Wavelets and Continuous Wavelet Transform for Autostereoscopic Multiview Images

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Abstract: Recently, the reference functions for the synthesis and analysis of the autostereoscopic multiview and integral images in three-dimensional displays were introduced. In the current paper, we propose the wavelets to analyze such images. The wavelets are built on these reference functions as on the scaling functions of the wavelet analysis. The continuous wavelet transform was successfully applied to the testing wireframe binary objects. The restored locations correspond to the structure of the testing wireframe binary objects.

Key words: 3D display, autostereoscopic display, integral imaging, multiview image processing, wavelets.

1. Introduction

The synthesis and analysis of images are two main concepts of the image processing. Particularly, the real-time processing of the multiview images is probably most important problem of 3D (three-dimensional) imaging and image processing in autostereoscopic multiview and integral displays [1, 2].

The extraction of depth is one of particular problems of the analysis of multiview and integral images [3]. The depth can be extracted from such images by various methods, e.g., the computational reconstruction [4, 5].

The image processing can be based on a set of functions like sin/cos functions in the Fourier transform. For instance, the computer-generated holograms can be synthesized from the hologram patterns [6, 7].

It appears that the autostereoscopic multiview images can be also synthesized from the voxel patterns [8]. The visual observation of 3D images was confirmed in Ref. [8]. The synthesis based on patterns was intended for the real-time autostereoscopic

three-dimensional displays. The patterns were further developed as the reference functions [9]. Using these functions, the structure of multiview images can be analyzed, and locations of voxels of a 3D structure can be extracted. Basing on that, a depth map can be built and effectively used in the further processing of multiview 3D images, e.g., in 3D broadcasting.

The above functions were tested for depth extraction in noised grayscale images and showed a good validity [10, 11]. However, the functions [9] are not orthogonal; they do not comprise a basis.

The paper is organized as follows: Section 2 recalls the multiview reference functions. Section 3 describes the convolution analysis of the testing image. Section 4 introduces the multiview scaling functions and wavelets. Section 5 presents results of the wavelet analysis. Section 6 gives conclusions.

2. Reference Functions

According to Ref. [9], the 1D reference functions can be written as

$$F_k(x) = \sum_{i=0}^{|k|-1} r\left(\frac{x - x_{kj}}{w_k}\right) \quad (1)$$

where k is the number of discrete plane, x is the coordinate, i is the running index, $r(x)$ is the rectangular

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unit impulse function (the unit impulse of the unit width centered at the origin), $x_{i,j}$ is the coordinate of the center of the pulse, w_k is the width of the pulse. The graphs of functions are given in Fig. 1.

Eq. (1) works in the projective space as described in Ref. [9]. The functions defined by Eq. (1) can be applied to the autostereoscopic displays with the HPO (horizontal parallax only).

For sake of convenience, the numeration of the depth planes in this paper is changed as compared to Ref. [9]. Namely, the index of the function is equal to the index of the reference function minus one. The sign of the index is matter of convenience; the functions with the positive numbers in a barrier 3D display become negative ones in a lenticular 3D display.

In the case of a full-parallax display, the following 2D (two-dimensional) functions can be used,

$$F_k(x, y) = \left[\sum_{i=0}^{|k|-1} r \left(\frac{x - x_{ki}}{w_k} \right) \right] \cdot \left[\sum_{j=0}^{|k|-1} r \left(\frac{y - y_{kj}}{w_k} \right) \right] \quad (2)$$

Fig. 1 1D reference functions $-2 \leq k \leq 2$.

Fig. 2 2D reference functions $-4 \leq k \leq -2$ (from left to right).

where y is the coordinate, j is the index, $y_{i,j}$ is the coordinate of the center of the pulse along the y -axis.

A 2D function Eq. (2) is the product of two 1D functions Eq. (1) along x - and y - axes. Examples of 2D functions are drawn in Fig. 2.

3. Convolution Analysis

To analyze the multiview image and recognize the location of voxels in space using the functions Eqs. (1) and (2), the convolution technique was applied [9]. A location is considered as recognized, when the convolution at the current point is higher than a predefined threshold. This way, the spatial 3D coordinates of testing objects were restored [10].

For the testing purpose, we use a binary wireframe object tetrahedron (its edges) as shown in Fig. 3a. This testing object was initially proposed in Ref. [8] and since that it was used several times; its dimensions are $40 \times 40 \times 9$, so it occupies the 3D space of approx. 15 thousands voxels.

The testing image is a computer-generated multiview image of the above object; the image is built from the reference functions. The computer-generated multiview image of the tetrahedron (observed “from the top”) is shown in Fig. 3b. This testing image additionally includes two points in the corners of the 5th plane. One image cell (elemental image) of the testing image contains 60×60 pixels.

