Mathematical Model Prediction of Heat Losses from a Pilot Sirosmelt Furnace

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Abstract: A mathematical model was developed for simulating heat transfer through the sidewall, bottom and top of a pilot scale TSL (Top-Submerged-Lance) Sirosmelt furnace. With a feed rate of about 50 kg/h, the furnace has been used for investigating the technical feasibility of a variety of pyrometallurgical processes for smelting nonferrous and ferrous metals and for high temperature processing of solid wastes including electronic scraps, etc. The model was based on numerical solution of energy transport equations governing heat conduction in multi-layered linings in the sidewall, bottom and top lid of the furnace as well as convection and radiation of heat from the furnace outer surfaces to the ambient. Imperfect contacts between two neighboring solid lining layers due to air gap formation were considered. Temperature profiles were determined across the furnace bottom, top lid and three sections of the furnace sidewall, from which the heat loss rates through the corresponding parts of the furnace were calculated. The modelling results indicate that approximately 88% of heat is lost from the furnace sidewall, 7-8% from the bottom and 4-5% from the top lid. With increasing melt bath temperature, the proportion of total heat loss from the bottom decreases whereas that from the top lid increases and that from the sidewall is little changed. For a bath temperature of 1,300 °C, total absolute heat loss rate from the furnace was found to be close to 12 kW.

Key words: Heat transfer, heat loss, TSL, smelting, modelling, simulation.

1. Introduction

The pilot-scale Sirosmelt furnace is a versatile TSL (Top-Submerged-Lance) type high temperature smelting reactor located at the Clayton laboratories of CSIRO (Commonwealth Scientific and Industrial Research Organization) in Australia. This reactor is used to help assess the technical feasibility of a variety of pyrometallurgical processes, including copper and nickel smelting and converting, and the recovery of value from waste materials including lead blast furnace slags and electronic scraps, etc. The development of the Sirosmelt process occurred at CSIRO during the 1970s and the process was commercialized by the Ausmelt and Isasmelt processes during the 1980s [1-3]. The key to the Sirosmelt process is the delivery of combustible and smelting gases below the surface of a melt bath. In its present manifestation, natural gas along with

oxygen enriched air is injected through a 32 mm outer diameter lance comprised of stainless steel co-axial tubes. Pulverized coal or fuel oil can also be used as a fuel. Highly turbulent conditions are created by the submerged injection and combustion of the gases inside the melt bath.

Smelting investigations usually aim to create steady state conditions where feed materials are added to the furnace at a rate of around 50 kg/h. Natural gas and oxygen-enriched air are injected at sufficient flow rates to maintain a constant bath temperature and provide oxygen for the smelting reactions to occur. Process feasibility studies rely on steady state Sirosmelt test work as well as on heat and mass balance models to assess material flows and energy requirements. A restriction in the ability to conduct accurate heat and mass balance studies has been the lack of knowledge of the amount of heat losses through the sidewall, the base (bottom) and the top lid of the Sirosmelt furnace. This paper presents a mathematical model of heat transfer in sidewall, bottom and top lid of the Sirosmelt furnace so

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as to predict the heat loss rates during steady state smelting operations.

2. Sirosmelt Furnace Construction

The Sirosmelt furnace is a cylindrical shaped vessel made from specially cut refractory bricks formed into an annular shaped sidewall. The bricks are held in place by a mild steel shell. Layers of mineral wool and castable refractory help insulate the reactor and take up the gap between the bricks and the steel shell. As schematically shown in Fig. 1, the vessel is constructed in five main parts which consist of a base (bottom), three sections of sidewall, i.e., lower (Wall Section 1), middle (Wall Section 2) and upper (Wall Section 3), and a top lid. Fig. 1 also gives the dimensions of the furnace and the thickness of each lining material layer (shown in inset tables). As indicated in Fig. 1, on the top lid, there are a number of openings that are used as injection lance port, offgas exit, and ports for material charging, sampling and observation. An injection lance is inserted along the centerline of the furnace through the lance port located at the centre of the top lid. The lance which is immersed into the melt bath to a certain depth is commonly known as a TSL. In normal operations, the melt bath is contained in the lower section while the middle and upper sections are used to separate the gas phase from the molten splashes.

