Low-Cost GPS/INS Integrated Land-Vehicular Navigation System for Harsh Environments Using Hybrid Mamdani AFIS/KF Model

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Abstract: Nowadays, GPS (global positioning system) receivers are aided by INS (inertial navigation systems) to achieve more precision and stability in land-vehicular navigation. KF (Kalman filter) is a conventional method which is used for the navigation system to estimate the navigational parameters, when INS measurements are fused with GPS data. However, new generation of INS, which relies on MEMS (micro-electro-mechanical systems) based low-cost IMUs (inertial measurement units) for the land navigation systems, decreases the accuracy and the robustness of navigation system due to their inherent errors. This paper provides a new method for fusing the low-cost IMU and GPS measurements. The proposed method is based on KF aided by AFIS (adaptive fuzzy inference systems) as a promising solution to overcome the mentioned problems. The results of this study show the efficiency of the proposed method to reduce the navigation system errors in comparison with KF alone.

Key words: GPS/INS integration, KF, AFIS.

1. Introduction

The last two decades have shown an increasing trend in the use of location based services in automotive vehicles applications including car tracking for theft protection, fleet management services, automated car navigation and emergency assistance. Most of these applications rely entirely on a GPS (global positioning system) receiver to provide ubiquitous navigational solution of the monitored vehicle. In order to provide such a reliable solution, GPS requires an optimal operating condition that is a clean line of sight to at least four satellites. Unfortunately, this condition cannot be fulfilled at all times, especially when driving in severe urban environment where buildings, concrete structures, high passes and tunnels may attenuate, block or reflect incoming signals resulting in a poorly accurate navigation solution. Indeed, multipath is one of the main sources of positioning errors for standalone

GPS receivers used in severe urban environments. Multipath error on pseudo-range measurements can reach hundreds of meters [1], thus introducing significant errors in position computation.

To reduce the impact of multipath, GPS is often combined to an INS (inertial navigation systems), which is a self-contained dead reckoning system based on the integration of linear accelerations and angular rates. The procedure, in which raw inertial measurements are processed to compute the position, the velocity and the attitude of a mobile, is referred as the INS mechanization process. Unfortunately, the low-cost standalone INS positioning tend to diverge over time due to the significant inherent errors contained on low-cost MEMS (micro-electro-mechanical systems) based inertial sensors.

To overcome shortcomings of standalone GPS or INS, both systems can be coupled together to form an integrated navigation system. The INS/GPS integrated systems exploit beneficial features of each individual system (i.e., short term precision of INS, and long-term

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stability of GPS), thus providing precise and ubiquitous navigation. The INS/GPS integration is a mature topic that is widely covered in the literature when it comes to high-end systems [2-5]. However, the recent studies reported several shortcomings related to the use of low-cost MEMS based IMUs (inertial measurement units), particularly with regard to the use of linearized error models.

Various kinds of KF (Kalman filter) have been extensively utilized for the integration of GPS with INS, and they can determine the optimal estimation of the system with minimum errors [6-9]. KF needs an adequate knowledge on the dynamic process of the system and measurement model. Another problem is that the precision of MEMS-based IMUs is significantly decreased by their drift and bias errors. This problem ruins the system performance, when employing a KF in car navigation [10]. Moreover, the critical problem of the KF is the big degree of covariance divergence due to its modeling error [11].

There are two different solutions to solve the covariance divergence of KF: The first one is using the un-modeled states in KF. However, this solution increases the computational complexity of the system [11]. The second one is usage of the process noise to develop the confidence, which can intercept the KF to dismiss new values for estimating the state vector [10, 12, 13]. This paper uses Mamdani AFIS (adaptive fuzzy inference system) based on the employing the process noise. The proposed model can be used for tuning phase of the KF by tracking the covariance values.

The paper is organized as follows: Section 2 presents

the GPS/INS integration methods; Section 3 describes the proposed model to fuse the INS and GPS measurements in deeply; The results are provided in Section 4, whereas Section 5 draws the conclusion and future works.

