Optimization of Rotating Machines Vibrations Limits by the Spring - Mass System Analysis

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Abstract: The present work consists to calculate the parameters of vibrations response of the rotating machines, it consists to development a new calculation method. We will calculate the response of the spring-mass system with finished differences according to operation parameters of the model elements used (rigidity, damping and frequency). An estimate of the absolute values of norms limits of machines vibrations will be started.

Key words: vibration, damping, stiffness, free movement.

1. Introduction

The rotating machines present particular behaviors in stage transitional. The free response of the system is to be evaluated.

A calculation of these parameters will be started of which we will try to determine this response according to the parameters such as the stiffness and the damping, which represent the parameters of forces opposite to the excitation effort.

2. Model of the Machines Vibrations

The rotating machines vibrations will be modeled by the system with one degree on Fig. 1 [1].

Rotating machine can be compared with this system: (Fig. 2)

- the base is replaced by the bearings
- the Springs is replaced by the rotor
- the mass is replaced by the body of the turbine or the pump
- Damping is ensured by the fluid passing in the turbine or the pump.

3. Basic Equation

The differential equation translating the behavior of

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the system (displacement of mass $x(t)$ compared to its rest position after release) in a direction can be written:

$$M \ddot{x}(t) + C \dot{x}(t) + kx(t) = F(t) \quad (1)$$

where, k is the rigidity of the spring which expresses that the force of recall is proportional to elongation x and the force F applied on the basis.

A viscous damping (or fluid) of coefficient C exerts on the movement of the mass (m), a force of damping $-C \frac{dx}{dt}$ is proportional to the instantaneous speed.

The total response of the movement is:

$$x(t) = x_L(t) + x_F(t) \quad (2)$$

where, $x_L(t)$ is response of the free movement, solution of the equation:

$$M \ddot{x}(t) + C \dot{x}(t) + kx(t) = 0 \quad (3)$$

$x_F(t)$ is the response of the movement forced, solution of the equation:

$$M \ddot{x}(t) + C \dot{x}(t) + kx(t) = F(t) \quad (4)$$

The response of the free movement [2] in the case or $\xi < 1$ is:

$$x_L(t) = X_0 e^{-\xi \omega_0 t} \sin(\omega_0 \sqrt{1 - \xi^2} t + \phi_0) \quad (5)$$

where, $\xi = \frac{C}{C_c}$ and C_c are critically damping.

The natural frequency of the free movement is:

$$\omega_0 = \sqrt{\frac{k}{M}} \quad (6)$$

Fig. 1 Mechanical system with one degree.

Fig. 2 Rotating machines system.

Fig. 3 One degree system.

When $\xi = 1$, then $C = 2\sqrt{kM} = 2M\omega_0 = C_c$, this case is rare, and when $\xi > 1$, the free response is an exponential function. $x_L(t)$ is represented on Fig. 3.

By holding account that $F(t) = F_0 e^{j\omega t}$, the response of the forced movement is:

$$x(t) = X e^{j(\Omega t - \phi)} \quad (7)$$

where,

$$X = \frac{F_0 / k}{\left[\left(1 - \left(\frac{\Omega}{\omega_0} \right)^2 \right)^2 + \left(2\xi \frac{\Omega}{\omega_0} \right)^2 \right]^{1/2}} \quad (8)$$

$$\tan \phi = \frac{2\xi \frac{\Omega}{\omega_0}}{1 - \left(\frac{\Omega}{\omega_0} \right)^2} \quad (9)$$

$$T = \frac{2\pi}{\omega_0 \sqrt{1 - \varepsilon^2}} \quad (10)$$

If $\varepsilon \ll 1$, then $T = \frac{2\pi}{\omega_0}$, with the natural

$$\text{pulsation } \omega_0 = \sqrt{\frac{k}{M}}.$$

We can have the harmonic response forced to one degree on Fig. 4.

3.1 Case of a Rotating Machine

(Case of excitation by a static unbalance, rigid rotor), that is to say a rigid rotor with an excitation of static unbalance (Fig. 5). e is eccentricity, C is the gravity center and G is the inertia center. $z(t)$ and $y(t)$ are the co-ordinates of the centre of gravity along two perpendicular directions. The equations of the movement is:

$$m \frac{d^2}{dt^2} (y(t) + e \cos(\Omega t)) = -ky(t) - c \dot{y}(t) \quad (11)$$

$$m \frac{d^2}{dt^2} (z(t) + e \sin(\Omega t)) = -kz(t) - c \dot{z}(t) \quad (12)$$

Fig. 4 Harmonic Forced response with one Degree.

