Lateral Displacement of Retaining Walls

José Medina, Nicolás Sau and Jesús Quintana

Department of Civil Engineer and Mining, University of Sonora, Hermosillo, Sonora 83000, México

Abstract: When Rankine or Coulomb theories to design of retaining wall are used, it is accepted beforehand that the retaining wall will experience a lateral displacement. This displacement is normally not calculated when a retaining wall is designed. This paper describes a method to estimate the lateral displacement of retaining walls. A practical example in the lateral displacement of a gravity retaining wall is presented.

Key words: Lateral displacement, retaining wall, passive earth thrust, active earth thrust, Rankine theory, Coulomb theory.

1. Introduction

When Rankine or Coulomb theories are used to design a retaining wall, it is accepted beforehand that the retaining wall will experience a lateral displacement. By allowing the wall to move laterally, the earth pressure decreases on the active side of the wall and earth pressure increases on the passive side, and therefore more economical walls can be obtained when the lateral displacement is allowed. This lateral displacement is usually not calculated when designing a retaining wall. This article describes a method that estimates the lateral displacement of retaining walls.

2. Passive Side Reaction Equilibrium

When the wall slides laterally the active earth thrust develops and the frictional strength is mobilized at the base of the wall. The reaction on the passive side is determined from the balance of forces in the horizontal direction, by the Eq. (1), Fig. 1:

$$R_{EH} = E_{AH} - F_F \quad (1)$$

where, E_{AH} is the horizontal component of active earth thrust, F_F is the friction force that develops at the base of the wall and R_{EH} is the equilibrium reaction that takes place on the passive side of the wall.

Soil-retaining wall contact pressure on the passive side is obtained from Eq. (2):

$$Q_E = R_{EH} / D_F \quad (2)$$

where, Q_E is soil-wall contact pressure on the passive side wall and D_F is the depth of the retaining wall foundation.

3. Calculation of the Lateral Displacement

The lateral displacement of the retaining wall is obtained from Eq. (3), Zeevaert [1]:

$$\delta h = \sum_{i=1}^n M_{hi} \Delta \sigma_{hi} \Delta x \quad (3)$$

where, δh is the lateral displacement of the retaining wall, M_{hi} is the soil horizontal deformation module in section i , Δx is the width of the sections considered, $\Delta \sigma_{hi}$ is the increase in the horizontal stress at the center of section i and n is the number of sections considered in the calculation of the horizontal displacement (Fig. 2).

The increment in the horizontal stress, at the center of the retaining wall and at one half of D_F is obtained with Eq. (4), Medina [2]:

Fig. 1 Horizontal reaction on the passive side of the wall.

Fig. 2 Lateral displacement of the retaining wall.

$$\Delta\sigma_h = Q_E I_{FH} \quad (4)$$

$$I_{FH} = \frac{3C1}{2\pi} \left(\sin \alpha_0 - \frac{\sin^3 \alpha_0}{3} \right) \times \{ (\psi_1 - \psi_2) + \sin(\psi_1 - \psi_2) \times \cos(\psi_1 + \psi_2) \} +$$

$$\frac{3C2}{2\pi} \left(\sin \alpha'_0 - \frac{\sin^3 \alpha'_0}{3} \right) \times \{ (\psi'_1 - \psi'_2) + \sin(\psi'_1 - \psi'_2) \times \cos(\psi'_1 + \psi'_2) \} \quad (5)$$

$$\alpha_0 = \tan^{-1} \left\{ \frac{L}{2x} \right\}; \quad \psi_1 = \tan^{-1} \left\{ \frac{D_F}{2x} \right\}; \quad \psi_2 = \tan^{-1} \left\{ \frac{-D_F}{2x} \right\} \quad (6)$$

$$\alpha'_0 = \tan^{-1} \left\{ \frac{L}{2\sqrt{D_F^2 + x^2}} \right\}; \quad \psi'_1 = \tan^{-1} \left\{ \frac{3D_F}{2x} \right\}; \quad \psi'_2 = \tan^{-1} \left\{ \frac{D_F}{2x} \right\} \quad (7)$$

where, $\Delta\sigma_h$ is the increase in the horizontal stress, I_{FH} is the influence value at the center of the retaining wall and at one half of D_F , L is the retaining wall length, x is the horizontal distance from the origin of the X axis to the center of the corresponding section (Fig. 2), $C1$ and $C2$ are coefficients depending on Poisson's ratio of the material and are obtained from Fig. 3.

