

# Single-Trial Evoked Potentials Extraction Based on Sparsifying Transforms

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**Abstract:** Evoked potentials are widely used to diagnose diseases and disorders in the central nervous system. It is thus essential to develop fast algorithms which can track the variations of evoked potentials for a variety of clinical applications. The sparsity of signals in a certain transform domain or dictionary has been exploited in the extraction of noisy signal. However, it isn't effective enough to extract the evoked potentials because the signal-to-noise ratio is extremely low. In this paper, we present a novel approach to solving evoked potentials extracting problem. Before the sparsifying the observations of evoked potentials, the observations are transformed to enhance the signal-to-noise ratio and sparsity. Then we can use the sparse representation algorithm to extract the evoked potentials. The alternating minimization algorithms are applied to calculate the transformation matrix and the sparse coefficients. We show the superiority of our approach over some filtering and sparse representation methods.

**Key words:** Evoked potentials, sparsifying transforms, single-trial extraction.

## 1. Introduction

EP (evoked potentials) is bioelectrical signal generated by the central nervous system when it is stimulated by well-defined external events. Common types of stimulation include visual, audio, and electrical [1, 2]. In EP analysis, the peak component information is of great interest in clinical applications research. The peak amplitude and latency represents the intensity and speed of response to the external stimuli, so the peak component information of EP reflects objectively how healthy the nervous systems are. Generally, the peak amplitude and latency of EP belonging to the same person obtained by same stimulation of the nerves is within a range, unless his nerves are damaged in one particular case, such as some disease and intraoperative injury [3, 4]. If one suffer from a disease unfortunately, e.g., congenital amblyopia, the peak component information will be

outside the normal range in long term. So making use of the time-locked characteristics of EP, ensemble averaging (EA) is the most widely used method for the estimation of EP signals to diagnose the diseases [5].

SNR (since the signal-to-noise ratio) of EP is usually low, a conventional EA requires 100-1000 trials to estimate an EP. It would take a long time, about 50-1000 seconds. So EA cannot be used to track the abrupt changes of evoked potentials. Tracking EP changes could be important in critical patient monitoring and intraoperative monitoring. In addition, the study of EP variation over time has many potential applications. Over the past two decades, many single-trial EP estimation algorithms have been devised to tracking the trial- to-trial variations in EP. Various denoising algorithms can be applied into the EP extraction. The denoising algorithms think the measurement of EP is corrupted by noise and the main source of noise is the spontaneous EEG (electroencephalogram) [6]. Many researchers have

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applied traditional denoising methods, such as Kalman filter [7], adaptive filter [8], neural network [9] and so on, for estimating EP signals from the noisy measurements. But the SNR of EP is too low, thus the extraction effect of single-trial EP is not so satisfactory. It requires researchers to seek more effective processing methods. Recently, GSA (generalized subspace approach)-based extraction method is proposed by Wang et al. [3]. They project the EP and EEG into different coefficient subspace respectively by applying projection matrix, and then remove the EEG projection coefficients for estimating the EP signal with the coefficient weighting matrix. The method can realize the optimum estimate of EP based on MMSE (minimum mean square error). There is however a serious flaw in this method. It requires accurate apriori knowledge, i.e. noise variance and autocorrelation matrix of the signal. A little bit of the errors will produce a large influence on the extracted EP signal. The requirements are hard to be implemented in the practice, so these denoising algorithms couldn't work very well [10].

Recently, sparse representation is applied into the signal separation and denoising, and the good results were achieved [11, 12]. Xu et al. in [13] propose the MOSCA (mixed over-complete dictionary based sparse component decomposition algorithm) which decomposes EP and EEG on the wavelet dictionary and the DCT (discrete cosine transform) dictionary respectively. Some components belonging to EP and EEG actually overlap each other, such that a lot of components are represented by the wrong dictionary and the corresponding coefficients. So this algorithm can't separate the EP and EEG well. In our previous paper, we proposed a double-trial EP estimation method, based on JSR (joint sparse representation). This method takes full advantage of the similarity between EP signals in two consecutive trials and separated the common and unique components with JSR, making use of the dictionaries for EP and EEG. But this method can extract the single-trial EP

signal. If the EP changes abruptly, it will definitely suffer.