With $r(t) = y(t) + iz(t)$, thus:

$$m \ddot{r}(t) + c \dot{r}(t) + kr(t) = me \Omega^2 e^{j\Omega t}$$

We have the solution for $\Omega = \omega_0$:

$$r(t) = \frac{e(\frac{\Omega}{\omega_0})^2}{(1 - (\frac{\Omega}{\omega_0})^2) + j(2\varepsilon \frac{\Omega}{\omega_0})} e^{j(\Omega t - \varphi)} \quad (13)$$

That corresponds to the critical speed (Fig. 6).

4. Numerical Calculation

The expressions of the first and second derived by using the differences are finished to solve the equation of motion of a rotor:

$$\left(\frac{\partial x}{\partial t}\right)_{i,j} = \frac{1}{\Delta t} \left(x_{t-\frac{1}{2}\Delta t} - x_{t+\frac{1}{2}\Delta t}\right) = \frac{1}{2\Delta t} (x_{t+1} - x_{t-1}) \quad (14)$$

$$\frac{\partial^2 x}{\partial t^2} = \frac{1}{\Delta t^2} (x_{t+1} - 2x_t + x_{t-1}) \quad (15)$$

The response of the free movement is the solution of the Eq. (3). That is to say:

$$\frac{M}{\Delta t^2} (x_{t+1} - 2x_t + x_{t-1}) + C \frac{1}{2\Delta t} (x_{t+1} - x_{t-1}) + kx_t = 0 \quad (16)$$

$$x_{t+1} = 1 + \frac{(2M - k\Delta t^2)x_t - x_{t-1}}{\frac{C\Delta t}{2} + M} \quad (17)$$

We choose the boundary conditions:

(1) $t = t_0 = 0$; $x_t = 0$.

Fig. 5 Rigid liaison.

Fig. 6 Critical speed.

(2) $t = T$; $x_t = X_0$.

where, T is the period, it is equivalent to say that if $t = 0$, then $\Delta t = 0$,

$$x_{t+1} = 1 - \frac{x_{t-1}}{\frac{C\Delta t}{2} + M} \quad (18)$$

If $t = T$, then $\Delta t = t - t_0 = T$,

$$x_{t+1} = \frac{(2M - kT^2)X_0}{\frac{CT}{2} + M} + (x_{t+1} - 1)T \quad (19)$$

Then,

$$X_0 = \frac{M + \frac{CT}{2}}{2M - kT^2} = \frac{1 + \frac{CT}{2M}}{2 + \frac{kT^2}{M}} = \frac{1 + \delta \frac{T}{2}}{2 + \lambda T^2} \quad (20)$$

this while posing that $\frac{C}{M} = \delta$.

It is the value of damping per unit of mass. We indicate the stiffness per unit of mass by:

$$\frac{k}{M} = \lambda \quad (21)$$

The damping and the force of recalls represent opposite efforts to the force of the mass M . These forces are represented by the coefficients λ and δ .

By taking account of the expressions of the coefficient $\varepsilon \ll 1$ quoted on chapter 1, and that damping is far from that criticizes, and for one period of the movement deadened: $T = \frac{2\pi}{\omega_0}$. We have:

$$X_0 = \frac{1 + \frac{\pi}{\omega_0} \delta}{2 + \frac{4\pi^2}{\omega_0^2} \lambda} \quad (22)$$

If the natural frequency ω_0 is a constant, we can write:

$$X_0 = \frac{1 + A(\omega) \cdot \delta}{2 + 4A^2(\omega) \cdot \lambda} = B_1(\omega) \cdot \frac{\delta}{\lambda} \quad (23)$$

With

$$A(\omega) = \frac{\pi}{\omega_0} = Cte \quad (24)$$

The amplitude of the free vibrations is a function of the coefficients of stiffness and damping quoted above. In addition, if there is no damping, the free response is:

$$X_0 = \frac{B_2(\omega)}{\lambda} = B_2(\omega) \cdot \frac{M}{k} = \frac{F_0}{k} \quad (25)$$

Taking account of the equilibrium equation, with A , B_1 and B_2 , which depend on the natural frequency ω_0 .

