Strategies to Measure Direct and Indirect Effects in Multi-mediator Models

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The analysis of mediators, multi-mediators, confounders, and suppression variables often presents problems to the scientists that need to interpret them correctly. After clarifying main differences among these terms, this paper focuses on the techniques to conduct and estimate multi-mediation effects. Multi-mediator effects are very common in social science literature, however, many studies do not report their analysis, or even worse, do not explore the significance of the indirect effects in the outcome variable. In exploring the underlying mechanism of observed variables, mediation addresses a key important aspect: Mediation explains how the changes occur. The measurement of direct and indirect effects involves the combination of several techniques, especially under multiple mediators. The objective of this paper is to show different approaches that should be used to investigate indirect and direct effects in order to shed some light on how to conduct a mediation analysis, how to assess the model estimation, and how to interpret mediation effects. The main conclusion of this paper is that by applying traditional methodologies (causal steps, product of coefficients, and the indirect approach), the real mediation effect could be overestimated or underestimated. This paper explains new methods that overcome the difficulties of traditional approaches. Examples of Mplus syntax are provided to facilitate the use of these methods in this application.

Keywords: mediator, moderator, cofounder, multi-mediators

The Concept of Mediation and Approaches to Test Multi-mediation Effects

In social sciences, researchers are interested in explaining the mechanisms that illustrate the relationship between an independent variable (X) and a dependent variable (Y) (Figure 1). This paper extends previous studies by considering the difficulties of analyzing multi-mediation effects that appear in complex situations in which there is a chain of effects that mediate the relationship between X and Y. The main contribution of this paper is to provide a methodology that includes how to conduct and assess the mediation analysis and interpret results.

Essentially, mediation analysis is the set of techniques used in conducting and testing the mediation
hypotheses when one or more independent variables indirectly affect one or more dependent variables. Mediation variables (Figure 2) are usually confused with moderator variables (Figure 3). The moderator effect alters the effect of the independent variable (X) on the dependent variable (Y), so that its effect depends on the moderator (M). However, the mediation effect builds on the process by which the independent variable (X) influences the dependent variable (Y), which is twofold: the direct effect on (Y) and the indirect effect on (Y) through the mediator variable (M) (Preacher, Rucker, & Hayes, 2007). The total effect is the degree to which a change in an upstream (exogenous) variable (X) has an effect on a downstream (endogenous) variable (Y). A direct effect is the degree to which a change in an upstream (exogenous) variable produces a change in a downstream (endogenous) variable without “going through” any other variable. In contrast, an indirect effect is the degree to which a change in an exogenous variable produces a change in an endogenous variable by means of an intervening variable. Given that the variables are standardized, the indirect effect of X on Y through M is equal to the product of associated paths a and b (Cole & Maxwell, 2003).

Mediators (Figures 2, 4, 5, and 6), moderators (Figure 3), confounders (Figure 4), and suppression variables all influence or change the cause-effect relationship between the independent and the dependent variables. For this reason, the researcher should want to isolate the influence of those third variables. To assess or neutralize the influence of potential third variables, an appropriate comparison group is required (Levine, 1992).

The focus of this paper is on the mediator variable, which addresses the mechanisms through which an effect occurs (MacKinnon, Coxe, & Baraldi, 2012). The mediation hypotheses posit how X affects Y through one or more potential intervening variables, or mediators (M). This paper addresses only the situation in which the causal order of X, M, and Y can be established on theoretical or procedural grounds (Preacher & Hayes, 2008). If a logical ordering of X, M, and Y cannot be established, it is important to highlight that other methods, such as longitudinal models, should be used to investigate mediation (Azen, 2003; Preacher & Hayes, 2008).

When the independent or antecedent variable (X) affects the dependent variable (Y) through only one mediator or intervening variable, the term of simple mediation is used (see Figure 2). MacKinnon, Krull, and Lockwood (2000) pointed out that mediator reduces or filters (Little, 2013) the causal effect between the independent variable and the dependent variable, because the moderator explains part or even all of the relationship between X and Y, based on the fact that X causes M and M causes Y. Thus, the moderator has the opposite effect of the suppression variable, which increases the effects of X on Y. Although, the suppression and moderator models share the same graph (Figure 2), the difference between them is that c´(c) is higher than c (Figure 1) in the case of the suppression model, in contrast c´(c) is lower than c in the case of moderator effect. By including a mediator in a single model—a model with only two variables (X and Y), two paths are added (Figure 2). The first path, path (a) represents the effect of X on the proposed mediator M. The other path, path (b), represents the effect of M on Y partialling out the effect of X. The total effect of X on Y is represented by the unstandardized weights \( a \times b + c´(c) \) (Figure 2), in which the sum is equal to c in a simple model (Figure 1) (Kenny, 1979). When the indirect effect has the same sign as the direct effect, it is named consistent mediation model and it is an inconsistent mediation model, if it has an opposite sign (Davis, 1985; MacKinnon et al., 2000).

