Task Assignment Problem of Robots in a Smart Warehouse Environment*

Zhenping Li, Wenyu Li, Lulu Jiang
Beijing Wuzi University, Beijing, China

The task assignment problem of robots in a smart warehouse environment (TARSWE) based on cargo-to-person is investigated. Firstly, the sites of warehouse robots and the order picking tasks are given and the task assignment problem for picking one order is formulated into a mathematical model to minimize the total operation cost. Then a heuristic algorithm is designed to solve the task assignment problem for picking multiple orders. Finally, simulations are done by using the orders data of online bookstore A. The results show that using the heuristic algorithm of this paper to assign robots, the cost was reduced by 2% and it can effectively avoid far route and unbalanced workload of robots. The feasibility and validity of the model and algorithm are verified. The model and algorithm in this paper provide a theoretical basis to solve the TARSWE.

Keywords: smart warehouse, robots, cargo-to-person, task assignment, mathematical model, heuristic algorithm

Introduction

The smart warehousing system based on “cargo-to-person” is a new type of distribution center warehousing management mode. In the smart warehousing system, items are stored in the movable shelves, and picking workers stand in front of stationary picking platform. Controlled by the computer system, the robot can lift up the assigned shelf and transport it to the closest picking platform. When picking and replenishing tasks are completed, the robot transports the shelf back to its original location. In the warehousing management system based on cargo-to-person, warehouse robots can complete complex works instead of human and the picking efficiency will be improved to some extent (Zou, 2013).

However, in the smart warehousing system, more complex tasks need to be completed by the robots. In order to improve the picking efficiency, the multi-robots cooperation is used in the general smart warehousing system. The intelligent warehousing system needs to coordinate multiple robots with multiple tasks at the same time, so the task assignment problem of warehouse robots is one of the key factors affecting the efficiency of warehousing system and needs to be studied deeply. In this paper, according to the orders which will be picked in a period of time, the task assignment problem of robots in a smart warehouse environment

* Project Supported: National Natural Science Foundation of China (11131009, 71540028, F012408), Funding Project for Academic Human Resources Development in Institutions of Higher Learning Under the Jurisdiction of Beijing Municipality (CIT&TD20130327), and major research project of Beijing Wuzi University.

Zhenping Li, professor, School of Information, Beijing Wuzi University, Beijing, China.
Wenyu Li, master, School of Information, Beijing Wuzi University, Beijing, China.
Lulu Jiang, master, School of Information, Beijing Wuzi University, Beijing, China.

Correspondence concerning this article should be addressed to Zhenping Li, No. 1 Fuhe Street, Tongzhou District, Beijing 101149, China.
(TARSWE) based on cargo-to-person is investigated. Given the sites of warehouse robots and the tasks for picking one order, the task assignment problem of warehouse robots is formulated into a mathematical model to minimize the total operation cost. Then a heuristic algorithm is designed to solve the task assignment problem for picking multiple orders. Finally, simulations are done by using the simulated orders data of online bookstore A.

**Literature Review**

The “cargo-to-person”-based smart warehousing management mode has been proposed for several years and there are many theoretical issues remained to be studied (Wurman, Andrea, & Mountz, 2008). So far, the researches about the TARSWE are still in a starting stage. The researches are mostly on the task allocation method for the warehouse robots. It mainly includes the method based on the market mechanism and the method based on swarm intelligence.

**Method Based on the Market Mechanism**

For solving the task assignment problem of a smart logistic center with robots, Gao (2006) studied the task allocation problem of robots by using the single item auction algorithm. Guo (2010) proposed a task assignment method based on the market auction idea. For heterogeneous robot completing different types of tasks, Kaleci and Parlaktuna (2012) proposed the task allocation method based on market mechanism. Trigui, Koubaa, and Cheikhrouhou (2014) thought that it can improve the work efficiency by switching tasks in the process of warehouse robots completing tasks and then proposed the extended distributed market oriented algorithm to solve the robot task allocation problem.

**Method Based on Swarm Intelligence**


However, few studies are about the TARSWE.