2. GPS/INS Integration Methods

2.1 GPS/INS Coupling

There are different structures for the GPS/INS integration, namely, uncoupled, loosely coupled, tightly coupled and ultra-tightly coupled. Table 1 summarizes the advantages and the disadvantages of them. It is common to integrate the INS and GPS through loosely coupled structure. Because it not only maintains independency of stand-alone GPS and INS solutions, but it can also provide more robustness for the navigation solution [14]. The loosely coupled integration is generally preferable in the GPS/INS integration as being composed of three distinct entities, which are the stand-alone GPS solution, the stand-alone INS solution and the GPS/INS coupled solution. This architecture, which is presented in Fig. 1, is shared by several authors [15-17].

In general, the KF filters are recognized as the most conventional method to estimate the navigation systems (Fig. 1) [18]. The error model in the dynamical model includes state errors of position, velocity and attitude, augmented by sensor state errors [19].

2.2 KF (Kalman Filters)

An optimal estimator is a state estimator whose gain is dynamically calculated to optimize a certain system

Table 1 A brief summary of commonly used INS/GPS integration architectures [14].

Type	Advantage	Disadvantage
Un-coupled	Simplicity of algorithm	Instability in GPS outages
Loosely coupled	• Separate INS and GPS KF; • Small size of individual KF; • Less computation complexity	• Sub-optimal performance; • Min four satellite requirement
Tightly coupled	• Optimal accuracy; • Max 4 satellites requirement	• Large error size state model; • More complex processing
Ultra-tightly coupled	• Reduce GPS dynamic stress; • Less jamming errors	Special hardware requirement

Fig. 1 Loosely coupled GPS/INS integration.

performance criteria. One of the widely used optimization criteria is the mean square error of the estimate. The optimal estimator based on this optimization criterion is called the Kalman filter. KF has been accepted as a conventional method to estimate the GPS/INS integration. The derivation of an error model which are applied in the KF can start with the construction of full scale true error models [18, 19].

First, a definition of priori error in the estimated of the state vector and the associated covariance matrix can be presented with:

$$\underline{e}_k^- = \underline{x}_k - \hat{\underline{x}}_k^- \quad (1)$$

$$P_k^- = E[\underline{e}_k^- \underline{e}_k^{-T}] = E[(\underline{x}_k - \hat{\underline{x}}_k^-)(\underline{x}_k - \hat{\underline{x}}_k^-)^T] \quad (2)$$

where, $\underline{e}_k^-$ is priori error vector and P_k^- is priori error covariance matrix.

According to the equations of state estimator present in the previous section, the estimated a posteriori state vector can be obtained by a linear combination of the vector of noisy measurements and estimated as a priori:

$$\hat{\underline{x}}_k^+ = \hat{\underline{x}}_k^- + K_k(\underline{z}_k - H_k \hat{\underline{x}}_k^-) \quad (3)$$

The error covariance matrix associated with this estimate posteriori can be calculated from the following expression:

$$P_k^+ = E[\underline{e}_k^+ \underline{e}_k^{+T}] = E[(\underline{x}_k - \hat{\underline{x}}_k^+)(\underline{x}_k - \hat{\underline{x}}_k^+)^T] \quad (4)$$

$$P_k^+ = E\left[(\underline{x}_k - \hat{\underline{x}}_k^- + K_k(\underline{z}_k - H_k \hat{\underline{x}}_k^-))(\underline{x}_k - \hat{\underline{x}}_k^- + K_k(\underline{z}_k - H_k \hat{\underline{x}}_k^-))^T\right] \quad (5)$$

$$P_k^+ = (I - K_k H_k)P_k^-(I - K_k H_k)^T + K_k R_k K_k^T \quad (6)$$

The diagonal of the error covariance matrix contains the variance of the estimation error of all system states. Thus, the trace of this matrix represents the sum of the variance and it is thereby an unbiased indicator of the mean square error of the estimate. The gain of the Kalman filter, commonly called the Kalman gain, can be selected to minimize the trace of the matrix P_k . This can be achieved by the following equality:

$$\frac{d(\text{trace } P_k)}{dK_k} = 0 = -2(H_k P_k^-)^T + 2K_k(H_k P_k^- H_k^T + R_k) \quad (7)$$

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \quad (8)$$

Substituting K_k within Eq. (6) by the expression given by Eq. (8), it is possible to simplify the expression of the matrix P_k^+ as:

$$P_k^+ = (I - K_k H_k)P_k^- \quad (9)$$

After repeating the previously presented equations of the state estimator, spreading the error covariance matrix, and calculating the Kalman gain, the Kalman filter algorithm can be summarized by the equations presented in Table 2. A system level block diagram of the discrete KF is shown in Fig. 2.