The connection between the models represented by model 1 (Figure 1) and model 2 (Figure 2) is represented by the unstandardized regression coefficients (J. Cohen & P. Cohen, 1983). Thus, the indirect effect of X on Y through the mediator variable is calculated by the product of the unstandardized regression weights (a
× b), while the direct effect is represented by $c'(c)$ (Preacher et al., 2007). Then, to calculate the total effect of X on Y (model 2), the authors will add the direct ($c'(c)$) and the indirect effect ($a \times b$). These calculations are valid in regressions and structural equation modeling (SEM) where M and Y are continuous variables. But, it cannot be applied in those cases in which one or more dependent variables are binary. If there is more than one binary dependent variables, the right analysis is to perform a logistic or probit regression, instead of doing a regression or SEM (MacKinnon & Dwyer, 1993). In case of non-binary variables, the set of squares regression that describes the model is (Preacher et al., 2007):

$$
Y = \beta_0 + \beta_1 X + \beta_2 M + \epsilon \\
M = \alpha_0 + \alpha_1 X + \epsilon
$$

where $\beta_1 = b$, $\alpha_1 = a$, and $\beta_2 = c'(c)$ in model 2; $a$, $b$, $c'(c)$, and $c$ are unstandardized coefficients.

The empirical conditions of mediation are the followings: (1) There must be a significant relationship between X and Y; (2) there must be a significant relationship between X and M; and (3) the mediator must be a significant predictor of the outcome variable in an equation, including both the mediator and the independent variable (Alwin & Hauser, 1975; Baron & Kenny, 1986; Judd & Kenny, 1981; MacKinnon et al., 2000). Indeed, if the mediator variable (M) completely mediates the relationship between X and Y, the direct effect of X on Y (controlling for M) must approach zero (Cole & Maxwell, 2003). However, some authors (Cole & Maxwell, 2003; Little, 2013) considered those conditions to be too restrictive. Even when the first condition is not addressed, there is still a mediation effect that is represented in the model. In contrast, only when direct effects involve changes in the relations among a set of variables, the indirect effect could be considered as a mediation path (Little, 2013). Those models in which direct effects are non-significant are called full mediation models.
Figure 3. Simple moderator model (model 3).

Figure 4. Confounder model (model 4).

Figure 5. Multi-mediation model (model 5, type A).
Methods of Estimation

The mediation effect can be explained as a chain of causal effects. As such, M is endogenous relative to X, but exogenous relative to Y (Cole & Maxwell, 2003). Indeed, the authors are analyzing how one variable causes changes in another variable, which in turn, causes changes in an outcome variable (Little, 2013). The pattern of those causal relationships constituted needs to be analyzed, with the knowledge that the inclusion of mediation effects requires testing the additional hypotheses that mediators generate. To provide stronger evidence of mediation, an independent assessment of the impact of the stressor on the dependent variable is required (Baron & Kenny, 1986; MacKinnon, Lockwood, & Williams, 2004).

To assess and test the indirect effects, there are six main sets of techniques (Little, 2013; MacKinnon et al., 2012; MacKinnon et al., 2004).

The Causal Steps Approach

This approach is based on the analytical effects and causal relations of the variables, but it does not test the mediation paths. It also has the limitation that it does not consider suppression variables (Baron & Kenny, 1986). This limitation implies that direct effects in mediation models ($c'c$) are always lower than the total causal effects in simple models ($c$ in model 1) (Little, 2013).