**Research Methodology**

**Problem Description**

The task assignment problem of robots in the smart warehouse can be described as the follows: There are some isomorphic warehouse robots and several picking platforms in a smart warehouse with the picking mode of cargo-to-person; all the items are stored in the movable shelves according to their correlation (Z. Li & W. Li, 2014a), the movable shelves are neatly arranged in the warehouse, and each shelf is arranged in a determine site. There are several storages in each shelf and only one type of items stores in each storage. The coordinates of shelves, picking platforms, and all kinds of items stored in the warehouse are known. There are some aisles in the warehouse for the robots running smoothly. The fixed cost of calling each robot and the unit-distance cost of each warehouse robot walking loaded and unloaded are also known. During working time, each warehouse robot goes to an assigned shelf and transports it to the picking platform; after the workers picking items from the shelf, the robot transports it back to its original location; and the task is finished. Then, the robot can begin another task. Given a set of orders to be picked in a period of time $T$, the shelves to be moved for picking the
orders are known. Suppose that the orders are picked one by one, the number of robots is larger than the maximum number of shelves that need to be transported for picking one order. In order to minimize the total operating costs, how to assign tasks for each warehouse robot is studied in this paper.

Mathematical Model for Picking One Order

Since the orders are picked one by one, firstly it formulates the task assignment problem of picking one order.

The symbols and variables are defined as follows:

- \( L = \{ F_1, F_2, ..., F_m \} \): set of available warehouse robots;
- \( B = \{ b_1, b_2, ..., b_k \} \): set of shelves in the warehouse, \( b_i \) represents the \( i \)th shelf;
- \( Q = \{ t_1, t_2, ..., t_n \} \): set of picking platform, \( t_j \) represents the \( j \)th picking platform;
- \( p \): number of tasks for picking one order;
- \( S = \{ s_1, s_2, ..., s_p \} \): set of tasks for picking one order, \( s_i \) represents the task of transport shelf \( b_i \) to the picking platform;
- \( d_{ij} \): walking distance of a robot walking from shelf \( b_i \) to shelf \( b_j \), \( i = 1, 2, ..., p; j = 1, 2, ..., p \);
- \( u_j \): walking distance of a robot transporting shelf \( b_j \) to its corresponding picking platform, \( j = 1, 2, ..., p \);
- \( r \): fixed cost of each warehouse robot;
- \( c_l^1 \): unit-distance cost of a robot walking loaded with a shelf;
- \( c_l^2 \): unit-distance cost of a robot walking unloaded;
- \( x_{lh} \) = 1, assign warehouse robot \( F_l \) to complete task \( s_h \);
- \( 0 \), otherwise \( l = 1, 2, ..., m; h = 1, 2, ..., p \).

The total operation cost of warehouse robots completing all tasks is mainly composed of the related cost, the self-cost, and the fixed cost of calling each warehouse robot. Related cost refers to the cost that the robot needs to move unloaded from current position to position of a beginning task. Self-cost refers to the cost that the warehouse robot completes a task.

If the current position of a warehouse robot \( F_i \) is in shelf \( b_i \), assign \( F_i \) to complete task \( s_j \), then the cost of robot \( F_i \) to finish task \( s_j \) can be expressed as follows:

\[
    c_{ij} = RC_i + SC_i + FC_i
    = d_{ij}c_1^2 + u_jc_1^1 + r
\]

When \( d_{ij} \geq A \), \( c_{ij} = M \).

Among them, \( RC_i \) is the related cost, \( SC_i \) is the self-cost, and \( FC_i \) is the fixed cost.

In order to establish a mathematical model, the \( p \) tasks of an order are numbered from small to large according to the task subscript: \( 1, 2, ..., p \). All the \( c_{ij} \) form a vector matrix \( C_{mp} \) of \( m \) row and \( p \) column, among them, the element \( c_{lh} \) refers to the cost of warehouse robot \( F_l \) completing the task \( s_h \).

Based on the above analysis, after the robots completing all tasks of an order, they will be assigned to complete new tasks. In order to minimize the total operation cost, the new tasks assignment problem of warehouse robots can be formulated into the following integer linear programming model.
Objective function (1) minimizes the total operation cost; constraint (2) refers to that each task can be assigned to one warehouse robot; constraint (3) represents that every warehouse robot can complete no more than one task; and constraint (4) indicates that the variables are binary.

The mathematical model of one order picking task assignment problem is a 0-1 programming model, which can be solved in polynomial time, for example, the improved Hungarian algorithm can be used to solve the integer programming model and obtain the exact optimal solution (Wu & Dong, 2008).