$$F_0 = Mg = kX_0 \quad (26)$$

For a rated frequency Ω_1 , if taking the Eq. (4),

$$X_1 = \frac{F_0 / k}{\left[1 - \left(\frac{\Omega_1}{\omega_0}\right)^2\right]^{1/2}} = \frac{B_2 / \lambda}{\left[1 - \left(\frac{\Omega_1}{\omega_0}\right)^2\right]^{1/2}} \quad (27)$$

We will have:

$$\frac{X_1}{B_2 / \lambda} = \frac{X_1 \cdot k}{B_2 \cdot M} = B_3 = \text{Cons} \tan te \quad (28)$$

$$B_3 = \frac{1}{\left[1 - \left(\frac{\Omega_1}{\omega_0}\right)^2\right]^{1/2}} \quad (29)$$

Same manner of the response with damping (Eq. (4)) we have:

$$\frac{X_1}{B_1 \cdot B_3} = \frac{\delta}{\lambda} = \text{Cons} \tan te \quad (30)$$

Let us evaluate this constant:

$$X_0 = \frac{X_1}{B_3} = B_1 \cdot \frac{\delta}{\lambda} \quad (31)$$

In the case of nominal frequency Ω_1 constant, we will thus have in general if there is damping:

$$\frac{X_0}{B_1(\omega)} = \frac{1}{\left[\left(1 - \left(\frac{\Omega_1}{\omega_0}\right)^2\right)^2 + \left(2\xi \frac{\Omega_1}{\omega_0}\right)^2\right]^{1/2}} = \frac{\delta}{\lambda} \quad (32)$$

The free response varies according to the same manner as the ratio of the coefficients of stiffness and damping. It varies according to the efforts opposed to that of the excitation. The force of damping and stiffness vary according to the effort F_0 . With Eq. (32) and If there is weak damping we will have the equation:

$$\frac{\delta}{\lambda} = \frac{1}{1 - \left(\frac{\Omega_1}{\omega_0}\right)^2} \quad (33)$$

If $\Omega = \chi \Omega_1$, then:

$$\frac{\delta}{\lambda} = \frac{1}{1 - \frac{1}{\chi^2} \left(\frac{\Omega}{\omega_0}\right)^2} \quad (34)$$

Thus:

$$\frac{X}{X_0} = \frac{1}{\chi^2 \left(\frac{\lambda}{\delta} - 1\right) + 1} \quad (35)$$

Same manner if there is no damping we will have:

$$\frac{1}{\lambda} = \frac{\chi^2}{\chi^2 - \left(\frac{\Omega}{\omega_0}\right)^2} \quad (36)$$

Thus:

$$\frac{X}{X_0} = \frac{1}{\chi^2 (\lambda - 1) + 1} \quad (37)$$

This ratio varies according to the oscillation frequency. If $\chi = 1$, then:

$$\frac{X_1}{X_0} = \frac{1}{\lambda} \quad (38)$$

With damping:

$$\frac{X_1}{X_0} = \frac{\delta}{\lambda} \quad (39)$$

According to the Eq. (4), if $\Omega = \omega_0$, Then $\frac{X}{X_0}$

tends towards infinite thus the denominator of Eq. (23) tends towards zero, that is to say:

$$\chi \approx \sqrt{\frac{\delta}{\delta - \lambda}} \approx \sqrt{\frac{C}{C - k}} \quad (40)$$

4.1 Case of Rotating Machines

The expression of the vibrations in the case of rotating machines is:

$$\frac{r}{e} = \frac{\left(\frac{\Omega}{\omega_0}\right)^2}{\left[\left(1 - \left(\frac{\Omega}{\omega_0}\right)^2\right)^2 + \left(2\varepsilon \frac{\Omega}{\omega_0}\right)^2\right]^{1/2}} \quad (41)$$

Same manner and for nominal speed of rotation Ω_1 , we can write:

$$\frac{r_1}{e_1} = \frac{\left(\frac{\Omega_1}{\omega_0}\right)^2}{\left[\left(1 - \left(\frac{\Omega_1}{\omega_0}\right)^2\right)^2 + \left(2\varepsilon \frac{\Omega_1}{\omega_0}\right)^2\right]^{1/2}} = \frac{\delta}{\lambda} \quad (42)$$