The Product of Coefficients Approach

This approach considers that sample indirect effects are the product of estimates of regression coefficients. This method uses the Wald test “r” to test the null hypothesis that the product of the indirect paths, $a$ and $b$, significantly differs from zero (Baron & Kenny, 1986; Preacher et al., 2007). The main limitation of this approach is that it assumes the normal distribution of the product of the coefficients $a$ and $b$. However, although $a$ and $b$ are asymptotically independent and normally distributed, the product of both paths, $a$ and $b$, should not be normally distributed, indeed, they are usually highly skewed (Aroian, 1947; Goodman, 1960; Preacher et al., 2007; Sobel, 1982). Because this method is based on the faulty assumption that $a$ and $b$ are normally distributed, this approach can only be recommended in the case of very large samples. Under the product of coefficients approach, three main tests to estimate the standard error of $a$ and $b$ could be calculated. These three tests, the Sobel test, the Aroian test, and the Goodman test, consider the value of the standard errors of $a (s_a^2)$ and $b (s_b^2)$ (MacKinnon, Warsi, & Dwyer, 1995).
The main formulas for testing mediation effect are the followings (model 2):

- Sobel test equation (1982): $z\text{-value} = \frac{a \times b}{\text{SQRT}(b^2 \times s_a^2 + a^2 \times s_b^2)}$
- Aroian test equation (1947): $z\text{-value} = \frac{a \times b}{\text{SQRT}(b^2 \times s_a^2 + a^2 \times s_b^2 + s_a^2 \times s_b^2)}$
- Goodman test equation (1960): $z\text{-value} = \frac{a \times b}{\text{SQRT}(b^2 \times s_a^2 + a^2 \times s_b^2 - s_a^2 \times s_b^2)}$
- Freedman and Schatzkin test equation (1992): $t_{N-2} = c - c'/\text{SQRT}[s_c^2 + s_c^2 - 2s_c \times s_c \times \text{SQRT}(1 - \rho^2 \times m)]$
- where $\text{SQRT}$ is squared root; $m$ is mediator; $s$ is standard error; $\rho^2 \times m$ is correlation coefficient.

Taking into account the adaptation of the formulas for testing multi-mediation effects, this paper provides the formulas for testing the indirect effects, which are the followings (model 5, type B):

- Sobel test equation: $z\text{-value} = \frac{a \times b \times d}{\text{SQRT}(b^2 \times s_a^2 + a^2 \times s_b^2 + d^2 \times s_d^2)}$
- Aroian test equation: $z\text{-value} = \frac{a \times b \times d}{\text{SQRT}(b^2 \times s_a^2 + a^2 \times s_b^2 + d^2 \times s_d^2 + s_a^2 \times s_b^2 + s_a^2 \times s_d^2 + s_b^2 \times s_d^2 + s_d^2 \times s_d^2 + s_d^2 \times s_d^2)}$
- Goodman test equation: $z\text{-value} = \frac{a \times b \times d}{\text{SQRT}(b^2 \times s_a^2 + a^2 \times s_b^2 + d^2 \times s_d^2 - s_a^2 \times s_b^2 - s_a^2 \times s_d^2 - s_b^2 \times s_d^2 - s_d^2 \times s_d^2 \times s_d^2)}$
- Freedman and Schatzkin test equation: $t_{N-2} = c - c'/\text{SQRT}[s_c^2 + s_c^2 - 2s_c \times s_c \times \text{SQRT}(1 - \rho^2 \times m)]$

The SEM Estimation of Indirect Effect Approach

This approach has the disadvantage of considering the standard errors of $a$, $b$ and the product $a \times b$ to be normally distributed. In contrast, with the causal effect approach, every chain of effects is tested by estimating the model with the method of estimation of maximum likelihood (ML). However, using the SEM method, authors do not know if the indirect effects are significant.

The syntax in Mplus under SEM estimation of simple mediation approach is as follows:

**TITLE:** Testing Simple Mediation Effects

**DATA:** FILE IS data.dat;

**VARIABLE:** NAMES ARE X Y M; !X: independent variable, Y: dependent variable, M: mediator variable

**USEVARIABLES ARE** X Y M;

**MODEL:** Y ON X;

Y ON M;

M ON X;

**OUTPUT:** TECH1 STANDARDIZED;

The syntax in Mplus under SEM estimation of multiple mediation approach is (model 5, type B):

**TITLE:** Testing Multi-Mediation Effects

**DATA:** FILE IS data.dat;

**VARIABLE:** NAMES ARE X Y M1 M2; !M1 and M2: mediators variables

**USE VARIABLES ARE** X Y M1 M2;

**MODEL:** Y ON X;

Y ON M2;

M2 ON M1;

M1 ON X;

**OUTPUT:** TECH1 STANDARDIZED

The Monte Carlo Approach
This approach simulates the distribution of $a$ and $b$ to produce a confidence interval for the indirect effect to determine if $a$ and $b$ are significant (MacKinnon et al., 2004). The main advantage of this method is that it does not assume that $a$ and $b$ are normally distributed, and also provides a test for indirect effects. However, it generates a complex matrix to calculate the confidence interval. In this approach, a model is estimated for each random sample and the analysis results are summarized over samples. This approach is also useful to test the power of the model.