**Heuristic Algorithm for Picking Multiple Orders**

The set of orders to be picked in a period of time T is given and the orders are usually divided into several batches: Each batch is picked at the same time, since the number of tasks for picking all orders is much larger than the number of warehouse robots. This paper assumes that every batch consists of one order, which means that the orders are picked one by one.

Based on this assumption, a heuristic algorithm for picking multiple orders is designed.

Suppose there are e orders to be picked, which are listed in a set:

$$\{O_1, O_2, ..., O_e\}$$

The tasks for picking each order are known.

The heuristic algorithm for picking all orders can be described as follows:

Step 1: Select the first order from order list set O; according to the warehouse robot’s position and the tasks for picking the first order, calculate the cost matrix $C_{lp}$. Then, the optimal task assignment solution for picking the first order can be obtained by solving the integer programming model in section 2.2.

Step 2: Update the positions of warehouse robots. The positions of robots that participate in picking tasks of the first order are updated as their corresponding task position. Positions of warehouse robots that do not participate in picking the first order remain unchanged. Delete the first order set.

Step 3: If the order set is empty, go to step 4; else go to step 1.

Step 4: Output the tasks assignment solution for picking all orders.

**Results Analysis**

The methods are tested by analyzing the orders data of online bookstore A.

**Description of Online Bookstore A**

There are 96 classes of books sold in online bookstore A stored in 24 movable shelves and each movable shelf has two layers. Each layer has two storage spaces and each storage space stores one class of books, namely, there are at most four classes of books stored in each shelf. At present, A bookstore uses the smart warehousing system based on cargo-to-person to manage picking and replenishment operation. There are three
picking platforms in the warehouse. The warehouse has 10 warehouse robots walking along a specific landmark. In the current operation mode, the orders are picked one by one, which means that the next order would not be picked until the picked order is packaged, so each order requires a separate picking time.

As shown in Figure 1, Cartesian coordinates system is established. The warehouse is abstracted as a grid map and each box in the figure is abstracted into a position coordinates (x, y). Square boxes represent shelves, the cycles with arrow represent warehouse robots, and the rectangular boxes represent picking platforms.

![Figure 1. Warehouse layout simplified diagram of online bookstore A at 10 a.m. (coordinate unit: 2 m).](image)

It simulated 10 orders to be picked between 10 a.m. and 12 a.m. The corresponding tasks sets of 10 orders are listed in Table 1. Coordinates of shelves and picking platforms are listed in Table 2. Thus, the walking distance among any pair of shelves and between the shelf and its corresponding picking platform can be calculated. The detailed information of 10 orders is known. The unit-distance cost of each warehouse robot walking loaded and unloaded are 3 dollars and 1.9 dollars and the fixed cost of calling each warehouse robot is 10 dollars. In order to avoid robot collision, assuming that $c_{ij}$ is infinity if $d_{ij} \geq 10$. According to the existing
resource in the warehouse, in order to minimize the total operation cost, how are tasks assigned for each warehouse robot?

