Flipping Calculus Using Mobile Technology^{*}

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In this paper, we introduce the flipping pedagogy and the exploratory team project of adopting this new teaching method in our Calculus education based on an international collaboration. The statistical comparison of exam grades of the flipped class and non-flipped class by the same instructor is included, as well as the summary of attitude survey to the flipping section. Moreover, we introduce how to use mobile technology to make a traditional classroom to be flipped friendly. Finally, we summarize the lessons learned from this project and the flipping plan for the future.

Keywords: flipping, Calculus, science, technology, engineering, and mathematics (STEM) education, mobile technology, iPad

Introduction

Flipping is a new teaching method that is becoming increasingly popular globally. The general idea of flipping is to move some basic knowledge of a course, such as definitions, facts, and problem-solving skills, outside of class, so that the students can grasp basic ideas prior to class. Currently, this knowledge acquisition is usually through pre-recorded online videos. Then, the majority of class time is devoted to affording students an opportunity to work on more complicated or deeper understanding questions or applying the knowledge in real-world applications assimilated through structured group problem-solving activities, small-group discussions, and whole-class discussions. These activities are under the guidance of an experienced and knowledgeable instructor. Because the instructor can interact with the students in small groups or even one-on-one, the questions the students have can be addressed instantly and individually. Thus, the students benefit from more frequently structured practices and in-class feedbacks from a knowledgeable teacher. Meanwhile the instructor can adjust the pace or class material based on the improved feedbacks from the class, which will lead to a more dynamic class.

Although the formal expression “flipping a course” is relatively new, this pedagogical strategy has been around for a number of years. The idea of flipping is similar to “just-in-time teaching” (Novak, Patterson,

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Gavrin, & Christian, 1999) and “inverted instruction” (Lage, Platt, & Treglia, 2000). The key is to have students previewed some basic knowledge, so that they can be more engaged in class. With the development of technology, flipped classes that are based on pre-recorded online videos appeared around 2009 (Demetry, 2010; Deslauriers, Schelew, & Wieman, 2011; Toto & Nguyen, 2009; Warter-Perez & Dong, 2012). These flipped classes were mostly in the fields of sciences and technology, and the whole process was done by individual instructors.

In 2012, the Mathematics Department of the University of Hartford in the United States started a departmental project—Flipping Calculus (McGivney-Burelle & Xue, 2013; Schroeder, McGivney-Burelle, & Xue, 2015; Schroeder, Xue, & McGivney, 2013). A whole package of materials were created by a group of six faculty members, including videos, in-class problem sets, entrance quizzes, and WebWork homework sets. Then, they flipped half of the 10 sections of Calculus I which they offered that semester and did a pedagogical study. Their study obtained encouraging statistical results showing that the students in flipped classes not only did better in the common final exam, but also continuously achieved better grades in the subsequent Calculus II course in which all sections were taught in a traditional lecturing way.

In China, the flipping method started to appear in higher education in 2012 (Zhang, Wang, & Zhang, 2012; Zou, 2012), and then spread to a few different fields (Zhang, He, & Li, 2013; Zhang & Chen, 2014; Shen, Liu, & Xie, 2013; Zhong, Song, & Jiao, 2013). However, flipping is still not popular in most universities.

Calculus at Guangdong University of Technology

At the college level, Calculus is a foundation course for all science, technology, engineering, and mathematics (STEM) majors. It is a hub-course that connects numerous different subjects. If taught well, it will provide students with a positive and successful first-year experience and gateway into more advanced courses.

Guangdong University of Technology is a public university located in Guangzhou, China, and has been selected as one of the 10 high-level universities development project of the Guangdong province. It is a national enrollment university with focus on Guangdong province. At the university (Guangdong University of Technology), the goal of Calculus education is to help students lay deep mathematics foundation, gain logical analysis experience, and foster critical thinking ability. Moreover, we hope to provide students with a Calculus experience that will transform them from traditional learners to independent and intellectual members of the community.