4. Example

4.1 Problem Statement

For the retaining wall shown in Fig. 4, calculate:

- (1) Lateral displacement at the center of the wall;
- (2) Lateral displacement at the edge of the wall;
- (3) Lateral displacement at the center of the wall for the passive earth thrust;

(4) Lateral displacement at the edge of the wall for the passive earth thrust.

Consider the following:

- $B = 3.40$ m, $b = 2.20$ m, $H = 6.50$ m, $D_F = 1.20$ m;
- Wall with vertical drain over an impervious material;
- Retaining wall with a unit weight of 21.60 kN/m³;
- Uniform overload, $q = 15$ kN/m²;
- Unit weight of saturated soil, $\gamma_{\text{sat}} = 20.85$ kN/m³;
- Angle of internal friction of soil, $\phi = 36^\circ$;
- Soil cohesion, $C = 0$ kN/m²;
- Soil Poisson's ratio, $\nu = 0.30$;
- Soil-wall friction angle, $\delta = 12^\circ$;
- Supporting soil-wall friction coefficient, $\mu = 0.36$;

Fig. 3 Coefficients C1 and C2 as functions of the Poisson's ratio.

Fig. 4 Example retaining wall.

- Soil deformation modulus, $M_h = 0.0006 \text{ m}^2/\text{kN}$, considered constant;
- Retaining wall length, $L = 20 \text{ m}$.

4.2 Solution

4.2.1 Active Earth Thrust

The earth thrust against the retaining wall, for the active case, is given by Eq. (8), Lambe and Whitman [3]:

$$E_a = \frac{(W - U \cos \beta) \tan(\beta - \phi) + U \sin \beta}{\sin \theta + \tan(\beta - \phi) \cos \theta} \quad (8)$$

where, E_a is the earth thrust against the retaining wall for the active case, W is the weight of the wedge which includes the load q on it, U is the porewater force acting against the sliding surface, P is the resultant of the friction force on the failure plane and normal force on this plane, ϕ is the angle of internal friction of soil, β is the angle between the failure plane and the horizontal, and θ is the angle between the thrust E_a line of action and the vertical (Fig. 5).

The weight of the wedge plus the overload q on it, is obtained from Eq. (9):

$$W = \left[\frac{\gamma_m H}{2} + q \right] \left(\frac{H}{\tan \beta} \right) \quad (9)$$

Fig. 5 Wedge free body diagram, active case.

where, γ_m is the soil unit weight, H is the height of the retaining wall and q is the charge per unit area due to the overload acting above the active fill.

The porewater force U is obtained from Eq. (10), Medina et al. [4]:

$$U = k H^2 \quad (10)$$

where, k is a coefficient depending on the angle β (Table 1).

Table 1 shows the calculation of active earth thrust against the retaining wall.

From Table 1, the active earth thrust, E_A is:

$$E_A = 161.591 \text{ kN/m}$$

The horizontal component of the active thrust is:

$$E_{AH} = E_A \cos \delta = 158.059 \text{ kN/m}$$

4.2.2 Passive Earth Thrust

The earth thrust against the retaining wall, for the passive case, is given by Eq. (11) obtained from the balance of forces in the vertical and horizontal direction of the failure wedge shown in Fig. 6:

$$E_p = \frac{W \tan(\beta + \phi)}{\sin \theta + \tan(\beta + \phi) \cos \theta} \quad (11)$$

Table 2 shows the calculation of passive earth thrust.

