Critical Buckling of Drill Strings in Cylindrical Cavities of Inclined Bore-Holes

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Abstract: Notwithstanding the fact that the problem of drill string buckling (Eulerian instability) inside the cylindrical cavity of an inclined bore-hole attracts attention of many specialists, it is far from completion. This peculiarity can be explained by the complexity of its mathematic model which is described by singularly perturbed equations. Their solutions (eigen modes) have the shapes of boundary effects or buckles (harmonic wavelets) localized in zones of the bore-hole that are not specified in advance. Therefore, the problem should be stated in the domain of entire length of the drill string or in some separated part including an expected zone of its buckling. In the paper, a mathematic model for computer analysis of incipient buckling of a drill string in cylindrical channel of an inclined bore-hole is elaborated. The constitutive equation is deduced with allowance made for action of gravity, contact, and friction forces. Computer simulation of the drill string buckling is performed for different values of the bore-hole inclination angle, its length, friction coefficient, and clearance. The eigen values (critical loads) are found and modes of stability loss are constructed. The numerical results for the case when the inclination angle equals friction angle coincide with ones obtained analytically.

Key words: Deep drilling, inclined bore-holes, drill strings, stability, singularly perturbed problem.

1. Introduction

At the present time, most of the easy oil and gas is produced. Inasmuch as the readily accessible deposits of hydrocarbon fuels are practically depleted in the result of their intensive extraction in preceding two centenaries, their drawing out from deeper reservoirs holds much promise.

An effective means to enlarge the oil and gas extraction efficiency is to drill the bore-holes in inclined and horizontal directions. Yet, drivage of bore-holes of this type is associated with great technological difficulties. They are conditioned by geometric and structural complication of these systems, as well as by redistribution of internal and external forces providing basic cause of decrease of the DS (drill string) mobility in the bore-hole cavity. These factors may be responsible for the next negative features of the drilling process:

• orientation of the gravity forces relative the DS axis changes, bringing to reduction of axial tensile forces (stabilizing the DS tube) and enlargement of longitudinal compressive forces (destabilizing the system);
• in this connection, the forces of contact interaction of the DS tube with the bore-hole wall enlarge essentially;
• during drilling and tripping operations, the axial and rotary motions of the DS lead to generation of essential distributed axial friction forces and torques;
• these friction forces and torques entail the deterioration of the WOB (weight on bit) and TOB (torque on bit) permeability to the bore-hole bottom and reduce the drilling efficiency;
• in the bore-hole sections, where the DS is compressed, it can lose its Eulerian stability with subsequent buckling and so additional stability analysis should accompany the drilling maintenance;
• as shown below, the problem of the DS buckling...
in the channel of a directed bore-hole is multiparametric, its principal feature is associated with variability of the internal compressive axial force. Therefore, the zone of the DS buckling is not known in advance and so the stability analysis should be performed in the entire length of the DS. This problem is singularly perturbed, that is why the modes of stability loss represent localized or distributed harmonic wavelets;

- the generated zones of enlarged contact and friction forces can be also responsible for dead or locked states of the DSs.

The outlined peculiarities of the bifurcational buckling of the DSs in the inclined DSs channels render detection of this phenomenon hardly predictable that is why the problem of their theoretic modeling is of current importance.

The first systematic analysis of helical buckling of a DS was performed by Lubinski et al. [1]. They discovered the mechanism of the DS buckling in vertical bore-holes and established the critical buckling conditions. Since then, different models of the DS buckling in bore-hole channels under action of gravity, contact, and friction forces, as well as torque and axial external force have been considered. Special attention was paid to analysis of directed bore-holes. Inasmuch as the inclined and horizontal bore-holes permeate the oil- and gas-bearing strata along the stratified structure of the underground reservoirs, they cover larger zones of fuel deposits and are effective expedients to enlarge the extraction efficiency [2-5]. Analysis of the DS stability in the bore-holes of these types was initiated after the pioneer paper by Dawson and Pasley [6]. Later, a lot of refinements were introduced in theoretical models of the buckling effects [7-12]. Analysis of whirl interaction of a drill bit with the bore-hole bottom was performed by Musa et al. [13].

Detailed review of this problem state was presented by Cunha [14], Gao and Huang [15], and Mitchell [16]. It can be concluded from these surveys that, basically, the approaches used by researchers attacking this problem were based on the buckling mode approximation by sinusoidal or helical curves, while as shown by Gulyayev et al. [17-19], the problem of the DS buckling in channels of long bore-holes is singularly perturbed and therefore, the DS buckling occurs through forming harmonic or spiral wavelets of variable steps in unknown segments of the DS length. Sometimes, they represent high-frequency beating harmonics distributed throughout the total length of the DS. That is why the buckling analysis of these DSs should be performed with the use of their large segments.

