Mathematical Modeling of the Hydrodynamics of an EGSB Reactor

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Received: April 23, 2014 / Accepted: May 21, 2014 / Published: June 25, 2014.

Abstract: Generally, in the literature, the hydrodynamic behavior of an EGSB (expanded granular sludge bed) reactor is considered as a complete mix reactor. Few works study in detail the flow of such reactors. The aim of this work was to study, in detail, the hydrodynamics of an EGSB reactor and to propose a mathematical model to describe its flow. A 3.04 L reactor was used with HRT (hydraulic retention time) of 12 h, affluent flow rate of 4 mL·min⁻¹, and the recirculation flow rate was changed to study three different upflow velocities in the tube (6, 8 and 10 m·h⁻¹). The pulse input method was used, with the use of blue dextran as tracer. In order to consider the dimensional differences between the tube and the separator, the reactor was divided into two regions (tube and separator). Initially, a model with two tubular reactors with dispersion in series was proposed and the Peclet number was adjusted for the two regions. It was observed that the region of the tube shows the behavior of a tubular reactor with high dispersion, whereas the region of the separator shows the behavior of a complete mix reactor. In order to simplify the equation, and by knowing that the concentration profile along the reactor was almost constant, a model of two CSTRs (continuous stirred tank reactors) was proposed in series and the number of reactors (N) was set. The best combination was five CSTRs, three in the tube region and two in the separator region. The presented models were equivalent and can be used to describe the hydrodynamic behavior of the EGSB reactor.

Key words: EGSB reactor, hydrodynamics, mathematical modeling.

1. Introduction

The EGSB (expanded granular sludge bed) reactors emerged as an improvement on the UASB (upflow anaerobic sludge blanket) reactors. They were developed in an attempt to solve problems observed in practice in the UASB reactors, such as the occurrence of dead zones, preferential flow, short circuit, among others [1, 2]. It has higher height/diameter ratio, allowing the application of higher velocities, reaching values of 10 m·h⁻¹ or even higher, while in the UASB reactor, they should not exceed 1.5 m·h⁻¹ [3, 4]. Industrially, the EGSB reactors have heights in the range between 7 m and 14 m [5].

Due to the application of higher velocities in the EGSB reactors, there is a greater expansion of the medium, providing a greater biomass-effluent contact, which may positively affect the increase in efficiency of the treatment [1, 6, 7].

The EGSB reactors are basically composed of an expanded bed and a three-phase separator. One feature that, most of the times, makes them differ from the UASB reactors, is the presence of recirculation of the medium, which, according to Sant’ Anna [8], assists in diluting biodegradable but inhibitory or toxic
substances, originally present in the effluent to be treated.

Biological reactors, as the EGSB, have their operation based on the formation of microbiological aggregates, on a carrier or dispersed in a medium [2].

The EGSB reactor has been studied for the treatment of various effluents, for example domestic sewage [1, 9], sugar cane vinasse [2, 7] and food industries, such as production of palm oil [10, 11], parboiled rice [12], among others.

Hydrodynamic models are used to formulate a flow field [13]. According to Carvalho et al. [14], hydrodynamics plays an important role in the study of anaerobic reactors, because it can influence the rates of biological reactions through changes in the rate of mass transfer and in the distribution of reactions along the reactor.

According to Levenspiel [15], in the hydrodynamic characterization of reactors, generally, two types of ideal flows are considered in the modeling, the plug flow (tubular) and the complete mix, because, for the majority of cases, such flows result in different performances and usually one of them suits the chosen process. In the plug flow, the flow occurs neatly along the reactor, and if the concentration of a particular parameter is equal at any point of the reactor, it is an ideal complete mix [16].

In the literature, usually, the hydrodynamics of EGSB reactors is considered as complete mix, for example, in the works by Brito and Melo [17], Fuentes [2] and López and Borzacconi [7], among others.

Brito and Melo [17] performed a pulse input hydrodynamic test in an EGSB and in a UASB, both with the volume of 480 mL. Lithium chloride was used as tracer. In each reactor, 1 mL of a 3.2 g L⁻¹ Li⁺ solution was injected. The authors considered the UASB reactor as an ideal tubular reactor and the EGSB reactor as complete mix one.