Every Fall semester, we offer about 42 sections of Calculus I to the whole university with all of the students being freshmen. Most sections are normal sections with class size varying from 150 to 200. Students get enrolled in different sections based on their majors. We also have two types of honors sections, which are called “Creative Section” and “Outstanding Section.” Enrollment of these honors sections are based on students’ national matriculation examination grade and major.

Currently, we are using the textbook *Higher Mathematics* (7th ed.), edited by Tongji University and published by Higher Education Press. The Calculus materials, from single variable to multi variable Calculus, are taught in two semesters.

Although significant efforts have been made over the past decades to reform the teaching and learning of college-level Calculus, this course remains a difficult and boring course for the majority of students. We believe one main reason is that students experience very little participation and involvement during the class. Mathematics is a subject that is impossible for a student to master by just listening and overseeing the instructor. It is imperative that the students get involved in class in order to understand the concepts, apply the new

formulas and theorems, and deal with the challenges. However, under the traditional lecturing teaching style, especially with a large class size, students almost have neither the chance nor the motivation to participate in class. Therefore, when they have to work on the homework problems independently after class, they will face a large gap between watching the professor working on a problem and solving a problem by themselves. Without guidance and instant feedback, it is easy for students to make mistakes, misuse formulas, misunderstand concepts, or just have no idea to a problem at all. This will very possibly lead to a vicious circle in learning Calculus. This provides us the motivation for the flipping project.

The Flipping Project at Guangdong University of Technology

Guangdong University of Technology is always self-updated with new teaching methods and technologies. We do care about teaching, especially undergraduate education. With the development of technology and the trend of “active learning” all over the world, we decided to make changes in teaching. During the academic year of 2015-2016, we piloted the flipping pedagogy in Calculus I and II courses to test the feasibility and challenges of this new teaching method.

Preparation

With the support of the School of Applied Mathematics, a flipping team was formed in the Spring semester of 2015. It included administrative persons (two associate deans), three full-time instructors, and one mathematics professor from the University of Hartford in the United States who has extensive flipping experience. The flipping team agreed to flip Calculus as a pilot course in Fall semester of 2015.

The formal collaboration was kicked off by a university-wide presentation about flipping pedagogy by the team member who was from the United States. Then, the team spent two weeks in June 2015 on discussing the details of flipping plan and technology preparation. At first, we picked three chapters, Indefinite Integrals, Definite Integrals, and the Applications of Integrals, as the flipping topics. One reason we picked these chapters is that they are taught during the middle of the semester, when the professor and students have come to know each other, so that it will be easier to do group discussion in class. Moreover, these topics are a relatively independent and complete unit of Calculus, which are usually among the most significant challenges for first-year students. Since the flipping videos were to be created by three instructors, we also discussed the curriculum and decided the core materials to cover. Furthermore, we prepared and tried the technology that will be used for flipping. The major issue was the videos. The experienced team member from the United States provided hands-on technology training. We decided to use PowerPoint to prepare the video slides and use Camtasia to do screencast video recording and editing. We also planned to use QQ, a social media software, to distribute the videos to students.

During the summer of 2015, three instructors created all the flipping videos. All the videos were within 10 minutes' length. We only put the core part of each class's material in the video and used simple examples to demonstrate the new concepts or formulas. Note that a flipped course is very different from an online course. We did not try to teach everything to the students through videos. Instead, we just hoped students could get a little bit of the new material through videos, so that they could work on more challenging and deeper materials during the class. For each video, we made the slides first and shared with other team members. Then, we modified the slides based on comments and suggestions. We started to record the video only when the slides of a video had been reviewed and modified by all the team members.

Meanwhile, the in-class problem sets were also prepared during the summer of 2015. The key idea was to find or create a set of problems for each section of the material to be flipped. This not only will help the students to master the knowledge by working on the problem sets, but also challenge the students to have a deeper understanding than they have in traditional lecturing. Each set included problems from relatively easy to complicate. Some challenging questions in the problem sets were usually not covered in traditional classes because of time limitations. We also put in a number of real world problems in order that the students would experience some potential applications.