So in this paper, in order to track the variation of EP in time, a novel single-trial EP estimation algorithm based on sparsifying transforms and GSA is proposed. The extraction process is performed in three stages: first, according to the characteristic of EP, the sparse dictionary of EP is constructed; second, we use transformation matrix to enhance the signal-to-noise ratio and sparsity; third, the sparse coefficients are calculated by sparse representation. A series of experiments carried out on simulated and human test responses confirmed the superior performance of the method.

## 2. Methods

### 2.1 Single-Trial Evoked Potentials Extraction

In this paper, let the EP signal  $s$  to be extracted by corrupted by noise from background ongoing activities. A main source of noise is the spontaneous EEG  $e$ . The observation  $x$  is thus

$$x = s + e \quad (1)$$

### 2.2 Dictionary Constructing for EP

Lange in [12] assume that EP waveform consists of a superposition of  $K$  components

$$s^*(t) = \sum_{k=1}^K a_k s_k(t) \quad (2)$$

where  $s_k(t)$  represents the basic shape of the  $k$ th component, and  $a_k$  is the amplitude of the  $k$ th component. It can be seen that  $s^*(t)$  and  $s_k(t)$  is the template and sub-template for the single trial EP waveform. The sub-template  $s_k(t)$  can be extracted from the template signal  $s^*(t)$  using a certain filtering window function, such as Hamming window and Blackman window. The central location and width of window is determined by the location of point of peak amplitude and peak width of the  $k$ th component. According to the quasi-periodicity and similarity of EP, the single trial EP can be represented by

$$s(t) = \sum_{i=1}^I b_i s_i(t - \tau_i) \quad (3)$$

where  $\tau_i$  indicates the component latency, and  $b_i$  refers to the component amplitude. Assume the dictionary and sparse coefficients represented by  $D$  and  $\theta$ . So  $s$  can be represented, as follows:

$$s = D\theta \quad (4)$$

where  $D = [S_1 S_2 \cdots S_I]$  and

$$S_i^T = \begin{pmatrix} s_i(d) & \cdots & \cdots & s_i(N) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ s_i(2) & s_i(3) & \cdots & \cdots & s_i(N) & 0 \\ s_i(1) & s_i(2) & s_i(3) & \cdots & \cdots & s_i(N) \\ 0 & s_i(1) & s_i(2) & s_i(3) & \cdots & s_i(N-1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & s_i(1) & \cdots & s_i(N-d) \end{pmatrix} \quad (5)$$

### 2.3 Spasifying Transforms

The observation  $x$  of EP is contaminated by EEG, so the sparsity of  $x$  is poor. In order to enhance the sparsity, we use the transformation matrix  $G$  for sparse representation of EP. It suggests that the signal  $x$  is approximately sparsifiable using  $G$ , that is

$$Gx = Gs + Ge = GD\theta + Ge \quad (6)$$

$Gx$  can be represented sparsely on dictionary  $GD$ , and  $Ge$  is the representation residual. So

$$\{G, \theta\} = \arg \min_{H, \theta} \|Gx - GD\theta\|_2^2 \quad s.t. \|\theta\|_0 \leq T_0 \quad (7)$$

The minimization corresponds to fitting the transform model parametrized by  $G$  and  $\theta$ .

In order to avoid the trivial solution  $G=0$  and scale ambiguity, we add two penalties in the cost.

$$\{G, \theta\} =$$

$$\arg \min_{H, \theta} \|Gx - GD\theta\|_2^2 - \alpha \log \det G + \beta \|G\|_F^2 \quad s.t. \|\theta\|_0 \leq T_0 \quad (8)$$

In (8), the  $l_0$ -norm for sparsity could be substituted by a convex  $l_1$ -norm penalty in the cost. This problem is non-convex and NP-hard. However, heuristics have been proposed for its solution such as the analysis K-SVD algorithm which alternates between updating  $G$  and  $\theta$ .

#### 2.3.1 Initialization

Before updating alternately,  $G$  is assigned an

initial value. Some researches use GSA (Generalized Subspace Approach) algorithm to extract the EP signal from the observation. But because of the non-stationarity of EEG, the transformation matrix can't be estimated accurately. In this paper, we use GSA algorithm to determine the initial value of  $G$ .

#### 2.3.2 Estimation of the Sparse Coefficient

Our algorithm alternates between updating  $H$  and  $\theta$ . Firstly, we solve the formula (8) with fixed  $H$ .

$$\theta = \arg \min_{\theta} \|Hx - HD\theta\|_2^2 \quad s.t. \|\theta\|_0 \leq T_0 \quad (9)$$

The approximate solutions can be calculated with BP (Basis Pursuit), which suggests a convexification of the problem by replacing the  $l_0$ -norm with  $l_1$ -norm. In this paper, we use the OMP (orthonormal matching pursuit) algorithm to get an approximation solution of (9) because of its simplicity and fast execution.