In this paper, the DS stability in the lower rectilinear segment of an inclined bore-hole is analyzed with account made for constraining effect by cylindrical channel of the bore-hole wall with different clearances as well as distributed gravity, contact, and friction forces.

2. Geometric Prerequisites to the Bending Analysis of a Drill String inside Cylindrical Cavity

Let a rectilinear DS lie on the lower generatrix of the cylindrical surface of an inclined bore-hole. Its axis is located in the \( XOZ \) plane of fixed coordinate system \( OXYZ \) and is inclined under angle \( \beta \) with respect to vertical \( OZ \) (Fig. 1). The internal geometry of the DS is specified by coordinate \( s \), measured as the length of axial line from the initial to the current point. Assume that if the DS is immovable, it is loaded by external distributed gravity (\( gr \f \)) and contact (\( cont \f \)) forces, and at its ends axial forces \( F_z(0) \) and \( F_z(S) \) are applied. Here, \( S \) is the DS length.

In the state of the DS movement, unknown distributed frictional force \( fr \f \) is added which is determined through the contact force with the help of the Coulomb law

\[
f^{fr}(s) = \pm \mu f^{cont}(s)
\]  

where, \( \mu \) is the coefficient of friction and the choice of signs “+” and “−” depends on the direction of the DS movement.
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Fig. 1 Schematic of a drill string in an inclined bore-hole.

As the results of these factors action, compressive axial forces are induced in some zones of the DS. On attainment by them of critical values, the DS buckles and its axial line \( L \) assumes new equilibrium shape inside the cylinder of radius \( a \) equal the bore-hole clearance. It is required to establish by theoretic simulation the critical state and to construct the mode of the bifurcational stability loss. This study will be performed by methods of non-linear analysis with the use of the linearization procedure. For this purpose, construct the non-linear differential equations of the DS equilibrium and linearize them in the vicinity of the considered stress-strain state. The external loads, corresponding to degeneration of the linearized operator, are considered to be bifurcational and eigen mode of this operator is the mode of the DS buckling [17-19].

Feature peculiar to this effect lies in the fact that if angle \( \beta \) of the bore-hole inclination is not small, the DS is compressed to the bore-hole wall by gravitation forces \( f_{gr}(s) \) which impede its raising on the bore-hole channel surface during buckling process and stabilize its equilibrium. In consequence of this, the DS is kept on this surface and therefore its geometry cannot change arbitrary because now it is determined by the geometry of the cylindrical surface where it lies. In analysis of the geometric transformation of the buckling DS, that compels to invoke the methods of the surface theory and differential geometry with the use of the model of curvilinear flexible rods [19].

To chase the DS geometry transformation inside cylindrical channel of an inclined bore-hole, introduce a moving right-handed coordinate system \( \text{oxyz} \) with axis \( o \alpha \) oriented along internal normal to the surface \( G \) and axis \( o \gamma \) directed along the tangent to the curve \( L \). Unit vectors of this system are \( \mathbf{i}, \mathbf{j}, \mathbf{k} \) (Fig. 2). In parallel with this system, the Frenet trihedron is used [18]. Its tangent unit vector \( \mathbf{t} \), unit vector of principal normal \( \mathbf{n} \), and unit vector of binormal \( \mathbf{b} \) are calculated by the formulæ

\[
\mathbf{t} = \frac{\mathbf{p}}{ds}, \quad \mathbf{n} = R \frac{\mathbf{d}t}{ds}, \quad \mathbf{b} = \mathbf{t} \times \mathbf{n}
\]  

(2)

Here, \( R \) is the curvature radius of the DS axis line, \( \mathbf{p}(s) \) is the curve \( L \) radius-vector in the fixed coordinate system \( OXYZ \). These vectors determine orientations of the rod elements and the curve shape. With their use the Darboux vector can be introduced

\[
\Omega = k_x \mathbf{b} + k_r \mathbf{t}
\]

(3)

where, \( k_x = 1/R \) is the curve curvature and \( k_r \) is its torsion. These parameters are calculated with the help of formulæ [20]

\[
k_x = \mathbf{n} \frac{d\mathbf{p}}{ds}, \quad k_r = \mathbf{t} \left( \mathbf{n} + \frac{dn}{ds} \right)
\]

(4)