The time update equations can be calculated thought the estimation equations, while the measurement update equations can be presented thought the correction equations. Therefore, the final estimation algorithm resembles an estimation-correction algorithm to solve the numerical problems as shown in Fig. 3.

The multiple dynamic models used in navigation are

Table 2 Description of KF variables.

Parameter	Description
Φ	State transition matrix of a discrete linear
X	State vector of a linear dynamic system
$\hat{X}(-)$	Prediction or a priori value of the estimated state vector of a linear dynamic system
$\hat{X}(+)$	Corrected or a posteriori value of the estimated state vector of a dynamic system
Z	Measurement vector or observation vector
K	Kalman gain matrix
H	Measurement sensitivity matrix or observation matrix which defines the linear relationship between the state of the dynamic systems and measurements that can be made
$P(-)$	Predicted or priori value of estimated covariance of state estimation uncertainty in matrix form
$P(+)$	Corrected or posteriori value of the estimated covariance of state estimation uncertainty in matrix form
w	Process noise
η	Measurement noise

Fig. 2 Discrete-time KF diagram. **Fig. 3** Recursive process of prediction.

non-linear, and, therefore, they can not be directly expressed in terms of the general form. The linearization of these systems around an optimal state vector is first requirement, so they can be expressed with using this standard form. First, it is defined a non-linear system such that:

$$\dot{\underline{x}} = f(\underline{x}) + G \cdot \underline{\eta}_p \quad (10)$$

$$\underline{z} = h(\underline{x}) + \underline{\eta}_m \quad (11)$$

where, f and h are non-linear functions. It is assumed that an optimal state vector $\underline{x}^*$ is known and it is defined as follows:

$$\underline{x}^* = \underline{x} - \delta \underline{x} \quad (12)$$

where, $\underline{x}$ is optimal state trajectory and $\delta \underline{x}$ is state of error vector. Thus, Eqs. (8) and (9) can be rewritten as a function of the optimal state vector and the error state vector as:

$$\dot{\underline{x}}^* + \delta\dot{\underline{x}} = f(\underline{x}^* + \delta\underline{x}) + G \cdot \underline{\eta}_p \quad (13)$$

$$\underline{z} = h(\underline{x}^* + \delta\underline{x}) + \underline{\eta}_m \quad (14)$$

It is assumed that the status error vector can be neglected, and it is possible to approximate the non-linear functions f and h using a Taylor series. Considering only the terms of first order, the result is:

$$\dot{\underline{x}}^* + \delta\dot{\underline{x}} \approx f(\underline{x}^*) + \left. \frac{\partial f(\underline{x})}{\partial \underline{x}} \right|_{\underline{x}=\underline{x}^*} \delta\underline{x} + G \cdot \underline{\eta}_p \quad (15)$$

$$\underline{z} \approx h(\underline{x}^*) + \left. \frac{\partial h(\underline{x})}{\partial \underline{x}} \right|_{\underline{x}=\underline{x}^*} \delta\underline{x} + \underline{\eta}_m \quad (16)$$

By selecting an optimal state vector as equality $\dot{\underline{x}}^* = f(\underline{x}^*)$, the following linearized general form is obtained:

$$\delta\dot{\underline{x}} = F \cdot \delta\underline{x} + G \cdot \underline{\eta}_p \quad (17)$$

$$\delta\underline{z} = H \cdot \delta\underline{x} + \underline{\eta}_m \quad (18)$$

where:

$$F = \left. \frac{\partial f(\underline{x})}{\partial \underline{x}} \right|_{\underline{x}=\underline{x}^*} \quad (19)$$

$$H = \left. \frac{\partial h(\underline{x})}{\partial \underline{x}} \right|_{\underline{x}=\underline{x}^*} \quad (20)$$

$$\delta\underline{z} = \underline{z} - h(\underline{x}^*) \quad (21)$$

2.3 AFISs (Adaptive Fuzzy Inference Systems)

AFISs (adaptive fuzzy inference systems) is a rule-based expert method for its ability to mimic human thinking and the linguistic concepts rather than the typical logic systems [20]. The advantage of the AFIS appears when the algorithm of the estimation states becomes unstable due to the system high complexity [21].