#### 2.3.3 Updating of the Transform Matrix

With fixed  $\theta$ , we solve the formula (8). This problem involves the unconstrained minimization

$$H = \arg \min_H \|Hx - HD\theta\|_2^2 - \alpha \log \det H + \beta \|H\|_F^2 \quad (10)$$

We can solve the formula (10) using some optimization algorithm such as steepest descent, or conjugate gradients. For convenience, we define

$$\Omega = \|Hx - HD\theta\|_2^2 - \alpha \log \det H + \beta \|H\|_F^2 \quad (11)$$

The gradient expressions for  $\Omega$  are

$$\nabla_H \Omega = 2(Hx - HD\theta)(x - D\theta)^T - \alpha H^{-T} + 2\beta H \quad (12)$$

Let  $H_j$  represent the transformation matrix in  $j$ th iteration, so

$$H_{j+1} = H_j - \lambda \nabla_H \Omega \quad (13)$$

### 2.4 EP Estimating

We can reconstructed the EP by  $D$  and  $\theta$ ,

$$s = D\theta \quad (14)$$

## 3. Results and Discussion

In the following simulations, the EP signals are simulated by the superimposition of four basic

components respectively which can be represented by Gauss distribution functions, thus

$$s(t) = -0.8 \exp\left(-\frac{(k-80)^2}{32^2}\right) + 0.90 \exp\left(-\frac{(k-107)^2}{32^2}\right) + 0.4 \exp\left(-\frac{(k-230)^2}{28^2}\right) \quad (15)$$

The background EEG superimposed on the EP signal is simulated by an autoregressive process, as follows:

$$q(t) = 1.5084q(t-1) - 0.1587q(t-2) - 0.3109q(t-3) - 0.0510q(t-4) + w(t) \quad (16)$$

where  $w(t)$  is the Gaussian white noise. The EP signal and the observation are shown in Fig. 1.

Then, the single-trial EPs in different SNR

estimated by our method are shown in Fig. 2. It can be seen from this figure that the estimation performance degrades with decreasing SNR.

In order to verify the performance of our algorithm for EP signal extraction, two single-trial extraction algorithms for EP signals, namely MOSCA and GSA are compared in the following simulations. The performance of the five algorithms under various SNR conditions is examined. The SNR of the observations is changed from +10 dB to -10 dB in increments of -2 dB. For each SNR value, 100 pairs of observations are generated. The average results of correlation coefficient with 100 independent runs are shown in Fig. 3 respectively.