But it is more rational to express the \( k_x \) and \( k_r \) parameters and to study the DS bending in the movable coordinate system \( \text{oxyz} \) because its use permits to
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Fig. 2  Axial line $L$ of the DS in its buckling in the cylindrical channel of a bore-hole.

convent the problem difficulty, connected with existence of constraining surface $G$, into advantage, conditioned by the fact that the $L$ curve geometry can be described in the known terms of the surface $G$ parameters. With this feature in mind, introduce vector

$$\mathbf{w} = k_i \mathbf{i} + k_j \mathbf{j} + k_k \mathbf{k},$$

representing angular velocity of the $oxyz$ system in its movement along the $L$ curve. Here, $k_i$ and $k_j$ are the curvatures of the $L$ curve in planes $yoz$, $xoz$, correspondingly; $k_k$ is its torsion. They are expressed via the known curvatures of the cylindrical surface $G$ parameterized by means of parameter $u$ directed along the surface generatrix and parameter $v$, determining the position of a point at the generating circle of radius $a$ (Fig. 2). Then, the $G$ surface metrics is determined by the formula

$$(ds)^2 = (du)^2 + a^2 (dv)^2$$

In this case, the $k_i$ curvature represents the geodesical curvature $k^\text{mod}$ of the $L$ curve in the $G$ surface and it can be expressed as follows [17, 20]:

$$k_i = k^\text{mod} = a(u^\prime v^\prime - v^\prime u^\prime),$$

where, symbol prime denotes the operation of differentiation with respect to $s$.

Plane $xoz$ is orthogonal to the $G$ surface, therefore, the $k_j$ curvature of the curve $L$ equals normal curvature $k_n$ of cylindrical surface with the use of the Euler theorem:

$$k_j = k^\text{nor} = k_j \cos^2 \theta + k_k \sin^2 \theta,$$

where, $k_i = 0$, $k_j = 1/a$, and $\theta$ is the angle between the direction of unit vector $t$ and coordinate line $u$.

Inasmuch as $\sin \theta = adv/ds$, it follows:

$$k_j = a(v)^2.$$  

Torsion $k_k$ of the $L$ curve is calculated with the use of equality [19]

$$k_k = u^\prime v^\prime$$

The constructed geometric correlations permit one to
deduce constitutive equations of the DS bending in a cylindrical cavity.

3. Constitutive Equations of the DS Bending inside Inclined Bore-Hole Cavity

Consider the problem about elastic bending of a DS inside the cylindrical channel of an inclined bore-hole. The DS is preloaded by distributed gravity forces \( f^g(s) \) throughout its length and by axial compressive force \( F_z \) at its lower end \( s = S \). Action of these forces is associated with generation of internal force \( F(s) \) and moment \( M(s) \), as well as external contact \( f^c(x) \) and friction \( f^f(x) \) forces.

Particular emphasis should be placed on the question of friction forces effects. In our case, two situations are considered when the DS is immovable at the precritical state and when it buckles during lowering and drilling. It can be assumed that at the former case the DS is exposed to action of different dynamic perturbations (for example, by mud flows) which relieve friction forces and then \( f^f = 0 \). Whereas, during axial motion of the DS this force is defined by Eq. (1). So, the challenge is to establish typical regularities of friction forces influence on the buckling process. In the result of these forces action, the DS can acquire complicated shape, so the theory of curvilinear flexible rods is used for its analysis. The bifurcation modeling is based on the analyses of non-linear equations of the curvilinear rod equilibrium. They are represented in scalar form in the moving coordinate system \( oxyz \) [18, 19] as equations of equilibrium of forces

\[
\begin{align*}
\frac{dM_x}{ds} &= -k_x M_x + k_x M_y + F_x, \\
\frac{dM_y}{ds} &= -k_y M_x + k_y M_y - F_y, \\
\frac{dM_z}{ds} &= -k_z M_x + k_z M_y, \\
\frac{dF_x}{ds} &= -k_x F_x + k_x F_y - f^c_{x}, \\
\frac{dF_y}{ds} &= -k_y F_x + k_y F_y - f^c_{y}, \\
\frac{dF_z}{ds} &= -k_z F_x + k_z F_y - f^c_{z},
\end{align*}
\]

and moments

\[
\begin{align*}
\frac{dM_x}{ds} &= -k_x M_x + k_x M_y + F_x, \\
\frac{dM_y}{ds} &= -k_y M_x + k_y M_y - F_y, \\
\frac{dM_z}{ds} &= -k_z M_x + k_z M_y.
\end{align*}
\]