**The Bootstrapping Approach**

Bootstrapping is a nonparametric technique that takes a large number of sub-samples with replacements from the original sample data. The bootstrapping approach produces a distribution based on the samples and does not impose assumptions of normality of the variables’ distribution, but it requires that $a$ and $b$ should be uncorrelated. Thus, bootstrapping treats a given sample of the population to get more accurate estimations than the previously discussed approaches. Bootstrapping is widely considered to be the best approach to analyze the mediation effects (Bollen & Stine, 1990; Lockwood & MacKinnon, 1998; MacKinnon et al., 2004; 2012; Preacher & Hayes, 2008; Shrout & Bolger, 2002; Preacher et al., 2007; Little, Preacher, Selig, & Card, 2007). Under the bootstrapping approach, bootstrapping samples are taken to build the bootstrapping distribution, and standard errors and confidence intervals are determined empirically. Bootstrapping allows building a two-tailed test in which a confidence interval is formed by the observation ranked, as the result of the sample size multiplied by the alpha squared value as the lower critical bound. In contrast, the upper bound of the interval is formed by the observation ranked, as the result of the sample size multiplied by 1-alpha squared. If zero is not in the confidence interval of the indirect effect, it means that the indirect effect is different from zero. In other words, the indirect effect is statistically significant. The main advantage of the bootstrapping test is its accuracy, which minimizes type I error and the power of the model.

The syntax in Mplus under bootstrapping approach with a simple mediation variable is as follows:

```
TITLE: Testing Indirect Effect in the Simple Mediation Model
DATA: FILE IS data.dat;
VARIABLE:
  NAMES ARE X Y M; ! X: independent variable, Y: dependent variable, M: mediator variable
USE VARIABLES ARE X Y M;
ANALYSIS: BOOTSTRAP = 10000;
MODEL:
  Y ON X;
  Y ON M (b);
  M ON X (a);
MODEL CONSTRAINT: I (indirect_effect);
  Ind_effect = a*b;
OUTPUT: TECH1 STANDARDIZED
         CINTERVAL(BOOTSTRAP);
```

The syntax in Mplus under bootstrapping approach with multiple mediation variables is as follows (model 5, type B):

```
TITLE: Testing Indirect Effect in the Multi-Mediation Model
DATA: FILE IS data.dat;
VARIABLE:
  NAMES ARE X Y M1 M2; ! X: independent variable, Y: dependent variable, M1, M2: mediator variables
USE VARIABLES ARE X Y M1 M2;
ANALYSIS: BOOTSTRAP = 10000;
MODEL:
  Y ON X;
  Y ON M1 (b);
  M1 ON X (a);
  Y ON M2 (c);
  M2 ON X (a);
MODEL CONSTRAINT: I (indirect_effect);
  Ind_effect = a*b;
OUTPUT: TECH1 STANDARDIZED
         CINTERVAL(BOOTSTRAP);
```
Model 5, type B describes a causal chain of mediation effects that should be applied in two different cases. The first case should be applied in situations in which the variables in the chain of mediators act as the conditional effect, if a previous effect occurs (Zhang, Zyphur, & Preacher, 2009). An example of a multi-mediation model could be the chain of effects that can answer these questions: Do efficiency of logistics and the costs to trade mediate the influence of investment in infrastructures on international trade flows? How much do the two mediator effects (efficiency and costs) differ in strength? The second case is when the individuals of the sample have similar within-group behavior and simultaneously different among-groups behavior. The combination of both effects, within-group and among-groups, is named the confounding effect (Zhang et al., 2009; Klein & Kozlowski, 2000; Ostroff, 1993). An example of confounding effect should be a model that is designed to answer the following questions: Regarding international trade flows at the economic area level, do efficiency of logistics and the costs to trade at country level mediate the influence of investment in infrastructures at the economic area level? In other words: Do efficiency of logistics and the costs to trade at the individual level mediate the relationship between the group-mean-centered investment in infrastructures on group-mean-centered international trade flows? How much do the two mediator effects (efficiency and costs) differ in strength? What is the effect that is more relevant: the within-group or among-groups? In both situations, the results of the tests (Sobel test, Aroian test, Goodman test, and Freedman & Schatzkin test) do not suffer from the confounded effects generated by differences within-group and among-groups or by the subsequent effects of a chain of mediators (Zhang et al., 2009). However, it should be interesting for researchers to pay attention to which level the mediation effect is taking place.

