Convex Optimization-Based Rotation Parameter Estimation Using Micro-Doppler

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Abstract: We present a novel algorithm that can determine rotation-related parameters of a target using FMCW (frequency modulated continuous wave) radars, not utilizing inertia information of the target. More specifically, the proposed algorithm estimates the angular velocity vector of a target as a function of time, as well as the distances of scattering points in the wing tip from the rotation axis, just by analyzing Doppler spectrograms obtained from three or more radars. The obtained parameter values will be useful to classify targets such as hostile warheads or missiles for real-time operation, or to analyze the trajectory of targets under test for the instrumentation radar operation. The proposed algorithm is based on the convex optimization to obtain the rotation-related parameters. The performance of the proposed algorithm is assessed through Monte Carlo simulations. Estimation performance of the proposed algorithm depends on the target and radar geometry and improves as the number of iterations of the convex optimization steps increases.

Key words: Micro-Doppler, FMCW radar, STFT (short-time Fourier transform), convex optimization, rotation parameter.

1. Introduction

To find the characteristics of a target with a radar, we transmit probing signal to the target, and then measure the time-delay and Doppler frequency shift from the echo signal back from the target, to estimate the position and velocity of the target. This is the conventional use of the radar. Recently, however, radars are also being used to analyze the micro-motion of a target using micro-Doppler information extracted from the echo signal.

The relationship between the micro-motion and micro-Doppler frequency has been derived analytically [1-2] for each micro-motion such as rotation, precession and nutation. In the application side, micro-Doppler information obtained from the Doppler spectrogram has been used to classify human activities or targets [3-4] using SVM (support vector machine)-based algorithms. Furthermore, micro-motion parameters have been estimated utilizing the inertia of the target [5-6].

In this paper, we analyze the relationship between the rotation of the wing or rotor and the micro-Doppler frequency obtained from Doppler spectrogram, and propose a rotation parameter estimation algorithm. Three or more monostatic radars are needed and the maximum Doppler frequency and angular velocity are extracted from the Doppler spectrogram of each radar. The proposed algorithm is based on the convex optimization to obtain the rotation-related parameters.

This paper is organized as follows. FMCW radar signal modeling and a flow of generating Doppler spectrogram are described in Section II. The proposed algorithm is given in Section III. Performance of the proposed algorithm is demonstrated in Section IV through simulation, and conclusion is made in Section V.

2. Doppler Spectrogram Generation

Doppler spectrogram extracted from the range-Doppler map is widely used to analyze the distribution of Doppler frequency as a function of time for the motion analysis or activity classification. In this

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section, we briefly review FMCW radar signal and the procedure for Doppler spectrogram generation.

2.1 FMCW Radar Signal Modeling

In the case of an FMCW radar, transmitted signal $s(m, t_s)$ and received signal $s_r(m, t_s)$ scattered from the target may be modeled as Eq. (1) [7]:

$$\begin{aligned} s(m, t_s) &= \exp\left(j2\pi\left(f_c t_s + \frac{1}{2}\alpha t_s^2\right)\right), \\ s_r(m, t_s) &= \exp\left(j2\pi\left(f_c(t_s - \tau(t)) + \frac{1}{2}\alpha(t_s - \tau(t))^2\right)\right) \\ 0 \leq t_s < T_{PRI}, \quad t &= mT_{PRI} + t_s, \quad m = 0, \dots, M-1 \end{aligned} \quad (1)$$

where m, f_c, t_s, α and T_{PRI} are respectively, index of the pulse, the center frequency, slow-time, chirp rate and PRI (pulse repetition interval). The time delay $\tau(t)$ is assumed to be slowly changing, i.e., there is no range cell migration during the coherence processing interval (CPI), MT_{PRI} , where M is the number of coherently integrated pulses. Note that $\tau(t)$ can be written as $\tau(t) = 2r(t)/c \approx 2(r_0 + v_r t)/c$ for a monostatic radar. Also $r(t)$, r_0 and c denote, respectively, the range to the target as a function of time, initial range and speed of light. Radial velocity is denoted with v_r .