Here, the benefits of the chosen reference frame \( oxyz \) and vector \( \omega \) became distinct. Only one component \( f^c_{x} \) of the unknown contact force \( f^c(s) \) is present in the first equation of system (11) and one component \( f^c_{z} \) of friction force \( f^f(s) \) enters into its third equation. These factors permit to simplify Eqs. (11) and (12). Since, bending moments \( M_x, M_y \) in Eq. (12) are expressed through curvatures \( k_x, k_y \), with the use of Eqs. (7) and (9), one has \( M_x = E k_x, M_y = E k_y \) and shear forces \( F_x \) and \( F_y \), can be calculated proceeding from two first equations of system (12)

\[
\begin{align*}
F_x &= -2E I a v^\gamma \beta - a(M_x - E a v^\gamma \beta) v^\gamma - v^\gamma u^\gamma \\
F_y &= -E I a \frac{d}{ds}(u^\gamma - v^\gamma u^\gamma) + a(M_x - E a v^\gamma \beta)(v^\gamma) \quad (13)
\end{align*}
\]

The terms of the right side of the third equation of system (12) equal zero and so torque \( M_z \) is constant throughout the DS length.

From Eqs. (6), (7), (9)-(13), the constitutive system of six first order equations is received

\[
\begin{align*}
\frac{dF_x}{ds} &= 2E I a u^\gamma \beta - (M_x - E a v^\gamma \beta) u^\gamma k_x + k_x F_x - f^c_{x} \\
\frac{dF_y}{ds} &= -k_y F_x - 2E I a v^\gamma \beta v^\gamma + a(M_x - E a v^\gamma \beta)(v^\gamma) k_x - f^c_{y} - f^c_{z} \\
\frac{dk_x}{ds} &= -\frac{1}{E I} a(v^\gamma)^2 M_x + a(u^\gamma)^2 + \frac{1}{E I} F_y \\
\frac{dv}{ds} &= v^\gamma \\
\frac{du}{ds} &= (1 - a^2)(v^\gamma) \\
\end{align*}
\]

(14)

In this system, variables \( F_x \) and \( M_y \) are eliminated from consideration because they can be represented through geometry parameters of the \( G \) surface, whereas the components of the gravity force are determined by the formulas

\[
\begin{align*}
f^c_{x} &= -f^c_{y} \sin \beta \cos \nu \\
f^c_{y} &= f^c_{x} (\cos \beta \cdot a v^\gamma + \sin \beta \sin \nu \cdot u^\gamma) \\
f^c_{z} &= f^c_{x} (\cos \beta \cdot u^\gamma - \sin \beta \sin \nu \cdot a v^\gamma)
\end{align*}
\]

(15)
The first equation of system (11) is not included into system (12) but it is used for calculation of the contact force

\[ f_\alpha = -\frac{dF}{ds} - k_i F_i + u'v'F_i - f_\alpha'' \] (16)

Eq. (14) with appropriate boundary conditions describe non-linear bending of a DS under action of gravity and friction forces as well as external axial forces applied to the DS ends inside the channel of an inclined rectilinear bore-hole. The states of the DS loading when the linearized operator of system (14) is degenerated are critical (bifurcational). The eigenvalues and eigen modes of this system represent critical loads and modes of stability loss (buckling) at these states.

4. Equations of Critical States of the DS inside Inclined Bore-Hole

In this paper, it is assumed that a rectilinear DS is freely lying or sliding along its axis in the bottom of a rectilinear bore-hole. It is necessary to calculate its critical load and to construct its buckling mode. With this aim in view, linearize Eq. (14) in the vicinity of the state \( u(s) = s, v(s) = 0, k_i(s) = k_i(s) = k_i(s) = 0, M_i(s) = M_i(s) = 0 \) at this state, the contact force \( f_\alpha''(s) \) is constant and easily defined by Eq. (16)

\[ f_\alpha''(s) = -f_\alpha'' \] (17)

Then, in a similar manner, friction force \( f_\alpha'' \) is calculated as follows:

\[ f_\alpha''(s) = \pm f_\alpha'' = \mp f_\alpha'' \] (18)

Thereafter, function \( F_i(s) \) is found from the second equation of system (14)

\[ F_i(s) - F_i(0) = \int \left(f_\alpha'' + f_\alpha''\right) ds = -(f_\alpha'' + f_\alpha'')s - R \] (19)

Here, \( F_i(0) \) is the \( F_i \) force value at the suspension point \( s = 0 \). If the DS lowering regime is chosen, then \( F_i(0) = (f_\alpha'' + f_\alpha'')S \) and \( R \) is prescribed by compressive force, acting at the lower end of the DS.