Fuentes [2] modeled an EGSB reactor considering a flow pattern as completely mixed, due to the effects of recirculation, and they implemented and solved the model using gPROMS tool (Process Systems Enterprise Ltd.).

López and Borzacconi [7] worked with two EGSB reactors, of 6 L and 12 L using for the treatment of sugar cane vinasse. In the modeling, they considered the hypothesis of a complete mix with a simplified kinetic model consisting of two reactions in series with first-order kinetics. Matlab software was used.

Few researchers have sought to know in detail the behavior of EGSB reactors, e.g., Bhattacharyya and Singh [18], who modeled the EGSB reactor as a plug flow reactor with recirculation and dead space, with a high dispersion. The authors conducted tests using conductivity probes inserted through the sampling points along the reactor, and those were connected to a data acquisition system. The hydraulic retention time of 3.3, 5.5 and 9 h were used, and they tested different upflow velocities (1.1, 2.7, 5.3 and 8.7 m h⁻¹). Despite the data similarity to the ones from the complete mix model, the presence of plug flow in the lower upflow velocities was observed, and with the upflow velocity increase, the flow pattern was closer to a condition mix.

Regarding on UASB reactor, EGSB precursor, there are several studies for its operation and hydrodynamics besides many models with different flow combinations (tubular and complete mix).

For instance, Hanisch and Pires [19] considered a model where the UASB was divided into four parts, considering the inlet region as complete mixture, area part of it considered as a dead zone, the blanket part was considered as tubular reactor with low dispersion and the sedimentation region was considered as a tubular reactor with high dispersion. Wu and Hickey [20] modeled the UASB reactor as a stirred tank reactor followed by a tubular flow reactor. Bolle [21] divided the reactor into three compartments, considering the bed and the sludge mantle as complete mix and the separator as an ideal tube reactor. Ren et al. [22], Carvalho et al. [14], Rodríguez-Gómez et al. [23], among others, considered the UASB reactor as a
series of CSTR.

The main objective of this work was to study, in detail, the EGSB reactor hydrodynamics and to verify which hydrodynamic model best fits the reactor, so that it can be later used in the mathematical modeling of the operation of such reactors.

2. Materials and Methods

The EGSB reactor basically consists of a tube and a separator. In the tube, the biomass is kept dispersed, mainly due to the high recirculation rate; and the present microorganisms are responsible for the degradation of the wastewater that will be treated. In the three-phase separator, the solids (biomass) are maintained in the system; the liquid portion is divided between the part that recirculates to the reactor inlet and the part that leaves the reactor as a treated effluent; and the gas that leaves from the top of the reactor, where it can be quantified.

The EGSB reactor (Scheme 1) used in this study has a total effective volume of 3.04 L, tube and separator diameter of 0.05 m and 0.11 m, respectively, and height of the tube and separator, respectively, of 1.13 m and 0.11 m. For the hydrodynamic testing, the reactor was not inoculated, that is, it still contained no biomass.

The tests were performed at different upflow velocities in the tube (6, 8 and 10 m·h⁻¹). The theoretical HRT (hydraulic retention time) applied was 12 h. Affluent flow ($Q_a$) of approximately 4 mL·min⁻¹ and the recirculation flow ($Q_r$) of approximately 195, 260 and 320 mL·min⁻¹ were set in order to obtain the desired upflow velocities.

Hydrodynamic tests were performed by the stimulus-response method, by injecting 50 mL of the tracer blue dextran at a concentration of 10 g·L⁻¹. The absorbance readings were performed in a spectrophotometer ($\lambda = 650$ nm). The concentration of blue dextran was then obtained by the spectrophotometer calibration curve. The duration of the trials was approximately 3 times of the hydraulic retention time.

The choice of the tracer in this experiment was based on the work by Nardi et al. [24] and Souza [25] who have tested various tracers such as sodium chloride, Eosin Y, bromophenol blue, blue dextran, mordant violet, rhodamine WT and bromocresol green. They concluded that the blue dextran is the most suitable tracer for use in hydrodynamic tests in heterogeneous reactors. Nardi et al. [24] found that, for the blue dextran, the wavelength with the highest absorption was 650 nm.

3. Mathematical Modeling

3.1 Model of Tubular Reactor with Dispersion

The tubular reactor model is an example of model with distributed parameters, in which it is assumed that the properties vary along with the spatial coordinates, that is, along the reactor [26, 27].

