anticipated to establish a more precise CSSM describing the relation between corresponding pixels of images on each scale which would implement more accurate validation.

On the other hand, the three sensors have different imaging parameters (e. g. spectral one, temporal one and geometric one), and it would bring some uncertainty into the comparison of NDVI CSSMs for the four experimental images. Consequently, in the next work, the normalization of imaging parameters will be concerned about to capture precise results.

4. Conclusions

Based on the fractal theory, this study has established an NDVI CSSM to quantitatively describe the transformation relationships of NDVI on continuous scales, which could be applied into the NDVI's validation. For land surface parameters with summing rules by area, such as land surface radiance and soil moisture, the CSSM methodology (shown in Fig. 4) is readily available from the literature. Furthermore, taking the Shatian Byland as an experimental area and comprehensively utilizing four images (two ETM+ images with wider and smaller coverage, respectively, a GEOEYE-1 image, and an HJ-1B CCD1 image), the impacts of various image characteristics (coverage area, spatial resolution, and imaging quality) on determining the most suitable scale level of NDVI CSSM have been analyzed. The results obtained have led to a better

Table 3 Comparison of NDVI CSSMs based on the HJ-1 CCD1 image and the ETM+ image with smaller coverage.

Image	Mean <i>SHI</i>	Most suitable model	Fractal dimension	Scale range	Error range
HJ-1 CCD1 image	0.4186	$\log_2 NDVI = -0.0089 \log_2 \frac{1}{scale} - 0.7427$	2.0089	30 m to 720 m	2.3396×10^{-4} to 0.0068
ETM+ image with smaller coverage	0.3823	$\log_2 NDVI = -0.0211 \log_2 \frac{1}{scale} - 0.8675$	2.0211	30 m to 5,430 m	1.3217×10^{-5} to 0.0237

understanding of the characteristics of the NDVI CSSM.

Furthermore, according to the theory system of fractal geometry [19], fractal measurement and computation is only part of it; the generated mechanism or dynamic process of mathematical fractal and statistical one is another significant part of it. For example, Iterated Function System (IFS) is utilized to reveal the dynamic process of fractal phenomena (e. g. fractal downscaling of soil moisture) [20, 21]. While these researches are preliminary, there are many important problems to be addressed. It will be the research focus in the future work.

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References

- [1] Li, X., and Wang, Y. 2013. "Prospects on Future Developments of Quantitative Remote Sensing." *Acta Geographica Sinica* 68 (9): 1163-1169. (in Chinese).
- [2] Morisette, J. T., Privette, J. L., and Justice, C. O. 2002. "A Framework for the Validation of MODIS Land Products." *Remote Sensing of Environment* 83 (1): 77-96.
- [3] Liang, S. 2009. *Quantitative Remote Sensing*. Translated by Fan, W. J. Beijing: Science Press, 180-183. (in Chinese).
- [4] Wu, X., Wen, J., Xiao, Q., Li, X., Liu, Q., Tang, Y., et al., 2015. "Advances in Validation Methods for Remote Sensing Products of Land Surface Parameters." *Journal of Remote Sensing* 19 (1): 76-92.
- [5] Liang, S. 2000. "Numerical Experiments on the Spatial Scaling Of Land Surface Albedo and Leaf Area Index." *Remote Sensing Reviews* 19 (1-4): 225-242.
- [6] Liang, S., Fang, H., Chen, M., Shuey, C., Walthall, C., Daughtry, C., et al. 2002. "Validating MODIS Land Surface Reflectance and Albedo Products: Methods and Preliminary Results." *Remote Sensing of Environment* 83 (1): 149-162.
- [7] Aman, A., Randriamanantena, H. P., Podaire, A., and Frouin, R. J. 1992. "Upscale Integration of Normalized Difference Vegetation Index: The Problem of Spatial Heterogeneity." *IEEE Transactions on Geosciences and Remote Sensing* 30 (2): 326-338.
- [8] Bian, L., and Walsh, S. J. 1993. "Scale Dependencies of Vegetation and Topography in a Mountainous Environment of Montana." *Professional Geographer* 45 (1): 1-11.
- [9] Friedl, M. A., Davis, F. W., Michaelsen, J., and Moritz, M. A. 1995. "Scaling and Uncertainty in the Relationship between the NDVI and Land Surface Biophysical Variables: An Analysis Using a Scene Simulation Model and Data from FIFE." *Remote Sensing of Environment* 54 (3): 233-246.
- [10] VanDer Meer, F. D., Bakker, W. H., Scholte, K., Skidmore, A. K., De Jong, S. M., Clevers, J. G. P. W., et al. 2001. "Spatial Scale Variations in Vegetation Indices and Above-Ground Biomass Estimates: Implications for MERIS." *International Journal of Remote Sensing* 22 (17): 3381-3396.
- [11] Gu, Y. X., and Wylie, B. K. 2015. "Developing a 30-m Grassland Productivity Estimation Map for Central Nebraska Using 250-M MODIS and 30-M Landsat-8 Observations." *Remote Sensing of Environment* 171: 291-298.
- [12] Hu, Z. L., and Islam, S. 1997. "A Framework for Analyzing and Designing Scale Invariant Remote Sensing Algorithms." *IEEE Transactions on Geoscience and Remote Sensing* 35 (3): 747-755.
- [13] Zhang, X., Yan, G., Li, Q., Li, Z., Wan, H., and Guo, Z. 2006. "Evaluating the Fraction of Vegetation Cover Based

- on NDVI Spatial Scale Correction Model.” *International Journal of Remote Sensing* 27 (24): 5359-5372.
- [14] Martonchik, J. V., Bruegge, C. J., and Strahler, A. H. 2000. “A Review of Reflectance Nomenclature Used in Remote Sensing.” *Remote Sensing Reviews* 19 (1-4): 9-20.
- [15] Luan, H., Tian, Q., Yu, T., Hu, X., Huang, Y., Du, L., et al. 2013. “Modeling Continuous Scaling of NDVI Based on Fractal Theory.” *Spectroscopy and Spectral Analysis* 33 (7): 1857-1862. (in Chinese).
- [16] Luan, H., Tian, Q., Gu, X., Yu, T., and Hu, X. 2013. “Modeling Continuous Scaling of NDVI Based on Fractal Theory and GEOEYE-1 Image.” *Journal of Infrared and Millimeter Waves* 32 (6): 538-544 + 549. (in Chinese).
- [17] Zhang, R. H., Tian, J., Li, Z. L., Su, H. B., Chen, S. H., and Tang, X. Z. 2010. “Principles and Methods for the Validation of Quantitative Remote Sensing Products.” *Science China Earth Sciences* 53 (3): 741-751.
- [18] Zhang, X. H. 2012. “Study on Spatial Heterogeneity and Scale Effect of Eucalyptus Forest Based on High Resolution Remote Sensing.” Ph. D. thesis, Nanjing University, 49-50. (in Chinese).
- [19] Chen, Y., and Chen, L. 2005. *Fractal Geometry (2nd ed.)*. Beijing: Earthquake Press, 39-44 + 95-98. (in Chinese).
- [20] Kim, G., and Barros, A. P. 2002. “Downscaling of Remotely Sensed Soil Moisture with a Modified Fractal Interpolation Method Using Contraction Mapping and Ancillary Data.” *Remote Sensing of Environment* 83 (3): 400-413.
- [21] Xie, H. P., and Sun, H. Q. 1997. “The Study on Bivariate Fractal Interpolation Functions and Creation of Fractal Interpolated Surfaces.” *Fractals* 5 (4): 625-634.