In consequence of these remarks, after linearization of system (14), only four equations are suitable for further use. Rewrite them in the linearized form:

\[
\begin{align*}
\frac{d\delta}{ds} + \frac{1}{EI} \delta &= F_i \delta_k, \\
\frac{d\delta}{ds} &= -\delta_i'' = 0, \\
\frac{d\delta}{ds} &= (1/EI)\delta, \\
\frac{d\delta}{ds} &= 0.
\end{align*}
\] (20)

Take into consideration that \( \delta_k = a\delta' \), \( \delta_i = 0 \), \( \delta_i'' = 0 \), \( f_i = f_i \cos \beta \), \( f_i = g \rho_m \rho_a F \), where \( g = 9.81 \text{ m/s}^2 \); \( \rho_m, \rho_a \) are densities of the tube material and mud; \( F \) is area of the DS tube cross-section; sign “+” is selected for the hoisting operation, sign “−” corresponds to the DS lowering.

Further, the DS lowering regime will be considered, therefore

\[ F_i(s) = f_i (\cos \beta - \mu \sin \beta)(S - s) - R \] (21)

The second and sixth equations of system (20) are trivial and so they will not be considered, the other four equations can be recast in a more convenient form

\[
\begin{align*}
\delta'' + \left(-\frac{f_i \cos \beta - \mu \sin \beta}{EI} (S - s) + \frac{R}{EI}\right)\delta' + \frac{f_i \cos \beta - \mu \sin \beta}{aEI} \delta = 0.
\end{align*}
\] (22)

where, \( \delta(y) = a\delta(v) \).

This fourth order homogeneous differential equation is similar to the equation of stability of a beam on elastic foundation and so their solutions have similar properties. In the first place, both of them are singularly perturbed [17, 21] and for this reason, their modes of stability loss have the shapes of boundary effects, usually localized in boundary zones. Secondly, as a rule, these modes represent damping out oscillating harmonics. Very important property of this equation consists in the fact that torque \( M_i \), presenting in system (14), disappeared in the result of its
linearization in the vicinity of the considered state. This fact testifies the system insensitivity to the DS torsion in this position.

5. Bifurcation Buckling of a DS in the Channel of Inclined Bore-Hole

It should be emphasized that Eq. (22) has variable coefficients, therefore its eigen values and eigen modes can be analyzed only by numerical methods. In the case under consideration, the finite difference method was used for this purpose. In what follows, the results of computer simulation are represented. First and foremost, the remark should be concentrated on the fact that the stated problem is multiparametric. Indeed, the buckling proceeding depends on the DS length, cross-section dimensions, angle of inclination, clearance value, presence or absence of friction effects and friction coefficient value, value of the boundary compressive force \( R \), boundary conditions, and so on. Therefore, below particular cases are studied under next values of characteristic parameters:

\[
\begin{align*}
E &= 2.1 \times 10^{11} \text{ Pa}, \quad I = 2.7 \times 10^{4} \text{ m}^4, \quad \rho_s = 7.8 \times 10^{3} \text{ kg/m}^3, \\
\rho &= 1.3 \times 10^{4} \text{ kg/m}^3, \quad d = 0.2 \text{ m}, \quad d_t = 0.18 \text{ m}, \\
F &= \pi(d_s^2 - d_t^2)/4 = 0.00597 \text{ m}^2, \quad \mu = 0.2.
\end{align*}
\]

The boundary conditions at the top and lower edges correspond to pinned ends

\[
\begin{align*}
\delta y(0) &= \delta y(S) = 0, \quad \delta y'(0) = \delta y'(S) = 0 \quad (23)
\end{align*}
\]

Initially, stability of the DS of lengths \( S = 500 \) and \( 1,000 \) m was treated for the clearance value \( a = 0.166 \) m. In Table 1, the calculation results for the bore-hole of 500 m in length are demonstrated. Two left columns of this Table are related to the frictionless model, two right ones contain data associated with influence of friction effect. The diagrams of axial force \( F_z(s) \), lateral displacement \( \delta y(s) \) and critical value \( R_{cr} \) are given for every considered case under different values of inclination angle \( \beta \). It can be seen that if the \( \beta \) angle is not very large (\( \beta = 45^\circ \) and \( 60^\circ \)), the tensile gravity forces provoke decrease of axial force \( F_z(s) \) in the upper zone of the DS and the mode of bifurcational buckling \( \delta y(s) \) acquires the shape of boundary effect (Fig. 3). As this takes place, the friction forces only slightly influence on the critical value \( R_{cr} \) and function \( \delta y(s) \). But when the \( \beta \) angle tends to the value \( \beta = \arctg \mu \), the gravity and friction forces balance each other, the equality \( \cos \beta - \mu \sin \beta = 0 \) takes place, function \( F_z(s) \) becomes constant, and Eq. (22) takes the simplified form:

\[
\delta y'' + \frac{R}{EI} \delta y''' + \frac{f \sin \beta}{aEI} \delta y = 0 \quad (24)
\]

This equation with conditions (23) can be solved analytically.