3. Heat Transfer Model

3.1 Model Assumptions

In the present work, the following assumptions were made in the formulation of the heat transfer model:

- Steady-state heat transfer was considered;
- The inner enclosure of the furnace was divided into three sections, namely lower, middle and upper sections (corresponding to Wall Sections 1 to 3 defined in Fig. 1, respectively). They were assumed to be perfectly mixed reactors, i.e., uniform temperatures were assumed in the melt bath in Wall Section 1 and in the gas phase in Wall Sections 2 and 3;

- The bulk temperature in Wall Section 1 was defined to be equal to the measured melt bath temperature ($T_{bulk1} = T_{bath}$), and the bulk temperatures in gas phase in Wall Sections 2 and 3 (T_{bulk2} and T_{bulk3}) were linearly interpolated between the bath temperature (T_{bath}) and a temperature deduced from the measured off-gas temperature;

- In the calculation of heat transfer through the top lid, all the openings on the lid (i.e., offgas exit, lance port and charging, sampling and observation ports) were neglected, assuming they are perfectly sealed;

- The injection lance was excluded from the heat transfer calculations;

- All heat conduction was assumed to be one-dimensional, normal to and across lining material layers and air gaps;

- Natural and forced convection and radiation at furnace sidewall and bottom shell outer surfaces were considered;

- Heat loss from the top lid shell outer surface was assumed to be by natural convection and radiation only. Forced convection was neglected as the top lid is located above an operating platform that blocked cross-flows;

- Radiation between two neighbouring material layers in an air gap was considered;

- The thickness of all air gaps was assumed to be 0.1 mm.

3.2 Overall Heat Transfer Equation

The rate of heat loss through a multi-layered sidewall, bottom or top lid of the Sirosmelt furnace can be described by an overall heat transfer equation:

$$Q = \sum_j \frac{T_{bulk} - T_a}{\sum_i R_i} \quad (1)$$

where, Q is the heat transfer rate through furnace sidewall, bottom or top lid in Watts (W), T_{bulk} the average temperature of a bulk material (e.g., melt bath in Wall Section 1 and gas phase in Wall Sections 2 and 3) in Kelvin (K), T_a the ambient temperature (K); i

Fig. 1 Schematic illustration of pilot-scale Sirosmelt furnace.

denotes a material layer in furnace sidewall, bottom or top lid; j denotes the heat transfer location (such as furnace bottom, top lid or a section of sidewall); and R_i

the thermal resistance of the i th material layer or the interface between this material layer and air, e.g. furnace shell outer surface and air gap (K/W).

3.3 Modelling Principle

The modelling principle was to determine heat transfer resistances (R_i) of various material layers comprising the furnace sidewall, bottom and top lid so that the temperature distributions across the sidewall, bottom and top lid and the resultant heat transfer rates (heat loss rates) can be calculated using Eq. (1). Details of the heat transfer resistance calculations and the modelling procedure can be found in Appendix A.

3.4 Material Thermophysical Properties

The thermophysical properties of various materials involved in the model are listed in Tables 1 and 2. They are considered as temperature dependent, except for steel shell and top lid ramming mix. Appendix B provides the detailed forms of the equations used for

calculating the thermophysical properties, i.e., Eqs. (B.1)-(B.8) in Tables 1 and 2.

4. Results and Discussion

4.1 Experimental Measurement

High temperature smelting experiments were performed on the pilot-scale Sirosmelt furnace in order to calibrate and validate the heat transfer model. The furnace sidewall and bottom shell outer surface temperatures were measured using an infra-red pyrometer during a period of steady state operation where the process variables were kept constant and the bath temperatures stabilized to within ± 5 °C. The measurement positions are indicated in Fig. 1 and the measured average surface temperatures in three experiments are given in Table 3. These measurements were used to calibrate and validate the model.

Table 1 Solid layer thermal conductivity and emissivity.

Material	Thermal conductivity (W/mK)	Emissivity
Shell (mild steel)	32.889 ^a	0.8 ^d
Insulation (kaowool)	Eq. (B.1)	0.9 ^e
Castable (alumina-silica)	Eq. (B.2)	0.855 ^d
Brick (magnesia-chrome)	Eq. (B.3)	0.75 ^d
Top lid ramming mix	5.0 ^f	0.855 ^f

^aRef. [4], ^bRef. [5], ^cRef. [6], ^dRef. [7], ^eRef. [8], ^fEstimated.

Table 2 Air thermophysical properties [4].

Property	Unit	Equation
Thermal conductivity	W/mK	Eq. (B.4)
Heat capacity	J/kgK	Eq. (B.5)
Kinematic viscosity	m ² /s	Eq. (B.6)
Density	kg/m ³	Eq. (B.7)
Thermal expansivity	1/K	Eq. (B.8)

Table 3 Measured melt bath temperature and average shell outer surface temperature (°C)*.

Experiment No.	Symbol	EXP 1	EXP 2	EXP 3
Bath	T_{bath}	1,275	1,270	1,260
Bottom	T_{bottom}	172	170	180
Wall Section 1	T_{side1}	227	226	225
Wall Section 2	T_{side2}	149	158	167
Wall Section 3	T_{side3}	129	133	140

*The average was taken over the temperatures measured at four locations in each wall section while the bottom surface temperature (T_{bottom}) was measured at only one location (i.e., roughly the centre of the furnace bottom), c.f., Fig. 1.