In the case that $\Omega = \chi \Omega_1$ and ($\varepsilon \ll 1$), we will have:

$$\frac{r}{e} = \frac{\frac{\chi^2}{1 + \lambda \chi^2}}{\left[1 - \left(\frac{\chi^2}{1 + \lambda \chi^2}\right)\right]} = \frac{\chi^2}{1 + \lambda \chi^2 - \chi^2} = \frac{\delta}{\lambda} \quad (43)$$

If $\chi = 1$, then

$$\frac{r_1}{e_1} = \frac{1}{\lambda} \quad (44)$$

If there is damping:

$$\frac{r_1}{e_1} = \frac{\delta}{\lambda} \quad (45)$$

If $\Omega = \omega_0$, then $\frac{e}{r}$, tends towards infinite thus the denominator of Eq. (32) tends towards zero, that is to say:

$$\chi \approx \sqrt{\frac{1}{1-\lambda}} \quad (46)$$

or

$$\chi \approx \sqrt{\frac{\delta}{\delta-\lambda}} \approx \sqrt{\frac{C}{C-k}} \quad (47)$$

The same results are obtained, for spring masse system or for rotating machines. This value of χ calculated corresponds to critical speed of rotating machines, i.e.:

$$\chi = \frac{\omega_0}{\Omega_1} = \sqrt{\frac{C}{C-k}} \quad (48)$$

With ω_0 the natural frequency:

$$\chi = \frac{\omega_0}{\Omega_1} = \sqrt{\frac{1}{1-\frac{k}{C}}} = \sqrt{\frac{r}{r-e}} \quad (49)$$

5. Optimization of Vibrations Limits

From expression,

$$\frac{X}{X_0} = \frac{1}{\chi^2(\lambda-1)+1} \quad (50)$$

and if $\chi = 1$, we note:

$$\tau = \frac{X_1}{X_0} = \frac{1}{\lambda} \quad (51)$$

We built Table 1 from the standard norm ISO 10816-1, which includes the absolute values of rotating machines vibrations.

The value of coefficient τ is mentioned in this table. We suppose that the amplitude X_0 is relative to equilibrium corresponding to the admissible limit vibration according to norm. And X_1 corresponding to vibration amplitude at nominal speed (value still admissible). The coefficient τ tacked from absolutes vibrations, it will be the same for the measures, it always has a constant value for all machine groups.

Then $\lambda = \frac{1}{\tau}$, and $k = \frac{1}{\tau} \times M$.

Calculate the relative change in the real value of X_1 over its supposed value (value to nominal speed, still admissible according the norm). So we have :

Table 1 Admissible vibrations in mm/s; Norm ISO 10168-1 [3].

Group	No. 01	No. 02	No. 03	No. 04
Vibration X_0 (in mm/s)	0.71	1.12	1.8	2.8
Vibration X_1 (in mm/s)	4.5	7.1	11.2	18
Coefficient τ	6.33	6.33	6.33	6.33

Fig. 7 Equivalence of Spectrum vibration.

$$\Delta X_1 = X_0 \cdot \tau - X_0 \cdot \frac{1}{\lambda} = X_0 \cdot \left(\tau - \frac{1}{\lambda} \right) \quad (52)$$

or

$$\varepsilon_1 = \frac{\Delta X_1}{X_1} = \frac{X_0}{X_1} \cdot \left(\tau - \frac{1}{\lambda} \right) = 1 - \frac{X_0}{X_1} \cdot \frac{1}{\lambda} \quad (53)$$

The increased off X_1 with respect to the equilibrium condition (X_0) is:

$$\varepsilon_2 = \frac{\Delta X_1}{X_1} = \frac{X_1 - X_0}{X_1} \quad (54)$$

The difference of two variations is:

$$\Delta \varepsilon = \varepsilon_1 - \varepsilon_2 \quad (55)$$

The new value X_2 is:

$$X_2 = X_1 + \Delta \varepsilon \cdot X_1 \quad (56)$$

The admissible vibration are caused by unbalance with values in norm limits. The low frequency spectrum in Fig. 7a and that on Fig. 7b of vibrations calculation are admissible vibrations. With Figs. 7a and 7b, we can say that the two spectrum are for admissible vibrations. With amplitude of the fundamental frequency $a(1-\varepsilon)$, we have:

$$\begin{aligned} X_{2\text{RMS}} &= \sqrt{a^2(1-\varepsilon)^2} = \sqrt{a^2(1+\varepsilon^2-2\varepsilon)} \quad (57) \\ &= X_{1\text{RMS}}(1-\varepsilon) = X_{1\text{RMS}} - \varepsilon X_{1\text{RMS}} \end{aligned}$$

5.1 Calculations Applications

Our machine is a machine entrained by gas turbine series GE 5001P, used for electric energy production with power of 23 MW.