Indeed, assume

\[
\delta y(s) = \delta C \sin \frac{\pi ns}{S} \quad (25)
\]

Then, one can receive from Eqs. (24) and (25)

\[
\left( \frac{m}{S} \right)^2 - R \left( \frac{m}{S} \right)^2 + \frac{f \sin \beta}{a} \left( \frac{S}{\pi} \right)^2 = 0 \quad (26)
\]

This equality gives eigen values \( R \), for different numbers \( n \)

\[
R_n = EI \left( \frac{m}{S} \right)^2 + \frac{f \sin \beta}{a} \left( \frac{S}{\pi} \right)^2 \quad (27)
\]

To calculate critical value of \( R \), it is necessary to minimize \( R_n \) with respect to \( n \). Use the condition \( dR_n/dn = 0 \). Then,

\[
\frac{dR_n}{dn} = 2EI \left( \frac{m}{S} \right)^2 n - \frac{2f \sin \beta}{a} \left( \frac{S}{\pi} \right)^2 = 0 \quad (28)
\]

and

\[
n_{cr} = \frac{S}{\pi} \sqrt{\frac{f \sin \beta}{a}} \quad (29)
\]

Therefore,

\[
R_{cr} = 2 \sqrt{EIf \sin \beta / a} \quad (30)
\]

Note that this force does not depend on the DS length.

In this instant, \( \beta = 90^\circ - \arctg \mu = 78.69^\circ \) and the values of these parameters are equal to \( n_{cr} = 22.45 \), \( R_{cr} = 225856 \) N. The considered state is represented by position 4 in Table 1. As may be seen, the critical force \( R_{cr} \) and buckling mode do not exhibit any essential changes for the frictionless model, but under conditions
Fig. 3  Boundary effect in the bifurcational mode of a DS in an inclined bore-hole.

Fig. 4  Expanding of the boundary effect with enlargement of the bore-hole inclination angle.

Fig. 5  Harmonic mode of stability loss of the DS inclined under friction angle.
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Table 1  Critical force $R_{cr}$ and buckling mode $\delta y(s)$ for the case $S = 500$ m and $a = 0.166$ m.

<table>
<thead>
<tr>
<th></th>
<th>Frictionless model</th>
<th>Frictional model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Inclination angle $\beta = 45^\circ$</td>
<td>$R_{cr} = 203.772$ kN</td>
</tr>
<tr>
<td></td>
<td>Frictionless model</td>
<td>Frictional model</td>
</tr>
<tr>
<td></td>
<td>$\delta y(s)$</td>
<td>$\delta y(s)$</td>
</tr>
<tr>
<td>2</td>
<td>$\beta = 60^\circ$</td>
<td>$R_{cr} = 221.771$ kN</td>
</tr>
<tr>
<td>3</td>
<td>$\beta = 78.495^\circ$</td>
<td>$R_{cr} = 230.962$ kN</td>
</tr>
<tr>
<td>4</td>
<td>$\beta = 78.69^\circ$</td>
<td>$R_{cr} = 230.942$ kN</td>
</tr>
<tr>
<td>5</td>
<td>$\beta = 79.06824^\circ$</td>
<td>$R_{cr} = 230.997$ kN</td>
</tr>
<tr>
<td>6</td>
<td>$\beta = 85^\circ$</td>
<td>$R_{cr} = 230.962$ kN</td>
</tr>
</tbody>
</table>
of the friction force presence, the boundary harmonic wavelet expands with the $\beta$ enlargement (Fig. 4). It propagates throughout the whole length of the DS (position 3) and becomes a simple sinusoid when angle of its inclination equals the friction angle (position 4). Schematic of the bifurcation mode for this state is shown in Fig. 5. After transition through this state, the buckling modes are less ordered (positions 5 and 6). At the same time, if friction forces equal zero, the buckling mode appears as boundary wavelet and the $R_c$ value enlarges with angle $\beta$ enlargement.

It issues from Eq. (29) that sinusoid semi wave lengths are defined by the formula

$$\lambda_c = S/n = \pi \sqrt{aE I / (f'' \sin \beta)}$$

As this formula and numeric calculations testify, the stability loss mode semi step $\lambda_c \approx 22.72$ m for all examples represented in Table 1.