The Flipping Process

We flipped about five weeks from early November of 2015 to the middle of December 2015, which is 1/3 the length of the whole semester. The topics we flipped were Indefinite Integrals, Definite Integrals, and the Application of Integrals. These are covered in Chapter 4 to Chapter 6 of the textbook.

Two classes taught by the same instructor back to back were chosen for the study of flipping. Class A had 44 students majoring in Mechanical Engineering and Automation, and Class B had 45 students majoring in Software Engineering. Class A was flipped for five weeks starting at mid-semester, while Class B was taught by the traditional lecture method throughout the semester. Both classes were given two mid-semester exams to test the material before flipping and the one using flipping, and a common final exam. The results of comparison will be discussed in the consequence sections.

Here is a typical flipped class. One or two days before the class, the instructor put the associated video(s) on QQ and notified the students to watch before the class. Students could ask questions within the QQ group. The instructor or other students could answer or make comments about the questions.

The first thing in class was usually a five-minutes mini quiz. The quiz usually contained only one question that was the same or very similar to an example from the videos. The purpose of the entrance quiz was to make sure that the students watched videos before class. Because of the size of the class, we randomly picked four to eight students' entrance quizzes to grade. The grade of entrance quizzes became a small part of their overall attendance grade.

After the quiz, the instructor projected the problem set on screen. The students worked on the problems in groups. Each group contained three to four students and the groups were formed by themselves. They were encouraged to work together on the problems. However, it turned out that the majority of the class chose to work individually and then compared the solutions to create the "group solution." Compared to the students in the United States, the students in China have a lot less experience in teamwork. Therefore, the instructor needed to pay extra attention and spend more time in leading students to work together.

The instructor walked around in the classroom to help the students directly and give them feedback. When most groups had finished a problem, the instructor invited one group to project their work on screen and explain to the whole class. Sometime we invited the group with correct answers, but sometimes we chose the one that had made typical mistakes. In order to encourage the students to present, we gave bonus points to the student who did the presentation.

The key idea of flipping is to have students participated in class. Mathematics is a subject that students have to get involved and practice before they can truly understand the material. In traditional teaching, when students work on problems after class by themselves, they cannot get help and feedback. It is very possible that they struggle on one question for a few hours or believe they get the answer but in fact do not. It will take a

long time to get feedback for their homework (or even no feedback), which is inefficient. However, in a flipped class, students' questions and confusions will get addressed instantly, individually, and specifically. More importantly, by working with other group members, they start to communicate about mathematics using their own language. They ask, explain, argue, and present. This is a significant training for the students not only for mathematics learning, but also in other subjects, and even for their life after college.

After class, students were expected to finish regular homework that was assigned from the book. Most of the homework questions were similar to the problem sets they have practiced in class. The homework was required to be completed independently and was graded.

The Technology Used in Flipping

Technology played an important role in flipping. We have explored and used different types of technologies in this project, which are summarized below.

Video creating software and hardware. There is no doubt that creating the flipping videos is the most time-consuming and challenging job in a flipping project. Many instructors hesitated in adopting flipping because of the difficulty in making videos. We tried different software and hardware and finally adopted Camtasia to create and edit the flipping videos. This is a kind of screencast software and it can trim and join video clips. Using the animations in Microsoft PowerPoint, we first created a video clip for each slide. Then, we edited the video clips and added effects, such as highlighting and text comments. Finally, we joined these clips together using Camtasia. We used regular laptops to create the videos. The member from the United States used a tablet Personal Computer (PC) that can write on the screen. It had a better effect, i.e., students appreciated the systematic handwriting problem-solving process.

Each video did not exceed 10 minutes. The aim of the videos was not to teach the students all the material of one section. Instead, we just wanted to let them master some basic definitions, theorems, and skills. The main learning process was still going to happen in classroom. Each video had at least one or two examples. We used animations of PowerPoint to show systematic solving.

If a tablet PC is available, a handwriting systematic problem-solving process using the digital pen will be more intuitive and efficient. Some Apps on tablet, such as Explain Everything, can also record screencast videos. They are easier to use and the learning curve is shorter. However, the power of editing on tablets is weaker than using a computer and the video result is not that precise.