From Table 2, the passive earth thrust, E_p is:

$$E_p = 91.279 \text{ kN/m}$$

The horizontal component of the passive thrust is:

$$E_{PH} = E_p \cos \delta = 89.284 \text{ kN/m}$$

4.2.3 Friction Force at the Base of the Wall

The frictional force at the wall base is obtained from Eq. (12):

$$F_F = \mu W_m \quad (12)$$

where, F_F is the frictional force at the wall base, W_m is the weight of the wall and μ is the friction coefficient between the wall and the supporting soil.

$$W_m = 408.672 \text{ kN/m}$$

$$F_F = 147.122 \text{ kN/m}$$

4.2.4 Equilibrium Reaction Passive Side

The equilibrium reaction on the passive side of the wall is obtained from Eq. (1); and the soil-wall contact pressure on the passive side is obtained from Eq. (2):

$$R_{EH} = 10.938 \text{ kN/m};$$

$$Q_E = 9.115 \text{ kN/m}^2$$

4.2.5 Lateral Displacement at the Wall Center

In the calculation of the lateral displacement, 8 sections of the same width, $\Delta x = 2.40 \text{ m}$ were considered. Table 3 shows the calculation of the lateral displacement at the center of the retaining wall.

From Table 3, the lateral displacement at the center of the retaining wall is the following:

$$\delta h_c = \sum_{i=1}^8 \Delta \delta h_i = 0.90 \text{ cm}$$

Table 1 Calculation of active earth thrust.

β_i (°)	W_i (kN/m)	k_i (kN/m ³)	U_i (kN/m)	E_{Ai} (kN/m)
53	405.379	1.657	70.021	160.289
54	390.848	1.578	66.658	160.840
55	376.681	1.496	63.207	160.987
56	362.856	1.446	61.100	161.591
57	349.353	1.367	57.753	161.127
58	336.152	1.301	54.957	160.668
59	323.237	1.266	53.468	160.678

Fig. 6 Wedge free body diagram, passive case.

Table 2 Calculation of the passive earth thrust.

β_i (°)	W_i (kN/m)	E_{pi} (kN/m)
17	49.102	92.790
18	46.202	91.898
19	43.598	91.401
20	41.245	91.279
21	39.108	91.522
22	37.156	92.129
23	35.366	93.113

Table 3 Lateral displacement calculation at the center of the retaining wall.

Section	1	2	3	4	5	6	7	8
Xi (m)	1.2000	3.6000	6.0000	8.4000	10.8000	13.2000	15.6000	18.0000
α_0 (rad.)	1.4514	1.2252	1.0304	0.8721	0.7470	0.6483	0.5700	0.5071
ψ_1 (rad.)	0.4636	0.1651	0.0997	0.0713	0.0555	0.0454	0.0384	0.0333
ψ_2 (rad.)	-0.4636	-0.1651	-0.0997	-0.0713	-0.0555	-0.0454	-0.0384	-0.0333
α_0' (rad.)	1.4027	1.2081	1.0217	0.8672	0.7439	0.6464	0.5687	0.5062
ψ_1' (rad.)	0.9828	0.4636	0.2915	0.2111	0.1651	0.1355	0.1149	0.0997
ψ_2' (rad.)	0.4636	0.1651	0.0997	0.0713	0.0555	0.0454	0.0384	0.0333
I_{FH}	2.8269E-01	1.3759E-01	8.5547E-02	5.9235E-02	4.3334E-02	3.2852E-02	2.5598E-02	2.0405E-02
$\Delta\sigma_{hi}$ (kN/m ²)	2.5766	1.2541	0.7797	0.5399	0.3950	0.2994	0.2333	0.1860
$\Delta\delta_{hi}$ (cm)	0.3710	0.1806	0.1123	0.0777	0.0569	0.0431	0.0336	0.0268

Table 4 Lateral displacement calculation at the center of a 40 m long wall.

Section	1	2	3	4	5	6	7	8
xi (m)	1.200	3.600	6.000	8.400	10.800	13.200	15.600	18.000
α_0 (rad.)	1.5109	1.3927	1.2793	1.1732	1.0757	0.9874	0.9084	0.8380
ψ_1 (rad.)	0.4636	0.1651	0.0997	0.0713	0.0555	0.0454	0.0384	0.0333
ψ_2 (rad.)	-0.4636	-0.1651	-0.0997	-0.0713	-0.0555	-0.0454	-0.0384	-0.0333
α_0' (rad.)	1.4861	1.3833	1.2739	1.1695	1.0731	0.9855	0.9069	0.8369
ψ_1' (rad.)	0.9828	0.4636	0.2915	0.2111	0.1651	0.1355	0.1149	0.0997
ψ_2' (rad.)	0.4636	0.1651	0.0997	0.0713	0.0555	0.0454	0.0384	0.0333
I_{FH}	2.8272E-01	1.3830E-01	8.7932E-02	6.3580E-02	4.9245E-02	3.9711E-02	3.2854E-02	2.7663E-02
$\Delta\sigma_{hi}$ (kN/m ²)	2.5769	1.2605	0.8015	0.5795	0.4488	0.3619	0.2994	0.2521
$\Delta\delta_{hi}$ (cm)	0.3711	0.1815	0.1154	0.0834	0.0646	0.0521	0.0431	0.0363