It is mentioned above that the stated problem is singularly perturbed and so its solutions, as a rule, have the shapes of boundary effects concentrated in small vicinity of the DS end. The problems of this type are known as non-classical ones [22]. That is why one would expect that they are not sensitive to enlargement of the DS length. Really, this conjecture is corroborated by results of analysis of the DS 1,000 m in length. As evident from Table 2, the critical axial forces $R_c$ retained their values just as for frictionless model, so also for the case of the friction presence, though they slightly enlarged with the angle $\beta$ enlargement. The modes of stability loss likewise did not change their shapes and step lengths.

Of some interest is the question of how the clearance magnitude influences on the stability loss process. In Tables 3 and 4, the calculation results for the DS buckling in the bore-hole channels of clearance 0.08 m are presented. As the data of these Tables suggest, the clearance decrease is associated with increase of the critical forces $R_c$, and again, the bifurcational values of the $R$ force nearly coincide for the short ($S = 500$ m) and long ($S = 1,000$ m) DSs. Particular attention should be given to the effect of the bifurcation mode transformation of the DS 500 m in length. When the DS is immovable and friction forces are eliminated, the $R_c$ forces enlarge with the $\beta$ angle increase, though the modes of stability loss transform insignificantly. But in the presence of friction forces, the effect of gradual transformation of the bifurcation mode from boundary effect to disordered harmonic is more obvious with the $\beta$ enlargement.

It is also interesting to note that with the $\beta$ enlargement, the distance between two zero points of the bifurcation mode reduces from $\lambda \approx 40.5$ m for $\beta = 45^\circ$ to $\lambda \approx 31$ m for $\beta = 85^\circ$.

The noted peculiarities of the DS buckling are associated with several factors. When the $\beta$ angle is not large, the DS is preloaded by tensile axial force in its upper zone and by compressive force in its lower part. In this case the stress-strain state of the DS is essentially heterogeneous. At the same time, the normal component the gravity force, compressing the DS to the bore-hole wall and impeding its buckling, is not large and the DS freely buckles below. With enlargement of the $\beta$ angle the tensile forces diminish, the DS stress-strain state becomes more homogeneous, and stabilizing effect of the tensile force decreases. But contact and friction forces, as well as the normal component of the gravity force, enlarge. This feature is responsible for the stabilizing effect up growth. Interaction of these conflicting properties brings to insignificant enlargement of critical force $F_c$ with angle $\beta$ rise.

6. Conclusions

(1) In the paper, a mathematic model for computer analysis of bifurcational buckling of a drill string in cylindrical channel of an inclined bore-hole is elaborated. The constitutive equation is deduced with allowance made for action of gravity, contact, and friction forces;

(2) On the basis of this equation, the eigen value problem is stated. The techniques for its numerical solution are proposed;
Table 2  Critical force $R_c$ and buckling mode $\delta y(s)$ for the case $S = 1,000$ m and $a = 0.166$ m.

<table>
<thead>
<tr>
<th>Inclination angle $\beta$</th>
<th>Frictionless model</th>
<th>Frictional model</th>
</tr>
</thead>
<tbody>
<tr>
<td>45°</td>
<td>$R_c = 203.772$ kN</td>
<td>$R_c = 202.129$ kN</td>
</tr>
</tbody>
</table>

| 60°                       | $R_c = 221.771$ kN | $R_c = 219.441$ kN |

| 60.495°                   | $R_c = 230.942$ kN | $R_c = 227.993$ kN |

| 78.69°                    | $R_c = 230.962$ kN | $R_c = 225.856$ kN |

| 79.06824°                 | $R_c = 230.995$ kN | $R_c = 227.684$ kN |

| 85°                       | $R_c = 230.622$ kN | $R_c = 228.841$ kN |
Critical Buckling of Drill Strings in Cylindrical Cavities of Inclined Bore-Holes

Table 3  Critical force $R_c$ and buckling mode $\delta(s)$ for the case $S = 500$ m and $a = 0.08$ m.