Video distribution. We used QQ to distribute the videos of flipping to the students. In the beginning of the semester, a QQ group that included all the students of the class was set up. The instructor frequently used this group to post some extra homework questions or solution sheets, so that the students were used to checking it frequently during the flipping period.

The advantage of using QQ or other social network software as the video distribution tool is that students can ask questions or have a discussion in the group. It is also a very easy and convenient tool. However, it has limitations especially in organizing the videos. In addition, the instructor has to remember to upload the video manually before class and remind the students to watch. It cannot be shared to the public. In a new semester, the instructor will have to repeat this distribution process.

There are a few other ways to distribute videos. For example, some instructors set up a personal "video channel" on public video Websites, such as Youku.com or Youtube.com. If the institution has a class management system like blackboard, it is also a typical tool to manage the videos on these systems. Some

instructor could also put the entrance equation online using the same system.

Class projection. An important idea in our flipping teaching was to have students present their work and hereby lead to a whole class discussion. Toward this end, we needed to project students' work onto the board. Since we were using a regular classroom, we had only one main projector for the classroom, which cannot be connected to tablets, such as iPad. Thus, we brought our own equipment to every class to do a wireless projection on the side of the white wall (see Figure 1).

Figure 1. Using mobile devices to project students' work in a traditional classroom.

The set of equipment included a palm size mini projector, a wireless router (the classroom needs no Internet), an Apple TV, and an iPad Mini. We also used a regular tripod to set up the mini projector, but a tiny mini-tripod would also work for most classrooms, if it was put on the first row of desks. All the equipment fitted in a backpack and they were prewired inside the backpack. Therefore, it took very little time to set up before class. Such mobile equipment made it easy to carry and set up. Recently, mini projectors with built-in WiFi receiver are developed, so that it will require even less equipment.

When we wanted to have students explain their work to the class, we just needed to take a picture of their work using iPad Mini and then projected via Apple TV. We could also use some Apps, such as Notability, to insert the pictures in the problem sets and write comments on it. After class, we could send the notes to the whole class via QQ.

Student response (voting) system. In addition to group work, we also adopted an instant student response system (voting system) to get them more actively involved by individual work. Here, we used the free system Plickers. We only needed to print out the respond card and then used the App on iPad to scan and project the results instantly on screen (see Figure 2).

The instructor can get prompt instant feedback from the students by using this system. Thus, he or she can adjust the teaching material based on the result. Meanwhile, students get more engaged in class. Based on the experience, they really enjoyed the time when we did "Plickers." It is obviously a powerful tool to promote active learning. Moreover, it can increase faculty-students' interaction and foster students' cooperation and

collaboration. “Good questions”¹ (Mille, Santana-Vega, & Terrell, 2006; Pilzer, 2001; McGivney & Xue, 2013) provoke rich discussions and allow students to arrive at a better understanding of key concepts.

Figure 2. “Plickers” in a class.

The Sequel Project

In the following Spring semester of 2016, the instructor of the flipped class independently continued to flip part of her Calculus II classes. She developed a set of videos and in-class problem sets. This time she flipped both of her two classes and the students all appreciated this new teaching method. Meanwhile, the instructor felt more comfortable and confident during the second semester flipping. The time and effort in preparing the material was also considerably reduced in this round. It showed that once the instructor has found and mastered the necessary technology, the flipping process would be a lot smoother and more efficient.

Analyses of the Study

During the Fall semester of 2015, one coauthor of this paper taught two back-to-back honors sections of Calculus I. Class A had 44 students majoring in Mechanical Engineering and Automation, and Class B had 45 students majoring in Software Engineering. During the first eight weeks, both classes were taught in a traditional lecturing method. Then, starting from the ninth week, we flipped Class A for about five weeks. The materials flipped were Indefinite Integrals, Definite Integrals, and the Application of Integrals. Class B, the control group, was kept taught in non-flipping lecturing method.

Table 1

Class A Exams (Flipped Class)

	Exam 1 (non-flipped class)	Exam 2 (flipped class)
Average (SD)	73.18 (16.1)	73.68 (16.58)
Median	77	75

¹ Over 2,000 multiple choices or true-false classroom voting questions for many college level mathematics courses, including Calculus, Linear Algebra, Differential Equations, and Statistics, can be found at the Website: <http://mathquest.carroll.edu/>.