4.2.6 Lateral Displacement at the Edge of the Wall

For the calculation of the lateral displacement on the edge of the retaining wall, first the lateral displacement is calculated in the middle of a wall twice as long; i.e. a wall of 40 m in length. In the calculation of the lateral displacement, 8 sections of the same width, $\Delta x = 2.40$ m were considered. Table 4 shows the calculation of the lateral displacement at the center of a 40 meters long wall.

From Table 4:

$$\delta h = \sum_{i=1}^8 \Delta\delta_{hi} = 0.95 \text{ cm}$$

Therefore, the displacement on the edge of a 20 m length wall will be half the displacement at the center of a 40 m length wall:

$$\delta h_o = 0.48 \text{ cm}$$

4.2.7 Lateral Displacement at the Middle due to Passive Thrust

The lateral displacement at the edge of the wall for

the passive earth thrust, is obtained from Eq. (13):

$$\delta h_{cp} = \delta h_c \frac{E_{PH}}{R_{EH}} \quad (13)$$

E_{PH} is the horizontal component of passive earth thrust, i.e.:

$$E_{PH} = E_p \cos \delta = 89.284 \text{ kN/m}$$

From Eq. (13):

$$\delta h_{cp} = 7.36 \text{ cm}$$

4.2.8 Lateral Displacement at the Edge due to Passive Earth Thrust

The lateral displacement on the edge of the retaining wall for a contact pressure equal to that provided by the passive thrust is obtained from Eq. (14):

$$\delta h_{op} = \delta h_o \frac{E_{PH}}{R_{EH}} \quad (14)$$

From Eq. (14):

$$\delta h_{op} = 3.87 \text{ cm}$$

4.3 Discussion of Results

The displacement on the edge of the retaining wall is about half the displacement at the center of the retaining wall.

The displacement at the center of a 40 m length wall is slightly larger than the displacement of a wall of 20 m long.

Considering the example discussed, the displacement of the retaining wall, for a contact pressure equal to that provided by the passive thrust, is approximately eight times greater than the lateral displacement of the wall for a contact pressure equal to that provided by the active thrust.

5. Conclusions

It is discussed a procedure for estimating the lateral displacement of a retaining wall, which is an aid to determine the dimensions B and b (Fig. 4) and depth of foundation for wall design.

When a retaining wall is designed, if the project permits, it is very convenient to design the wall so that

it moves in a horizontal direction, decreasing the earth pressure on the active side and increasing the pressure of on the passive side, and therefore a more economic wall is obtained when lateral displacement is allowed.

As the wall moves in the horizontal direction, the earth pressure decreases on the active side and increases on the passive side, so that the balance of forces in the horizontal direction is reached.

References

- [1] Zeevaert, L. 1983. *Foundation Engineering for Difficult Subsoil Conditions*. New York: Van Nostrand Reinhold Company.
- [2] Medina, J. 2010. "Esfuerzo horizontal producido por una carga rectangular horizontal uniforme aplicada en el interior de un sólido." In *Proceedings of XXV Reunión Nacional de la Sociedad Mexicana de Ingeniería Geotécnica* 3: 871-6.
- [3] Lambe, T. W., and Whitman, R. V. 1979. *Soil Mechanics SI Version*. New York: John Wiley & Sons, Inc.
- [4] Medina, J., and Mendoza, N. Sau y E. 2014. "Cálculo de la Fuerza de Presión del Agua en el Análisis de Muros de Contención." XVIII Reunión nacional de Profesores de Ingeniería Geotécnica, Puerto Vallarta, Jalisco, México.