**The Phantom Variables Approach**

Under this approach, the likelihood-based confident interval on the mediating effect is estimated using structural equation modeling (Cheung, 2007). This is a new methodology that provides even more accurate estimations than other methods already mentioned. However, it is not yet widely used. As Little et al. (2007) and Preacher et al. (2007) pointed out building confidence intervals to assess the mediation effect has three main advantages over the alternative of only accept-reject methods (the causal steps approach and the product...
of the coefficients approach). The first advantage of any confidence interval methods is that it indicates not only if mediation effects exist, but also the magnitude and the signal or direction. Second, it provides an estimation about the population instead of a specific sample. Third, it shows the precision of the estimation.

**Longitudinal Models**

Mediators are variables through which the influence of an antecedent variable is transferred to a criterion (Mathieu & Taylor, 2007). As stated above, the logical ordering of $X$, $M$, and $Y$ has to be established to analyze mediation (Preacher & Hayes, 2008). The causal relation between $X$ and $Y$ through the mediation variable ($M$) must be established. It means that the indirect effect is only considered as a mediator when it involves changes in the relations among a set of variables. In other words, by testing the mediation effect, authors are looking for the evidence of causing change in the mediator and/or outcome variables. In multi-mediation models, the overall indirect effect would equal the sum of the product terms representing each of the tracings from the first independent variable to the outcome (Cole & Maxwell, 2003).

A requirement for a variable to have caused another is that the cause must precede the outcome in time (Cole & Maxwell, 2003; Gollob & Reichardt, 1987). The way to prove the mediation effect is to analyze different moments in time to support the relations between the variables, using longitudinal models (Cole & Maxwell, 2003; Little, 2013; Maxwell & Cole, 2007; Selig & Preacher, 2009). The main limitation of cross-sectional models to analyze causal and mediation effects is overcome by longitudinal models (Cole & Maxwell, 2003; Cheong, MacKinnon, & Khoo, 2003; Gollob & Reichardt, 1987; MacKinnon et al., 2012; Maxwell & Cole, 2007; Selig & Preacher, 2009). In summary, cross-sectional models describe relations among variables that occur instantaneously and this consideration biases the parameter estimation (Maxwell & Cole, 2007). However, to describe causality, the previous cause of an ulterior effect requires a gap of time to exert these effects, in which the variables will be measured in a set of at least two different moments (Cole & Maxwell, 2003; Gollob & Reichardt, 1987; Judd & Kenny, 1981; Kraemer, Wilson, Fairburn, & Agras, 2002; Selig & Preacher, 2009; Sobel, 1990).

The longitudinal multi-mediation models address the questions of: Which variable or variables initiate the chain of effects to the dependent variable(s)? How does a process of effects cause future outcomes? What is the sequence of effects that influence one another (Selig & Preacher, 2009)? Additionally, with the analysis of mediation effects and the patterns of those effects over time, it is also possible to use longitudinal models to analyze inter-individual differences (Selig & Preacher, 2009).

**Conclusions**

The use of mediation variables appears very often in the social science literature. Instead of analyzing theoretical and empirical evidence of mediation, the causal effects are too often only theoretically demonstrated. The inclusion of mediation effects requires testing the additional hypotheses that the model generates. This paper is inspired by the fact that some traditional approaches commonly used in the social science field could overestimate or underestimate the real multi-mediation effects. That is the case of the product coefficient approach test (Krull & MacKinnon, 2001; Zhang et al., 2009). The inclusion of mediation effects requires testing the additional hypotheses that the model generates. In recognition of this requirement, this article aims to analyze direct and indirect effects in multi-mediation model. Based on the difficulties that the most advanced methodologies overcome from previous methodologies, Monte Carlo and Bootstrapping are the best
approaches to test the mediation effects.

Also, the evidence that cross-sectional models describe relations among variables that occur instantly implies biased parameters estimations when the model includes mediation variables (Maxwell & Cole, 2007; Little, 2013). It is explained by the requirement for a variable to cause another, which is that the cause must precede the outcome in time. Thus, this paper recommended to use a longitudinal analysis for testing the pattern and sequence of effects in mediational models.

**References**