AFIS architecture includes three parts: fuzzification, fuzzy inference and defuzzification. The first part is responsible to convert the crisp input values to the fuzzy values, the second part formulates the mapping from the given inputs to an output, and the third part converts the fuzzy operation into the new crisp values. The AFIS are able to convert the inaccurate data to normalized fuzzy crisp which are represented by MF (membership functions) and the confidence-rate of the inputs. AFIS are capable to choose an optimal MF under certain convenient criteria meaningful to a specific application [22, 23].

Mamdani and Sugeno are the two practical AFIS types which were used in several studies [24-27]. The main difference between these two fuzzy algorithms is based on the process complexity and the rule definition. Another important aspect to take into consideration is that Mamdani needs more processing time than Sugeno. Sugeno type also provides less flexibility in the system design compared to the Mamdani type. In general, Mamdani type is more efficient and accurate than Sugeno type [24, 25]. All those reasons have motivated us to use the Mamdani type (Fig. 4) to design the fuzzy-part of the proposed GPS/INS integration model.

3. Proposed AFIS/KF System

This paper employs IAE (innovation adaptive estimation) concept in the fuzzy part of the proposed model [23, 28, 29]. The dynamic characteristics of the vehicle motion are based on the KF process. The AFIS can be exploited to increase the accuracy and the KF. Additionally, AFIS part can prevent the divergence in the tuning phase of KF. Hence, Mamdani AFIS is used as a structure for implementing the tuning of the nonlinear error model. Fig. 5 shows the proposed hybrid AFIS-KF model.

Fig. 4 General overview of AFIS using Mamdani type.

Fig. 5 The proposed hybrid Mamdani AFIS-KF model in navigation system.

Fig. 6 The proposed Mamdani AFIS part of the hybrid AFIS-KF in navigation system.

The proposed AFIS model is based on the covariance matrix for the input of the Mamdani AFIS, as well as the difference of the actual and the estimated covariance matrices. Fig. 6 presents the proposed AFIS overview that is used in this paper. The estimated covariance matrix based on the innovation process is computed partly in the KF by:

$$\hat{S}_k = H_k P_k H_k^T + R_k \quad (22)$$

where, $\hat{S}_k$ and H_k represent the estimated covariance and the design matrices, respectively. P_k and R_k are previous covariance update and the measurement noise covariance matrices. The actual covariance matrix (γ) according to Ref. [30] can be presented by:

$$\gamma_k = \frac{1}{W} \sum_{i=k-W+1}^k (z_i - H_i \hat{x}_i) \times (z_i - H_i \hat{x}_i)^T \quad (23)$$

where, W is the window size which is given by the moving window technique. Thus, the difference between the actual and estimated covariance matrices (ε) can be presented by:

$$\varepsilon_k = \gamma_k - \hat{S}_k \quad (24)$$

In fact, the value of ε can display the level of divergence between the actual and the estimated covariance matrices. When the value of ε is close to zero, the estimated and the actual covariance matrices will be very similar and the absolute value of the ε can be neglected. However, if the value of ε is not near to zero (smaller or greater than zero), an adaptation is considered in the algorithm and the value of R_k in Eq. (22) should be adjusted to compensate this difference.

The proposed rules assessment according to the difference between the actual and the estimated

covariance matrices is described as three scenarios of MF. If the value of ε is higher than zero, then the value of R_k is turned down in accordance with the value of δR_k ; If the value of the ε is less than zero, then the value of R_k is turned up in accordance with the value of δR_k ; If the value of ε is close to zero, then R_k is unchanged. where, $\delta R_k = R_k - R_{k-1}$.