<table>
<thead>
<tr>
<th></th>
<th>Frictionless model</th>
<th>Frictional model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Inclination angle $\beta = 45^\circ$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$F_z$, kN</td>
<td>$\delta(s)$, m</td>
</tr>
<tr>
<td></td>
<td>$R_c = 288.267$ kN</td>
<td>$R_c = 286.618$ kN</td>
</tr>
<tr>
<td>2</td>
<td>$\beta = 60^\circ$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$F_z$, kN</td>
<td>$\delta(s)$, m</td>
</tr>
<tr>
<td></td>
<td>$R_c = 315.273$ kN</td>
<td>$R_c = 312.936$ kN</td>
</tr>
<tr>
<td>3</td>
<td>$\beta = 78.495^\circ$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$F_z$, kN</td>
<td>$\delta(s)$, m</td>
</tr>
<tr>
<td></td>
<td>$R_c = 330.396$ kN</td>
<td>$R_c = 325.342$ kN</td>
</tr>
<tr>
<td>4</td>
<td>$\beta = 78.69^\circ$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$F_z$, kN</td>
<td>$\delta(s)$, m</td>
</tr>
<tr>
<td></td>
<td>$R_c = 330.450$ kN</td>
<td>$R_c = 325.342$ kN</td>
</tr>
<tr>
<td>5</td>
<td>$\beta = 79.06824^\circ$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$F_z$, kN</td>
<td>$\delta(s)$, m</td>
</tr>
<tr>
<td></td>
<td>$R_c = 330.548$ kN</td>
<td>$R_c = 327.035$ kN</td>
</tr>
<tr>
<td>6</td>
<td>$\beta = 85^\circ$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$F_z$, kN</td>
<td>$\delta(s)$, m</td>
</tr>
<tr>
<td></td>
<td>$R_c = 330.898$ kN</td>
<td>$R_c = 325.406$ kN</td>
</tr>
</tbody>
</table>
Table 4  Critical force $R_{cr}$ and buckling mode $\delta y(s)$ for the case $S = 1,000$ m and $a = 0.08$ m.

<table>
<thead>
<tr>
<th></th>
<th>Frictionless model</th>
<th>Frictional model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Inclination angle $\beta = 45^\circ$</td>
<td></td>
</tr>
<tr>
<td>(1)</td>
<td>$F_x$, kN</td>
<td>$R_{cr} = 288.267$ kN</td>
</tr>
<tr>
<td>(2)</td>
<td>$\delta y$, m</td>
<td>$\delta y$, m</td>
</tr>
<tr>
<td>(3)</td>
<td>$F_x$, kN</td>
<td>$F_x$, kN</td>
</tr>
<tr>
<td>2</td>
<td>$\beta = 60^\circ$</td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>$R_{cr} = 315.273$ kN</td>
<td>$R_{cr} = 312.936$ kN</td>
</tr>
<tr>
<td>(5)</td>
<td>$\delta y$, m</td>
<td>$\delta y$, m</td>
</tr>
<tr>
<td>3</td>
<td>$\beta = 78.495^\circ$</td>
<td></td>
</tr>
<tr>
<td>(6)</td>
<td>$R_{cr} = 330.390$ kN</td>
<td>$R_{cr} = 327.364$ kN</td>
</tr>
<tr>
<td>(7)</td>
<td>$\delta y$, m</td>
<td>$\delta y$, m</td>
</tr>
<tr>
<td>4</td>
<td>$\beta = 78.69^\circ$</td>
<td></td>
</tr>
<tr>
<td>(8)</td>
<td>$R_{cr} = 330.450$ kN</td>
<td>$R_{cr} = 325.342$ kN</td>
</tr>
<tr>
<td>(9)</td>
<td>$\delta y$, m</td>
<td>$\delta y$, m</td>
</tr>
<tr>
<td>5</td>
<td>$\beta = 79.06824^\circ$</td>
<td></td>
</tr>
<tr>
<td>(10)</td>
<td>$R_{cr} = 330.548$ kN</td>
<td>$R_{cr} = 327.171$ kN</td>
</tr>
<tr>
<td>(11)</td>
<td>$\delta y$, m</td>
<td>$\delta y$, m</td>
</tr>
<tr>
<td>6</td>
<td>$\beta = 85^\circ$</td>
<td></td>
</tr>
<tr>
<td>(12)</td>
<td>$R_{cr} = 330.898$ kN</td>
<td>$R_{cr} = 329.199$ kN</td>
</tr>
<tr>
<td>(13)</td>
<td>$\delta y$, m</td>
<td>$\delta y$, m</td>
</tr>
</tbody>
</table>
(3) The problem is shown to be singularly perturbed and hence its solutions (eigen modes) are represented as boundary effects (harmonic wavelets) localized at the vicinity of the lower end of the drill string;

(4) Influence of friction presence and values of the bore-hole inclination angle, its length and clearance on the evolution of the bifurcation mode is studied. The mode shape transformation is demonstrated can be accompanied by its expanding and displacing to upper zones of the drill string;

References