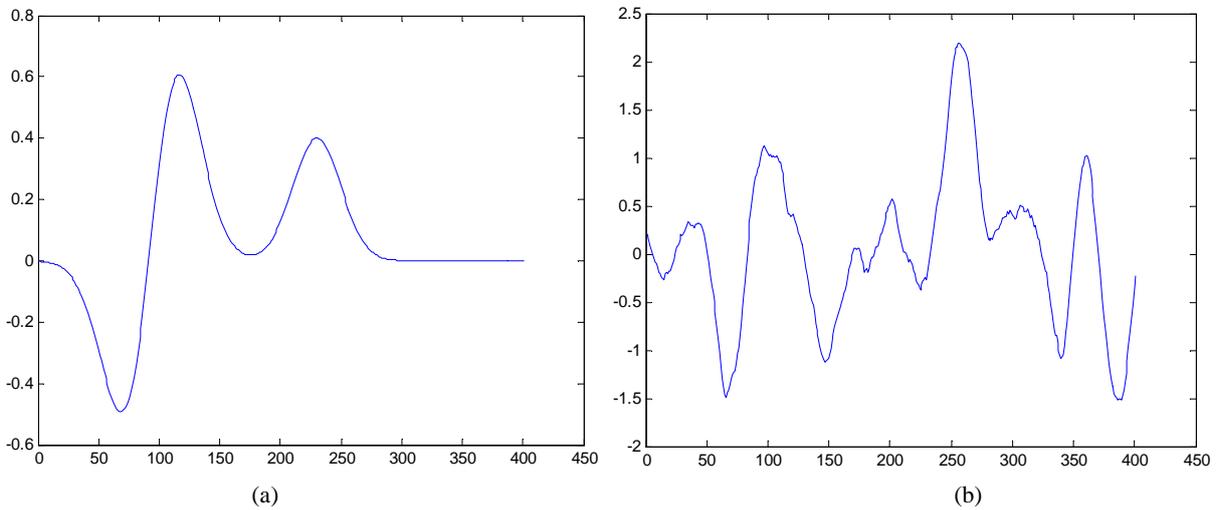
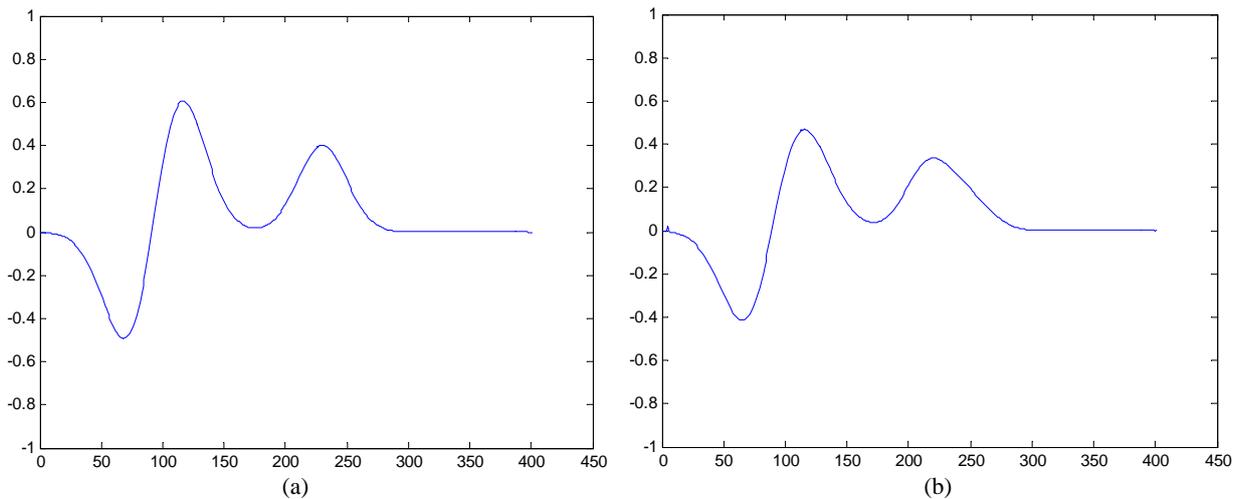
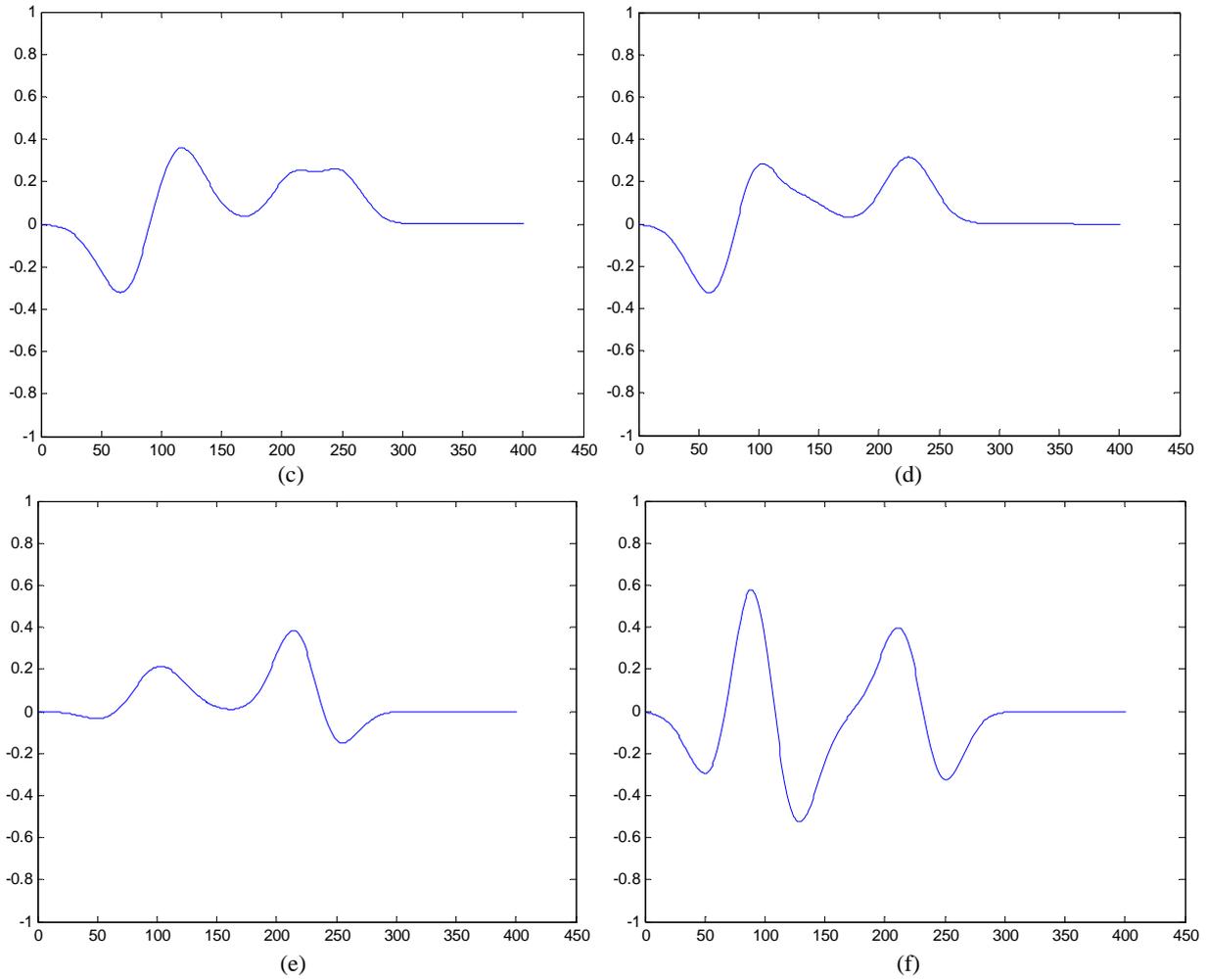