Now the deramped signal will be

$$\begin{aligned} s_d(m, t_s) &= s(m, t_s) \cdot (s_r(m, t_s))^* \\ &= \exp\left(j2\pi\left(f_c \tau(t) - \frac{1}{2}\alpha \tau^2(t)\right)\right) \exp(j2\pi \alpha \tau(t) t_s) \end{aligned} \quad (2)$$

2.2 Range and Doppler Estimation

In Eq. (2), $\alpha \tau(t)$ is the beat frequency f_b , which is proportional to the range to the target. Note that $\exp(j2\pi(f_c \tau(t) - 1/2 \alpha \tau^2(t)))$ may be considered as a constant. Therefore Eq. (2) may be written as

$s_d(m, t_s) = A \exp(j2\pi f_b t_s)$, and f_b can be found by applying the Fourier transform with respect to t_s . The range can be found from f_b by $r(t) = cf_b / 2\alpha$.

Doppler frequency can be obtained by applying the

Fourier transform to $s_d(m, t_s)$ with respect to $m (= 0, \dots, M-1)$ for constant t_s' where $s_d(m, t_s')$ is the maximum, i.e., at the target position. Because we assume that there is no range cell migration, t_s' may be considered identical for all m . Now the Fourier transform of $s_d(m, t_s')$ becomes:

$$FT(s_d(m, t_s')) = \sum_{m=0}^{M-1} s_d(m, t_s') \exp\left(\frac{-j2\pi km}{M}\right) \quad (3)$$

Since $f_d \approx f_c \frac{2v_r}{c}$, the radial velocity v_r can be found from f_d .

2.3 STFT (Short-Time Fourier Transform)

If the target has micro motion such as rotation, vibration, then its Doppler frequency also changes in a short time [1]. Hence, to obtain the micro motion information, the STFT needs to be applied for $M_{STFT} T_{PRI}$ where t_s' is constant. Since the resolution of Doppler frequency is proportional to $1/M_{STFT} T_{PRI}$, frequency resolution of STFT is much poorer than that of FT. To compensate this, we pad zeros at the end of $s_d(m, t_s')$, ($m = 0, \dots, M-1$) when the STFT is applied.

An example of the Doppler spectrogram is given in Fig. 1. This figure corresponds to the signal obtained from a radar located at $(1000, 0, 0)^T$ for a missile located at $(0, 0, 0)^T$. The missile is assumed to have a single scatterer at the edge of a 50cm-length wing, and rotate with the angular velocity of $20\pi \text{ rad/s}$ (10 Hz) with respect to $(0, 0, 1)^T$.

3. Rotation Parameter Estimation

We shall describe the proposed convex optimization-based rotation parameter estimation algorithm using the Doppler spectrogram and the relationship between Doppler frequency and object's dynamics.

3.1 Rotation Parameter

First, the position of the target is assumed to be known, for example by using the trilateration with three radars. Let us consider two coordinate systems;

Fig. 1 Doppler spectrogram.

Fig. 2 The reference frame and the local frame.

the reference coordinate frame and the local coordinate frame that have the same origin which is at the center of the gravity of the target.

We may regard the reference frame as the navigation frame (X, Y, Z) which is completely determined when the target position is known. The local frame is the body frame (x, y, z) which is attached on the target, and therefore, changes with respect to the heading of target. See Fig. 2 [8].

Referring Fig. 2, note that the target's angular velocity vector is $(0, 0, z_r)^T$ and the scatterer located at $(x_s, 0, 0)^T$ rotates around $(0, 0, z_r)^T$ in the local coordinate frame. These two vectors can be rewritten in reference coordinate using coordinate transform:

$$\begin{aligned}\bar{\omega} &= (X_r, Y_r, Z_r)^T = R_{rot} (0, 0, z_r)^T \\ \bar{r} &= (X_s, Y_s, Z_s)^T = R_{rot} (x_s, 0, 0)^T\end{aligned}\quad (4)$$

where R_{rot} is the direction cosine matrix (rotation matrix) that represents the attitude of the local frame with respect to the reference frame [8].