Since it is a model where the reaction term will not be taken into consideration, the concentration over time is the sum of the convection and diffusion.
phenomena. Model equations and boundary conditions are described in Eqs. (1)-(4). $C$ is the concentration of dextran in the reactor outlet; $t$ is the time; $v$ is the upflow velocity; $D$ is the axial dispersion coefficient; $C_{in}$ (Eq. (4)) is the inlet concentration; $Q_a$ is the affluent flow; $C_a$ is the affluent concentration, which in this case is null; $Q_r$ is the recirculation flow; $C_r$ is the concentration of recirculation, $Q_i$ is the flow rate of the tracer and $C_i$ is the concentration of the tracer. Since the test was carried out by using the pulse input, the $Q_i$ and $C_i$ parameters were calculated from the time of injection and injected volume.

In the resolution of the equations of the model, the method of central finite differences of order 2 was used, of which first and second-order approaches are described in Eqs. (5) and (6), respectively. The goal of this method is to transform a problem of differential equations in a problem of algebraic equations by the discretization of the domain of the independent variable, by dividing the calculation domain into subdomains and then generating approximations for the dependent derivatives [26, 28].

$$\frac{\partial C}{\partial t} = -v \frac{\partial C}{\partial x} + D \frac{\partial^2 C}{\partial x^2} \quad (1)$$

$\text{In } z = 0; \quad D \frac{\partial C}{\partial z} = v(C_t - C_{in}) \quad (2)$

$\text{In } z = L; \quad \frac{\partial C}{\partial z} = 0 \quad (3)$

$$C_{in} = \frac{Q_a C_a + Q_i C_i}{Q_a + Q_i} \quad (4)$$

In a convective-diffusive system, the Peclet number ($Pe$) expresses the ratio between convection and diffusion, and the higher $Pe$, the higher the influence of the convective process for mass transport. Moreover, when $Pe$ is very low, the process is handled as a diffusive process [29, 30].

Based on such information, in this work, the $Pe$, which can be expressed by Eq. (8), was the setting parameter.

$$Pe = \frac{vL}{D} \quad (8)$$

In the proposed model, the reactor was divided into two regions, two tubular reactors in series: the first, the region of the tube, and the other, the separator region. The length of each region is divided into $N$ values of $\Delta z$, and the equations were solved using the tool ode 15 s from the 2011 Matlab program. The $Pe$ was adjusted for each of the regions using the method of least squares. A schematic drawing of the presented model is described in Scheme 2, the inlets and outlets that were taken into consideration can also be seen.

### 3.2 CSTR Model

The CSTR model is also known as complete (or perfect) mix model. It is an example of a lumped model, where it is considered that properties, such as concentration, do not vary along with the position coordinates [27].

The modeling is based on mass balance, where the mass that is accumulated in the volume control ($V$) per time unit is equal to the mass that enters it per time unit minus the mass that leaves it per time unit, as described in Eq. (9).

$$V \frac{dc_i}{dt} = Q_o C_{i_0} - Q C_i \quad (9)$$

In this model, the reactor was divided into two regions, tube and separator. And each region was divided into several CSTR ($n$-CSTR). The $N$ in each region was adjusted by using the least squares method and the Matlab program 2011. The scheme of the presented model is described in Scheme 3.
4. Results and Discussion

Most existing flow models do not describe with reasonable accuracy the behavior of the flow of anaerobic digesters [31]. According to Bhattacharyya and Singh [18], the standard mixture of EGSB reactors has not been studied in detail. In general, the models found in the literature to characterize the EGSB reactors do not consider the recirculation effect, i.e., it is the type of black box models, where only input and output are considered. In order to further study the hydrodynamics of EGSB reactors, adjusts to the tubular flow with dispersion model. By adjusting the Peclet number, it can be concluded that the phenomenon, convective or diffusive, predominates in both parts of the reactor (separator and tube).

Fig. 1 shows the experimental concentration curve obtained over time for the three velocities studied. It can be seen that the hydrodynamic behavior of the system is unchanged when the velocity is increased.