The generator rotor mass is 12 tons and the nominal speed is 3,000 tr/min. We apply the results which use this machine on the other machine groups therefore;

A new form of the norm is presented in Table 2. It takes account the calculation in Table 3.

The amplitude calculation is compared to the norm values. We can calculate with the same manner the new vibration values X_n from X_1 , then:

$$\begin{aligned} X_2 &= X_1 + \varepsilon X_1 \\ X_3 &= X_1 + 2\varepsilon X_1 + \varepsilon^2 X_1 \\ X_4 &= X_1 + 3\varepsilon X_1 + 3\varepsilon^2 X_1 + \varepsilon^3 X_1 \end{aligned}$$

$$\begin{aligned} X_n &= X_1 + (n-1)\varepsilon X_1 + (n-1)\varepsilon^2 X_1 \\ &+ \dots + (n-1)\varepsilon^{n-2} + \varepsilon^{n-1} \end{aligned}$$

Thus,

$$X_n = X_1(1 + (n-1)\sum_{i=1}^{n-2} \varepsilon^i + \varepsilon^{n-1}) \quad (58)$$

$$X_n = X_1(1 + (n-1)\varepsilon \frac{1 - \varepsilon^{n-1}}{1 - \varepsilon} + \varepsilon^{n-1}) \quad (59)$$

We should search the number of operations n calculation to find the new values can take X_1 , this is by solving in equation: $X_n > X_1$, thus:

$$1 + (n-1)\sum_{i=1}^{n-2} \varepsilon^i + \varepsilon^{n-1} > 1 \quad (60)$$

The term sum is a geometric progression, so:

$$(n-1)\varepsilon \cdot \frac{1 - \varepsilon^{n-1}}{1 - \varepsilon} + \varepsilon^{n-1} > 0 \quad (61)$$

$$n > 1 - \left[\varepsilon^{n-2} \frac{(1 - \varepsilon)}{(1 - \varepsilon^{n-1})} \right] \quad (62)$$

thus $n > 1$. The limit value of X_n is for $n = 2$.

The term in brackets is equal to zero for $n = 2$ (Table 3). It corresponds to our calculations.

We took the lower limit of this range because an increase in vibration can cause deterioration of the equipment status. Machine design is the main judge of the limits of admissible vibrations.

Our study can be taken into account in machine design.

6. Numerical Calculation of Free Response

From equation of the free response without damping we have:

$$X_0 = \frac{1}{2 + 4A^2(\omega)\lambda} \quad (63)$$

We will have:

$$\frac{k}{M} X_0 = \frac{1}{2A^2(\omega)} X_0 = \frac{\omega_0^2}{2\pi^2} X_0 \quad (64)$$

For a system with several degrees (two degrees for example) were we having not damping:

$$[M]^{-1}[K] \cdot \{X\} = \frac{\omega^2}{2\pi^2} \cdot \{X\} \quad (65)$$

Table 2 Comparison vibration calculated in mm/s and those of the norm.

Groupe	Group No. 01	Group No. 02	Group No. 03	Group No. 04
Norm ISO 10816	4.5	7.1	11.2	18
Our calculation	5.21	8.22	12.96	20.84

Table 3 Calculation on real machines.**Machine characteristics:**

Machine: generator entrained by gas turbine model GE 5001P

Generator rotor mass (M): about 12 tons

Nominal rotation speed: 3,000 tr/min

First critical speed: A 1,500 tr/min

Factor τ for all groups of machines: $\tau \approx 6.33$

Group: machine group IV

Calculations :

$$\text{Factor } \lambda : \lambda = \frac{1}{\tau} = 0.158 = 0.158 \times 10 \times 10 = 15.8$$