Table 1

<table>
<thead>
<tr>
<th>Order</th>
<th>Task sets of the order (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>{s_2, s_5, s_10, s_{16}}</td>
</tr>
<tr>
<td>2</td>
<td>{s_2, s_5, s_7, s_8, s_{10}, s_{21}, s_{23}, s_{24}}</td>
</tr>
<tr>
<td>3</td>
<td>{s_4, s_5, s_{12}, s_{18}, s_{20}, s_{22}, s_{24}}</td>
</tr>
<tr>
<td>4</td>
<td>{s_1, s_3, s_{14}, s_{19}, s_{22}, s_{24}}</td>
</tr>
<tr>
<td>5</td>
<td>{s_1, s_3, s_8, s_{11}, s_{14}, s_{25}, s_{29}, s_{23}}</td>
</tr>
<tr>
<td>6</td>
<td>{s_2, s_5, s_9, s_{11}, s_{16}, s_{19}, s_{23}}</td>
</tr>
<tr>
<td>7</td>
<td>{s_4, s_7, s_{10}, s_{11}, s_{17}, s_{18}}</td>
</tr>
<tr>
<td>8</td>
<td>{s_2, s_3, s_{14}, s_{22}, s_{23}}</td>
</tr>
<tr>
<td>9</td>
<td>{s_2, s_6, s_{11}, s_{12}, s_{14}, s_{19}, s_{22}, s_{23}}</td>
</tr>
<tr>
<td>10</td>
<td>{s_1, s_6, s_7, s_{12}, s_{17}, s_{23}}</td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th>Shelf</th>
<th>(b_1)</th>
<th>(b_2)</th>
<th>(b_3)</th>
<th>(b_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinate (1)</td>
<td>(3, 7)</td>
<td>(5, 7)</td>
<td>(7, 7)</td>
<td>(9, 7)</td>
</tr>
<tr>
<td>Shelf</td>
<td>(b_6)</td>
<td>(b_6)</td>
<td>(b_6)</td>
<td>(b_6)</td>
</tr>
<tr>
<td>Coordinate (2)</td>
<td>(11, 7)</td>
<td>(13, 7)</td>
<td>(13, 9)</td>
<td>(11, 9)</td>
</tr>
<tr>
<td>Shelf</td>
<td>(b_{10})</td>
<td>(b_{11})</td>
<td>(b_{12})</td>
<td>(b_{12})</td>
</tr>
<tr>
<td>Coordinate (3)</td>
<td>(9, 9)</td>
<td>(7, 9)</td>
<td>(5, 9)</td>
<td>(3, 9)</td>
</tr>
<tr>
<td>Shelf</td>
<td>(b_{13})</td>
<td>(b_{14})</td>
<td>(b_{15})</td>
<td>(b_{16})</td>
</tr>
<tr>
<td>Coordinate (4)</td>
<td>(3, 11)</td>
<td>(5, 11)</td>
<td>(7, 11)</td>
<td>(9, 11)</td>
</tr>
<tr>
<td>Shelf</td>
<td>(b_{17})</td>
<td>(b_{18})</td>
<td>(b_{19})</td>
<td>(b_{20})</td>
</tr>
<tr>
<td>Coordinate (5)</td>
<td>(11, 11)</td>
<td>(13, 11)</td>
<td>(13, 13)</td>
<td>(11, 13)</td>
</tr>
<tr>
<td>Shelf</td>
<td>(b_{21})</td>
<td>(b_{22})</td>
<td>(b_{23})</td>
<td>(b_{24})</td>
</tr>
<tr>
<td>Coordinate (6)</td>
<td>(9, 13)</td>
<td>(7, 13)</td>
<td>(5, 13)</td>
<td>(3, 13)</td>
</tr>
<tr>
<td>Picking platform</td>
<td>(t_1)</td>
<td>(t_2)</td>
<td>(t_3)</td>
<td>(t_3)</td>
</tr>
<tr>
<td>Coordinate (7)</td>
<td>(4, 3)</td>
<td>(8, 3)</td>
<td>(12, 3)</td>
<td>(12, 3)</td>
</tr>
</tbody>
</table>

Results Analysis of Online Bookstore A

At first, according to the coordinates of shelves and picking platforms in Table 2, the walking distance among any two shelves and between the shelf and its corresponding picking platform are calculated. Then the algorithm in this paper is used to get the task assignment schemes of the warehouse robots.

For the first order, according to the coordinates of shelves and picking platforms, the vector matrix \(C_{10,4}\) can be obtained (\(M\) denotes infinity).
Then solving the integer programming model by using the LINGO program, the optimal task assignment schemes of order 1 can be obtained: \( F_1 \rightarrow b_3; F_2 \rightarrow b_5; F_{10} \rightarrow b_9; F_6 \rightarrow b_{16} \), and the cost is \( 43.8 + 43.8 + 52 + 64 = 203.6 \) dollars. Where, \( F_i \rightarrow b_j \) represents that warehouse robot \( F_i \) is assigned to transport shelf \( b_j \) to the corresponding picking platform.

After finishing all the tasks of order 1, the positions of warehouse robots are updated. Then using the same method, the optimal task assignment schemes of order 2 to order 10 can be obtained. The results are shown in Table 3.

According to the corresponding task assignment schemes and cost of robots for picking each order in Table 3, the total operation cost is 4,010.4 dollars, which the warehouse robots spend on completing all tasks within 10 a.m. and 12 a.m.

At present, the task assignment of warehouse robots of online bookstore A is made according to personnel experience, which might lead to the long route and unbalanced workload of different robots. The specific task assignment schemes within 10 a.m. and 12 a.m. are shown in Table 4.