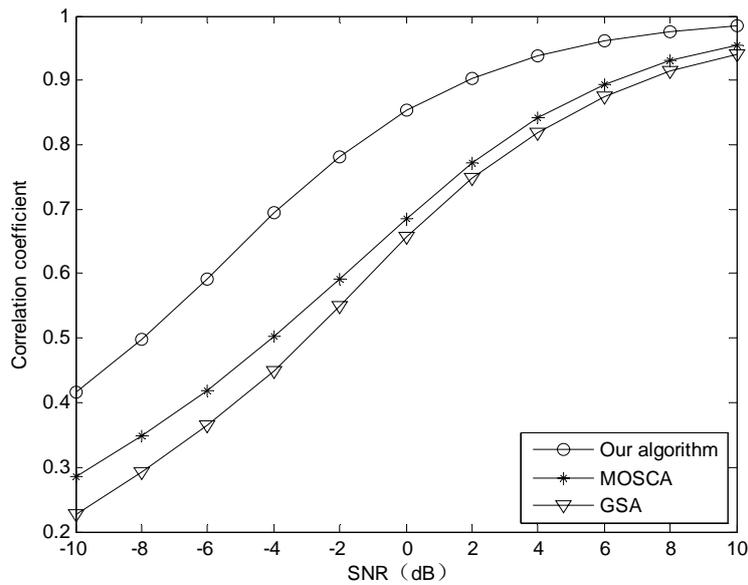
Fig. 1 (a) a simulated waveform and (b) the observation, SNR=-10Db.





**Fig. 2** Single-trial EP estimated by our method.

(a) the original EP (b) SNR=5 dB; (c) SNR=0 dB; (d) SNR=-5 dB; (e) SNR=-10 dB; (f) SNR=-15 dB.



**Fig. 3** Correlation coefficient and error power with different SNR.

It can be seen from Fig. 3 that the estimation performance degrades for all the methods with increasing strength of the noise. Our algorithm significantly bested other algorithms. This indicates our algorithm is powerful in resistance to noise.

#### 4. Conclusion

In this paper, we have presented a novel single-trial EP extraction algorithm based on sparsifying transforms. The proposed algorithm uses the alternative strategy to update the transformation matrix and sparse coefficients. The EP signal can be constructed by the dictionary and coefficients. The experimental results show that the proposed scheme has better estimation performance than the state-of-the-art algorithms.

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