Let the angular velocity vector and scatterer point vector in reference frame be $\bar{\omega}$ and $\bar{r}$, respectively. It will be convenient to decompose $\bar{\omega}$ and $\bar{r}$ into their norms $|\bar{\omega}|$ and $|\bar{r}|$, and the unit vectors $\bar{\omega}/|\bar{\omega}|$ and $\bar{r}/|\bar{r}|$. The magnitude $|\bar{\omega}|$ will be estimated from the Doppler spectrogram, and the unit vectors $\bar{\omega}/|\bar{\omega}|$ and $\bar{r}/|\bar{r}|$ are estimated using convex optimization. Before applying the convex optimization, we need to initialize $\bar{r}/|\bar{r}|$ appropriately. Detailed description of the algorithm is given in subsection 3.2.

3.2 Convex Optimization-Based Rotation Parameter Estimation

We shall explain the detailed procedure in four steps.

(1) Calculate $|\vec{\omega}|$ from the Doppler spectrogram

As shown in Fig. 1, Doppler variation appears in Doppler spectrogram. Since we are interested in the micro-motion only, the variation of micro Doppler is sinusoidal. From the Doppler spectrogram, 1D time-frequency signal can be obtained using the morphological technique [5]. As a result, if we extract the frequency component of 1D time-frequency signal, we can estimate $|\vec{\omega}|$. This well-known procedure is omitted in this paper.

(2) Set the initial value $|\vec{r}|$

The magnitude of scatterer point vector $|\vec{r}|$ differs depending on the size or type of the missiles. In our simulation study, the initial value of $|\vec{r}|$ is set as 5 cm or 10 cm.

(3) Calculate $\vec{\omega}/|\vec{\omega}|$ and estimation errors based on convex optimization

Now the maximum Doppler frequency obtained from Doppler spectrogram is used to calculate $\vec{\omega}/|\vec{\omega}|$ and its estimation error. Consider the i th radar. See Fig. 3. Since the i th Doppler frequency $f_{d,i}$ is proportional to the i th radial velocity $v_{r,i}$ between the i th radar and the target, we can write the i th Doppler frequency as:

$$f_{d,i} = 2 \frac{f_c}{c} v_{r,i} \approx 2 \frac{f_c}{c} \vec{x}_i^T \cdot (\vec{\omega} \times \vec{r}) \quad (5)$$

where $\vec{x}_i$ is the unit vector directed from the center of gravity of the target to the i th radar. The approximation present in Eq. (5) is due to that fact that the distance between the center of the target and the scatterer point is much closer than the distance between the center of the target and the radar.

As can be seen in Eq. (5), when the $|\vec{\omega}|$ and $|\vec{r}|$ are determined, note that the inner product of the $\vec{x}_i$ and $\vec{\omega} \times \vec{r}$ should be maximized to maximize $f_{d,i}$. To obtain the maximum inner product value, we apply convex optimization. In this paper, while changing $\vec{\omega}/|\vec{\omega}|$, $\vec{r}/|\vec{r}|$ is estimated to maximize $f_{d,i}$ and

estimation error compared to measured maximum Doppler frequency is calculated. At this step, to estimate $\vec{\omega}/|\vec{\omega}|$ and $\vec{r}/|\vec{r}|$ efficiently, setting initial $\vec{\omega}/|\vec{\omega}|$ close to true $\vec{\omega}/|\vec{\omega}|$ is very important. Initial $\vec{\omega}/|\vec{\omega}|$ can be computed for three radars as:

$$\begin{aligned} \begin{bmatrix} \vec{x}_1^T \\ \vec{x}_2^T \\ \vec{x}_3^T \end{bmatrix} \cdot \vec{\omega}/|\vec{\omega}| &= \begin{bmatrix} \cos \theta_1 \\ \cos \theta_2 \\ \cos \theta_3 \end{bmatrix} \rightarrow \begin{bmatrix} \vec{x}_1^T \\ \vec{x}_2^T \\ \vec{x}_3^T \end{bmatrix} \cdot \vec{\omega}/|\vec{\omega}| \approx \begin{bmatrix} 1/f_{d,1} \\ 1/f_{d,2} \\ 1/f_{d,3} \end{bmatrix} \\ \rightarrow \vec{\omega}/|\vec{\omega}| &\approx \begin{bmatrix} \vec{x}_1^T \\ \vec{x}_2^T \\ \vec{x}_3^T \end{bmatrix}^{-1} \begin{bmatrix} 1/f_{d,1} \\ 1/f_{d,2} \\ 1/f_{d,3} \end{bmatrix} \end{aligned} \quad (6)$$

Eq. (6) uses the relationship between radial velocity and its Doppler frequency, where θ_i is the angle between $\vec{x}_i$ and $\vec{\omega}/|\vec{\omega}|$.

Since the initial $\vec{\omega}/|\vec{\omega}|$ calculated from Eq. (6) is near to the true $\vec{\omega}/|\vec{\omega}|$, $\vec{\omega}/|\vec{\omega}|$ is modified by adding small amount of $\Delta x, \Delta y, \Delta z$ to the initial $\vec{\omega}/|\vec{\omega}|$. In our simulation, $\Delta x, \Delta y, \Delta z$ are set to ± 0.5 .

$$\vec{\omega}/|\vec{\omega}|' = \frac{\vec{\omega}/|\vec{\omega}| + [\Delta x \ \Delta y \ \Delta z]^T}{\left\| \vec{\omega}/|\vec{\omega}| + [\Delta x \ \Delta y \ \Delta z]^T \right\|} \quad (7)$$

Now, we can formulate a convex optimization problem [9] for each radar:

$$\begin{aligned} \min_{\vec{r}} [-f_{d,i}(\vec{\omega}, \vec{r})] &= \min_{\vec{r}} \left[-2 \frac{f_c}{c} \vec{x}_i^T \cdot (\vec{\omega} \times \vec{r}) \right] \\ &= \min_{\vec{r}} \left[-2 \frac{f_c}{c} |\vec{\omega}| |\vec{r}| \vec{x}_i^T \cdot (\vec{\omega}/|\vec{\omega}| \times \vec{r}/|\vec{r}|) \right] \\ \text{subject to } (\vec{\omega}/|\vec{\omega}|)^T (\vec{r}/|\vec{r}|) &= 0 \end{aligned} \quad (8)$$

Note that the maximization of $f_{d,i}(\vec{\omega}, \vec{r})$ has been changed to the minimization of $-f_{d,i}(\vec{\omega}, \vec{r})$ in Eq. (8). We use predetermined values for $|\vec{\omega}|, |\vec{r}|, \vec{\omega}/|\vec{\omega}|$ as explained before. Therefore, $f_{d,i}(\vec{\omega}, \vec{r})$ depends only on $\vec{r}_i/|\vec{r}_i|$.

We define the estimation error ε as:

$$\varepsilon = \frac{1}{N} \sum_{i=1}^N \left| \frac{f_{d,\max,i}^{est} - f_{d,\max,i}^{mea}}{f_{d,\max,i}^{mea}} \right| \quad (9)$$

where $f_{d,\max,i}^{est}$ and $f_{d,\max,i}^{mea}$ are respectively, the estimated

Fig. 3 Positions of radars and a target in the reference frame.

Table 1 Target characteristic.