Table 2

Class B Exams (Non-Flipped Class)

	Exam 1 (non-flipped class)	Exam 2 (non-flipped class)
Average (<i>SD</i>)	68.67 (20.05)	61.44 (22.42)
Median	74	60

We gave both classes two mid-semester exams. Exam 1 was given right before the flipping period to test their learning about the material before flipping, and Exam 2 was given right after the flipping period to test the flipping effect. The statistical results are summarized in Tables 1 & 2.

We can see that Class A in general is better than Class B. When both classes used the traditional lecturing method, in Exam 1, Class A is 4.51 points higher in average and 3 points higher in median than Class B. When Class A was flipped for the relatively more difficult part, while Class B still used traditional method, in Exam 2, Class A is 12.24 points higher in average and 15 points higher in the mean than Class B. Obviously, the flipped class did a lot better in learning for the chosen parts of the material.

In addition to the mid-semester exams, we have also given a survey to the flipped class to obtain students' feedback about flipping. Forty students finished the anonymous survey. In one question, we let the students rated the extent to which the videos helped them learn the material on a scale of zero (not at all) to five (a lot). The average response was 3.45. Then, we asked what did they like most about the videos. Students most often said that the flipping videos were a lot easier and more efficient in comparison to reading the book in previewing. They could pause and replay to study at their own pace. They appreciated that they could get the big idea of the section before class. They also liked the idea that there was a summary at the end of each video. With respect to what they disliked or how to improve the flipping videos, they would like more dynamical and graphical explaining rather than still PowerPoint slides, more systematic examples, and more different types of examples.

It is interesting that only 5% of students took notes while they watched the videos. In a similar flipped class in the United States, most students took notes when watching videos. We believe one reason is that, in China, most students watch the videos using cell phones, so they have to hold the phone, and in the United States, we found most students watch the videos using laptops or tablets, so they have their hands free.

The majority of students felt that the flipping teaching method was more efficient and enjoyable compared to traditional methods. They appreciated that they could work in class, when the instructor was around to answer their questions instantly. Also, they agreed that having the big idea in mind helped them learn better in class. They liked the group discussion and the instant response system (Plickers). Because, in general, the students in China do not have too much experience in working in groups, they appreciated the training of teamwork. They felt they get better self-learning ability. Flipping has definitely increased student-instructor interactions. Meanwhile, many students pointed out that to flip better, the hardware, such as classroom tables, projectors, and WiFi signals in campus, needed improvement. Moreover, they also hoped to have more different problems types, and more problems outside of the textbook to be included in the in-class problem sets.

Conclusion

From this one-year flipping pilot experiment in Calculus I and II, we learned the following lessons:

1. Flipping as a new teaching method is able to be implemented in universities in China by using mobile technology. However, in order to achieve better result in flipping, hardware and software updates and investments are necessary;

2. We are able to cover all the material using flipping as in a non-flipped class;
3. There exist some evidences showing that students have better learning outcomes in a flipped classroom;
4. Students are more active and engaged in a flipped class. Comparing other countries, like the United States, the instructor of a flipped class in China needs to pay more attention to prepare and work in leading the students' group work, discussion, and presentation;
5. Creating good videos are time-consuming. However, students appreciate the videos made by their own instructor. They feel more comfortable for the same style explaining in the videos and in class;
6. It is supportive and helpful if there is a group of faculty members to start the flipping project. Moreover, after the first semester, when a faculty has explored all the technology and challenges, he/she can continue flipping independently.

Overall, our project in flipping Calculus has been successful. We feel it is a more efficient and effective way in teaching. Students performed better in a flipped class. More importantly, they got training in independent learning, teamwork, and presentation, which are very important skills for their future lives. Therefore, it is definitely worth for our time and effort in exploring this new teaching method. We will surely continue to flip more materials and more classes. We also look forward to the day when more colleagues will start to adopt flipping in their teaching.

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