Table 3

<table>
<thead>
<tr>
<th>Order</th>
<th>Task assignment schemes</th>
<th>Cost (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( F_3 \rightarrow b_2; F_4 \rightarrow b_5; F_{10} \rightarrow b_9; F_6 \rightarrow b_{16} )</td>
<td>203.6</td>
</tr>
<tr>
<td>2</td>
<td>( F_3 \rightarrow b_2; F_6 \rightarrow b_5; F_5 \rightarrow b_2; F_4 \rightarrow b_9; F_{10} \rightarrow b_3; F_2 \rightarrow b_{21}; F_9 \rightarrow b_{25}; F_1 \rightarrow b_{24} )</td>
<td>483</td>
</tr>
<tr>
<td>3</td>
<td>( F_3 \rightarrow b_2; F_4 \rightarrow b_5; F_5 \rightarrow b_{22}; F_3 \rightarrow b_{23}; F_2 \rightarrow b_{24}; F_9 \rightarrow b_{25}; F_1 \rightarrow b_{24} )</td>
<td>458.8</td>
</tr>
<tr>
<td>4</td>
<td>( F_1 \rightarrow b_3; F_7 \rightarrow b_3; F_8 \rightarrow b_{14}; F_5 \rightarrow b_{15}; F_{10} \rightarrow b_6; F_2 \rightarrow b_{24}; F_9 \rightarrow b_{25}; F_1 \rightarrow b_{24} )</td>
<td>455</td>
</tr>
<tr>
<td>5</td>
<td>( F_7 \rightarrow b_1; F_1 \rightarrow b_3; F_4 \rightarrow b_2; F_{10} \rightarrow b_{11}; F_3 \rightarrow b_{14}; F_2 \rightarrow b_{15}; F_2 \rightarrow b_{23}; F_1 \rightarrow b_{23} )</td>
<td>482.4</td>
</tr>
<tr>
<td>6</td>
<td>( F_7 \rightarrow b_2; F_4 \rightarrow b_3; F_3 \rightarrow b_2; F_{10} \rightarrow b_{11}; F_9 \rightarrow b_{16}; F_3 \rightarrow b_{22}; F_9 \rightarrow b_{25}; F_1 \rightarrow b_{24} )</td>
<td>415.2</td>
</tr>
<tr>
<td>7</td>
<td>( F_4 \rightarrow b_4; F_6 \rightarrow b_6; F_3 \rightarrow b_{10}; F_8 \rightarrow b_{13}; F_2 \rightarrow b_{17}; F_9 \rightarrow b_{18} )</td>
<td>355</td>
</tr>
<tr>
<td>8</td>
<td>( F_7 \rightarrow b_2; F_6 \rightarrow b_3; F_5 \rightarrow b_{14}; F_5 \rightarrow b_{22}; F_3 \rightarrow b_{24} )</td>
<td>323.2</td>
</tr>
<tr>
<td>9</td>
<td>( F_4 \rightarrow b_2; F_3 \rightarrow b_{11}; F_1 \rightarrow b_{12}; F_8 \rightarrow b_{23}; F_9 \rightarrow b_{25}; F_3 \rightarrow b_{24}; F_1 \rightarrow b_{23} )</td>
<td>498.8</td>
</tr>
<tr>
<td>10</td>
<td>( F_4 \rightarrow b_1; F_5 \rightarrow b_2; F_6 \rightarrow b_3; F_{10} \rightarrow b_{12}; F_9 \rightarrow b_{17}; F_3 \rightarrow b_{24} )</td>
<td>335.4</td>
</tr>
</tbody>
</table>
In this case, the heuristic algorithm in this paper can also be used for the task assignment according to their similarity. When the scale of the warehouse is larger and the quantity of warehouse robots is enough, the orders can be batched together to reduce the total operation cost. Further, a heuristic algorithm is designed to solve the task assignment problem for picking multiple orders. At the end, by analyzing the orders data of online bookstore A, the feasibility and validity of the model and algorithm are verified. This paper assumes that the orders are picked one by one. When the scale of the warehouse is larger, i.e., the amount of orders is larger and the quantity of warehouse robots is enough, the orders can be batched according to their similarity (Z. Li & W. Li, 2014b), every batch of orders can be picked simultaneously, then in this case, the heuristic algorithm in this paper can also be used for the task assignment.

### References