In the convolution analysis, we use thresholding. The threshold is the same in all planes thanks to the partition of unity property of the reference functions. In 2D case, the threshold is numerically equal to the product of the cell area multiplied by the squared number

Fig. 3 Testing object tetrahedron and its computer-generated multiview image.

of the gray levels in the image. For instance, for 256 gray levels and cells of 60×60 pixels, the estimated threshold is 2.4×10^8 . It corresponds to the empirical value 1.7×10^8 .

The restored spatial structure of the testing image Fig. 3b is shown in Fig. 4 as a grey-scale map of recognized locations (a depth map in which the number of the depth plane is shown by the level of gray). The recognized locations correspond to the structure of the source object (Fig. 3a).

The influence of noise can be estimated by adding the background noise. The stability of the recognition of coordinates under noisy conditions was confirmed [11]. The grayscale images were analyzed as well; the edges of tetrahedron were painted in various colors and then transformed back to grayscale. According to Ref. [11], the convolution of the grayscale images became 30-40% asymmetric, and the recognition of locations in the grayscale images becomes less stable.

4. Scaling Functions and Wavelets

A promising technique of the real-time processing is the wavelet analysis. Generally, the wavelets can be built by various methods. One of them is to start from the scaling functions [12]. The scaling functions should satisfy certain conditions, i.e., the two-scale relation, stability (Rietz function), and the partition of the unity. Then, a wavelet can be built as a linear combination of the following scaling functions [13],

$$\psi(t) = \sum_k \beta_k \sqrt{2} \varphi(2t - k) \quad (3)$$

where ψ is a wavelet, φ is a scaling function, and β is a coefficient.

A known scaling function which satisfies all necessary conditions is the Haar function [14]. The functions [9] are based on rectangular unit pulse which actually is the shifted Haar function.

The Haar function itself defines a family of wavelets [15]. Similarly, the multiview reference functions based on it can define wavelets. Correspondingly, the reference functions Eq. (2) can be

considered as the scaling functions of a wavelet transform and thus the multiview wavelets can be built. It is important that the orthogonality is not a requirement for the scaling functions. Preliminary considerations were presented at the conferences [16, 17]. In the current paper, we propose the multiview wavelets and present the wavelet analysis of a multiview image.

The difference between the rectangular unit pulse $r(x)$ and the Haar function [15] is only in the coordinates of the edges of the pulse. Therefore, the reference functions Eq. (1) can be alternatively re-expressed as the linear combinations of the shifted Haar functions,

$$F_k(x) = \sum_{i=0}^{|k|-1} \text{Haar} \left(\frac{x - x_{kj}}{w_k} + \frac{1}{2} \right) \quad (4)$$

The 2D functions can be re-expressed in the similar manner. The corresponding multiview wavelets are the linear combinations of the shifted Haar wavelets.

The multiview scaling functions constructed this way are shown in Fig. 5. The corresponding multiview wavelets obtained from Eq. (3) are shown in Fig. 6.

The 2D wavelets are the product of two 1D wavelets for x - and y -directions. An example is shown in Fig. 7.

In Fig. 7, the grey color represents zero, while white and black colors mean ± 1 .

Fig. 4 Spatial structure of tetrahedron restored by convolution. Recognized depth is shown by levels of gray as in a depth map.

Fig. 5 Multiview scaling functions as functions of x -coordinate ($n = 2$ and $n = 4$).

Fig. 6 Multiview wavelets as functions of x -coordinate ($n = 2$ and $n = 4$).

5. Wavelet Analysis

Using the wavelets (Figs. 6 and 7), the continuous wavelet transform can be performed in 1D and 2D cases (HPO and full-parallax multiview images, resp.). An example of the wavelet analysis of the testing image in Fig. 3b is shown in Fig. 8 for four positive indexes n .

In Figs. 8a-8d, the displacement of the recognized point (which has the highest value of the squared wavelet transform and shown in the black color in the graph) along the left lower edge of tetrahedron can be clearly observed (from the corner to the center, as expected).

Fig. 7 2D wavelet $n = 2$ (The pixels are numbered; one cell = 60 pixels).

Fig. 8 2D wavelet analysis $n = 1, \dots, 4$ in (a), ..., (d), resp.

The wavelet analysis shows that, similarly to Fig. 4, the coordinates of the voxels of the testing object tetrahedron can be recognized using the proposed wavelets.

For the discrete wavelet transform, the functions with numbers 2^n can be used, i.e., the 1st, 2nd, 4th, etc. functions. An example of the multiresolution analysis of the same testing object can be found in Ref. [17].

6. Conclusions

The proposed wavelet transform shows the characteristic features of the testing object. The spatial structure can be successfully extracted from a multiview image. There is a good agreement in the restored locations. The wavelet analysis of a binary multiview image using the proposed wavelets confirms that the recognized locations coincide with the coordinates of voxels of the original binary wireframe image.

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