Since the diameter of the separator (0.11 m) is relatively larger than the diameter of the tube (0.05 m), causing significant change in the upward velocity, it was decided to divide the reactor into two tubular reactors in series. The $Pe$ was adjusted in accordance with the criteria of the least sum of squared errors (least squares method), using the fminsearch tool from 2011 Matlab.

For the three tested velocities, the separator behaved as a CSTR reactor, that is, $Pe$ presented values near zero, resulting in a predominance of diffusion process, in which the convective process can be disregarded. Whereas the tube showed low values of $Pe$ behaving as a tubular reactor with high dispersion, as observed in the work by Bhattacharyya and Singh [18].

The Peclet values obtained for the region of the tube were 5.1, 6.7, 13.7 for velocities of 6, 8 and 10 m·h$^{-1}$. In Fig. 2, the experimental curves and the model curves for the two tubular reactors in series for the three mentioned velocities are presented.
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The sum of squared errors in all tests showed values in the order of $10^4$, with an average correlation coefficient of 0.94.

The pulse time was estimated as 1 min, however, variations on this value may have been the cause of the difference between the peak and the experimental model. Another possible cause for this difference in peak is the beginning of data collection. In the experimental test, the time zero was considered immediately after the injection of the tracer, whereas for the modeling, time zero had to be considered the start of the injection.

According to Secchi [32], the more accurate the process description, the greater the resulting equations number, consequently, it is more difficult to treat. Thus, although they can be resolved, it is advisable to use approximations to reduce the equations and decrease their complexity.

By taking into consideration that the concentration profile along the reactor proved to be practically uniform, as can be seen in Fig. 3, it can be considered the approximation that a tube reactor shows the behavior of a series of $N$ complete mix reactors (NCSTR), with a very high $N$. Thus, it was chosen to test a simpler model.

The division of the reactor into two regions was maintained, however, for this model, the reactor was divided into two NCSTRs in series. Moreover, in order to seek which combination of reactors with equal volumes returns the least sum of squared errors, the value of $N$ was adjusted. The graphs of data from the experimental and NCSTR model for three studied velocities are shown in Fig. 4.

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**Fig. 2** Experimental and model curves of two tubular reactors with high dispersion in series, for the velocities of (a) 6 m·h$^{-1}$, (b) 8 m·h$^{-1}$ and (c) 10 m·h$^{-1}$.

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**Fig. 3** Concentration profile along the reactor to the three applied velocities.
It can be considered that the presented models are equivalent, and this similarity can also be observed by comparing the graphs of Fig. 2 to the graphs of Fig. 4.

Under the applied conditions, both models can be used to describe the hydrodynamic behavior of EGSB reactors. The advantage in using NCSTR model is that it is a simplified model equation, which makes it simpler to work with, and that will facilitate the resolution of the equations when adding the reactive terms.

Depending on the objective of the researcher, the tubular model, which uses the method of finite differences in its resolution, can be used to describe the hydrodynamics of EGSB reactor, however, if the $Pe$ presents higher values, oscillations caused by numerical problems can occur, as noted by Correia and Kwong [33].

5. Conclusions

The EGSB reactor has been extensively studied as an improvement on UASB reactor, however, its hydrodynamics is still little studied. In the present study, it was aimed to investigate the hydrodynamics of the EGSB reactors in detail, considering the influence of recirculation, not just dealing with it as a model where only input and output are considered (black box model).

The modeling dividing the reactor into two regions helps in the detailed study of hydrodynamics, because in most cases, the difference in dimensions between the tube and the separator is not considered, but they can influence the velocity and hence the flow pattern of the system.

The model of two tubular reactors in series showed, by adjusting the $Pe$, that the reactor can be described as a tubular reactor with high dispersion (region of the tube), followed by a complete-mix reactor (region of the separator).

Aiming at a simplification of the model equation, the model of two series of CSTR was tested, and it was equivalent to the model of two tubular reactors. It
is concluded that the flow behavior in the EGSB reactor can be considered as a series of five CSTR reactors, in which three reactors are in the tube region and two in the separator region.

Acknowledgments

The authors thank the Coordination for the Improvement of the Higher Level Personnel (CAPES) and the Agency of the São Paulo Research Foundation (FAPESP) for financial support.

References


[23] Rodriguez-Gómez, R.; Renman, G.; Moreno, L.; Liu, L.