Fig. 7a explains the grade of the membership parameter in the five fuzzy sets. For example, if δR_k is close to zero, the grade of MF is shown with I2 in the AFIS. As the degree of MF in the fuzzy set is I1 or I4, it approaches to minus one.

The MF of the output for the proposed AFIS model is presented in Fig. 7b. The output is the value of δR_k which changes from -1 to 1 . Moreover, it is fuzzified in seven levels from O1 to O7. These seven levels are important to differentiate between the likelihood levels. The next section presents the performance of the proposed AFIS/KF within simulation experiments.

4. Simulation Experiments and Analysis

Results of the AFIS/KF method are compared with those obtained from conventional KF to evaluate the performance of the proposed model. A loosely coupled structure for GPS/INS integration is considered in this study. The navigation system is shown in Fig. 8. The error state related to system can be presented as:

$$\delta x = [\delta n \quad \delta e \quad \delta v_n \quad \delta v_e \quad \delta \psi]^T \quad (25)$$

where, δn and δe are the north and the east position errors; and δv_n and δv_e are the north and the east

Fig. 7 MF (membership functions): (a) input; (b) output.

Fig. 8 Navigation system.

velocity errors, respectively. $\delta\psi$ represents the heading error.

The vehicle trajectory was generated in three main different motions (Fig. 9). Three main time intervals are considered for the straight and circle motions of the trajectory. The straight was performed 0~1,200 s and 2,401~3,600 s. Time segments and the counter-clockwise circle was done in 1,201~2,400 s to some segment. The vehicle moves with a constant speed (4.5π m/s).

Two main points are considered for GPS availability signals in the scenario. First, there are different error

sources, like multipath errors for satellite constellations. It is called “normal situation” that maximum GPS unavailability is 5 s. Second, five obstructed segments were considered in the GPS signals during the trajectory. It is called “bad situation” which are shown with the red points in Fig. 9. Table 3 shows the duration, start and stop point of each outage.

Fig. 10 shows the results in normal situation of GPS without GPS outage more than 5 s. Fig. 11 depicts the errors of the KF and AFIS-KF model during the five assumed outages. Fig. 12a shows a comparison of RMSE (Root Mean Square Errors) between KF and the proposed AFIS-KF model in the normal situation. This figure also shows that the AFIS-KF attenuates the errors by 30%, compared to the KF when the GPS is working in the normal situation. Fig. 12b presents a comparison of RMSEs between KF and the proposed hybrid AFIS-KF model. This figure obviously illustrates the significant improvement of navigation solution when using the proposed AFIS-KF model. It is observed that, after applying AFIS-KF, errors were decreased by 75% compare to KF.

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Fig. 9 Trajectory in three main time-segments due to the dynamics characteristic.

Table 3 Definition of the GPS outages based on their durations in the trajectory.

Outages (No.)	Period (s)	Start-point (s)	Stop-point (s)
1	60	296	356
2	90	896	986
3	92	1,510	1,602
4	83	2,160	2,243
5	48	3,230	3,278

Fig. 10 Position, velocity and heading errors in normal situation of GPS.

Fig. 11 Position, velocity, and heading errors during the bad situation of GPS.

Fig. 12 Comparison of position and velocity RMS errors via two approaches—KF and AFIS/KF: (a) normal situation of GPS; (b) bad situation of GPS.

5. Conclusions

This paper has proposed the GPS/INS integration

based on a hybrid Kalman filter and a Mamdani type of adaptive fuzzy inference system. The proposed model improves the performance of the navigation system,

whereas the traditional KF needs a priori knowledge of the system. The traditional KF does not perform precisely in GPS outages. Thus, the proposed AFIS-KF is exploited to prevent the divergence in the KF tuning process. The proposed AFIS model is based on an innovation process of the covariance matrix as the input of the AFIS. The major benefit of this model is that the positioning accuracy is increased in comparison with usual KF-based integration. The main reason is that the estimated value of the covariance matrix is adapted to its actual value in the proposed hybrid AFIS-KF model. The future work will consist to verify the proposed hybrid AFIS-KF model with tightly coupled INS/GPS integration in harsh environment.

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