Variables	Scenarios	
	1	2
Angular velocity vector	$20\pi(0,0,1)^T$	$20\pi\left(0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)^T$
Norm of scatterer point vector	3, 5, 10 [cm]	

and the measured Doppler frequency at the i th radar. The error ε is calculated whenever $\bar{\omega}/|\bar{\omega}|'$ changes and consider $\bar{\omega}/|\bar{\omega}|'$ which has the smallest ε as final estimation angular unit vector for $|\bar{r}|$. The convex optimization problem can be solved with any widely available package.

(4) Change $|\bar{r}|$ and calculate estimation error for each $|\bar{r}|$

In the previous step, a quite accurate angular unit vector can be obtained. In this step, $|\bar{r}|$ is altered based on this vector. As varying $\bar{\omega}/|\bar{\omega}|'$, the unit scatterer point vector is estimated by comparing ε for various $|\bar{r}|$ candidates. If ε is almost the same for varying $|\bar{r}|$, then reestimate $\bar{\omega}/|\bar{\omega}|'$ for $|\bar{r}|$ which has the smallest ε . Since $\bar{r}/|\bar{r}|$ varies quite rapidly compared to other rotation parameters, estimating $\bar{r}/|\bar{r}|$ during very short time is meaningless. So, in this paper, $\bar{r}/|\bar{r}|$ is used as a convex optimization variable only.

4. Simulation Results

4.1 Simulation Environments

Three monostatic FMCW radars are located as shown in Fig. 3. They are assumed to be operated at

10 GHz center frequency, with 750 MHz bandwidth, 1.5 GHz sampling frequency, $52.6\mu s T_{PRI}$ and 16 coherent pulses for Doppler frequency estimation.

Table 1 shows the true target characteristic for two scenarios.

4.2 Simulation Results

First, $|\bar{r}|$ is estimated at an interval of 1 cm. As shown in Fig. 4, for both scenarios, the relative error becomes the minimum at true $|\bar{r}|$. Note that there is estimation error even for noise-free environment. This is due to the resolution limitation of $\Delta x, \Delta y, \Delta z$ used at $\bar{\omega}/|\bar{\omega}|'$ estimation. If $\Delta x, \Delta y, \Delta z$ becomes smaller or convex optimization is run enough, the estimation error will also become smaller.

Fig. 5 shows the RMSE (root mean squared error) of $\bar{\omega}$ estimation for different iteration numbers of convex optimization. Iteration number is counted when all the ε for all $\bar{\omega}/|\bar{\omega}|'$ are compared. Thus, at the next iteration, $\bar{\omega}/|\bar{\omega}|'$ which has the smallest ε is set to the initial $\bar{\omega}/|\bar{\omega}|'$. As can be seen in Fig. 5, the RMSE of $\bar{\omega}$ becomes smaller, as the iteration number increases. These results also demonstrate that difference of the geometry affects the convergence

Fig. 4 Plots of ε versus $|\vec{r}|$ when true $|\vec{r}|$ is 3, 5, 10 cm for scenarios 1, 2.

Fig. 5 Estimation error and RMSE of $\hat{\omega}$ versus iteration numbers for scenarios 1 and 2.

Fig. 6 RMSE of $\hat{\omega}$ versus position error for scenario 1.

Fig. 7 RMSE of $\hat{\omega}$ with respect to frequency error for scenario 1.

speed of the proposed technique.

Figs. 6 and 7 depict the effect of measurements error in scenario 1. Fig. 6 is for the position error, and Fig. 7 is for the Doppler frequency error. The position error is set to 10^{-2} [m], and the frequency error to 10^2 [Hz]. The number of iterations for Monte Carlo trials is 100. Since the distance between a radar to the target is 1,000 m, the corresponding position error level may be considered to be small. However, in the case of frequency errors, note that they are relatively large.

5. Conclusions

We present a convex optimization-based rotation parameter estimation algorithm using micro-Doppler without knowing inertial characteristic. As the number of iterations increases, the estimation performance of the proposed algorithm improves. Also, the convergence rate of the proposed algorithm is related to the geometry of radars and target. Additionally, the performance of the proposed technique is related to the position and Doppler frequency measurement accuracy.

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