Unit:

$$\lambda = \frac{k}{M} = 0.158 \times \frac{1000}{9.81} = 16.10$$

$$\lambda = \frac{k}{M} = \omega_0^2 = \left[\frac{2 \times 3.14 \times 1500}{60} \right]^2 = (157)^2 = 24649$$

$$= 24649 \times 9.81 / 0.158 \times (1000 \times 10 \times 10) = 15.28$$

Error Calculation :Error: ε_1

$$\varepsilon_1 = \frac{\Delta X}{X_1} = \frac{X_0}{X_1} \cdot \left(\tau - \frac{1}{\lambda} \right) = 1 - \frac{X_0}{X_1} \cdot \frac{1}{\lambda} \approx 1$$

Error: ε_2

$$\varepsilon_2 = \frac{\Delta X}{X_1} = \frac{X_1 - X_0}{X_1} = \frac{18 - 2.8}{18} = 0.842$$

Global error:

$$\Delta \varepsilon = \varepsilon_1 - \varepsilon_2 = 0.158 = 15.8\%$$

New Value of X_2 :

$$X_2 = X_1 + 0.158 X_1 = 20.84 \text{ mm / s}$$

$$X_{1 \text{ RMS}} = \sqrt{a(1 - \varepsilon)^2} = a \times (1 - \varepsilon) =$$

$$a(1 - 0.158) = 20.84 \times 0.842 \approx 18 \text{ mm / s}$$

Value of X_n :

$$X_n = X_1 (1 + (n-1) \sum_1^{n-2} \varepsilon^n + \varepsilon^{n-1})$$

$$n > 1 - \left[\varepsilon^{n-2} \frac{(1 - \varepsilon)}{(1 - \varepsilon^{n-1})} \right] = 0$$

Example:

$$X_2 = X_1 (1 + (2-1) \varepsilon \frac{1 - \varepsilon^{2-2}}{1 - \varepsilon} + \varepsilon^1) = X_1 + \varepsilon \cdot X_1$$

This is the solution of the equation:

$$[M]^{-1} \left\{ \ddot{X} \right\} + [K] \cdot \{X\} = \{0\} \quad (66)$$

With:

$$\{x(t)\} = \{X e^{j\Omega t - \varphi}\} \quad (67)$$

To extract the forced answer we seek the particular solution of the system of differential equation with a second member [1]:

$$[M]^{-1} \left\{ \ddot{x}(t) \right\} + [C] \cdot \{x(t)\} + [K] \cdot \{x(t)\} = \{F_0\} e^{j\Omega t} \quad (68)$$

With

$$\{x(t)\} = \{X e^{j\Omega t - \varphi}\} \quad (69)$$

Thus,

$$[K - M\Omega^2 + jC\Omega] \cdot \{X e^{-j\varphi}\} = F_0 \quad (70)$$

For the free movement:

$$[K - M\Omega^2 + jC\Omega] \cdot \{X e^{-j\varphi}\} = 0 \quad (71)$$

7. Numerical Calculation Applications

In the system represented on Fig. 8, let us calculate the natural frequency and the proper

vectors:

Fig. 8 Calculation example.

$$\begin{bmatrix} 3m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (72)$$

We seek these vectors and pulsation while respecting:

$$(M^{-1}K - \omega^2 I)X = 0 \quad (73)$$

This expression can be derived from Eq. (70)

(For the free response without damping):

$$[K - M\Omega^2] \cdot \{Xe^{-j\varphi}\} = 0 \quad (74)$$

Thus,

$$\det \begin{vmatrix} 2k - 3m\omega^2 & -k \\ -k & k - m\omega^2 \end{vmatrix} = 0 \quad (75)$$

That is to say:

$$3m^2\omega^4 - 5km\omega^2 + k^2 = 0 \quad (76)$$

Thus:

$$\omega_1 = 0.482\sqrt{\frac{k}{m}} \quad (77)$$

$$\omega_2 = 1.197\sqrt{\frac{k}{m}} \quad (78)$$

The clean vectors associated each pulsation are:

$$\text{For } \omega_1: X_2 = 1.303X_1 \quad (79)$$

$$\text{For } \omega_2: X_2 = -2.303X_1 \quad (80)$$

8. Conclusions

In this study we determined the expression of the vibrations response by analysis of system spring masse according to the stiffness and damping parameters which represent the forces opposed to the effort of excitation.

The vibration responds calculated depends on the rigidity, damping and oscillation frequency.

The rest of this article is to revalue the limits of machine vibration by using an error calculation and justifying the reasoning by analytical calculation applied to a selected machine.

We hope that this work will be developed in future and will contribute to scientific research in this area.

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