4.2 Model Calibration

There exist some parameters in the model that are practically uncertain and have to be affirmed by means of calibrating the model against the experimental measurements. Among them, a typical parameter is the ambient air flow velocity, i.e., u in Eqs. (A.10) and (A.14), which was used for calculating forced convection heat transfer at the furnace bottom and sidewall outer surfaces. This velocity depends on local wind conditions around the furnace. In this work a constant air flow velocity (u) was chosen as a solely adjustable parameter for model calibrations. The model calibration was performed in such a way that, through adjusting u , the calculated furnace shell outer surface temperatures approximately matched measured ones in one experiment (EXP 1), and it was found that the magnitude of the ambient air velocity was around 0.8 m/s. Then, by setting $u = 0.8$ m/s in the model, further model simulations were carried out for two other experimental cases (EXP 2 and EXP 3). Fig. 2 shows a comparison of the predicted sidewall and bottom shell surface temperatures with the measured ones in the three experiments. As seen from Fig. 2, once calibrated with measured data from only one experiment (i.e., EXP 1), the model could give rather promising predictions of the shell surface temperatures for other two experiments (EXP 2 and EXP 3) within about $\pm 15\%$.

4.3 Predicted Temperature Distributions across Furnace Sidewall, Bottom and Top Lid

After the calibration procedure, the model was used to predict the temperature distributions across the linings of the furnace sidewall, bottom and top lid and to investigate sensitivities of furnace heat loss to a range of process parameters. Fig. 3 shows the predicted temperature profiles in the furnace bottom, sidewall and top lid for the experimental case EXP 1. The sidewall and top lid have the same material layers corresponding to those for the bottom (marked in Fig. 3a) except for the working layer in the top lid that was a

Fig. 2 Comparison between predicted and measured sidewall and bottom shell outer surface temperatures (solid symbols denote data from EXP 1 for model calibration).

ramming mix marked in Fig. 3c (see Fig. 1 for details of furnace construction).

4.4 Effect of Bath Temperature on Heat Loss Rate

Fig. 4 shows the effect of bath temperature on heat loss rates from furnace bottom, three sidewall sections and top lid. It can be seen from Fig. 4a that the absolute heat losses from different locations of the furnace all increase with increasing bath temperature. There is a linear relationship between the furnace total absolute heat loss rate and the bath temperature.

Within the investigated bath temperature range (1,250–1,350 °C), the predicted total absolute heat loss rate from the furnace ranges from 10.6–13.4 kW, and for a bath temperature of 1,300 °C, for instance, the model predicts the total heat loss rate to be close to 12.0 kW. When the bath temperature increases, the absolute magnitudes of the heat loss rates from the middle and upper sections (Wall Sections 2 and 3) increase faster than those from other parts of the furnace.

Fig. 4b shows the relative heat loss rates (as percentage of the total heat loss rate) from sidewall, bottom and top lid of the furnace as a function of the bath temperature. It can be seen from Fig. 4b that major heat loss occurs from the furnace sidewall representing

(a)

(b)

(c)

Fig. 3 Predicted temperature profiles across (a) furnace bottom, (b) three sections of sidewall and (c) top lid for experimental case EXP 1.

(a) Absolute heat loss rates

(b) Relative heat loss rates

Fig. 4 Predicted influence of bath temperature on heat loss rates from furnace bottom, top lid and different sidewall sections.

about 88% of total heat loss while only limited amount of heat is lost through the furnace bottom (7-8%) and top lid (4-5%). With increasing bath temperature the relative heat loss from the bottom decreases but that from the top lid increases while that from the sidewall is essentially unchanged.

5. Conclusions

A heat transfer model has been developed for predicting heat losses from a pilot-scale Siros melt furnace. The model is also capable of predicting temperature distributions across the furnace sidewall and bottom as well as top lid. The model has been

calibrated against the measured furnace shell outer surface temperatures. The modelling results indicate that about 88% of heat is lost from the furnace sidewall, 7-8% from the bottom and 4-5% from the top lid. The proportion of total heat loss from the bottom decreases with the increasing bath temperature whereas that from the top lid increases while that from the sidewall is little changed. For a bath temperature of 1,300 °C, the total heat loss rate was found to be close to 12 kW.

In future planned work these heat loss calculations will form part of a heat and mass balance model of the Sirosmelt furnace.

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References

- [1] Floyd, J. M., and Conochie, D. S. 1984. "Sirosmelt—The First Ten Years." In *Proceedings of Extractive Metallurgy Symposium*, 1-8.
- [2] Rankin, W. J., Jorgensen, F. R. A., Nguyen, T. V., Kou, P. T. L., and Taylor, R. N. 1989. "Process Engineering of Sirosmelt Reactors, Lance and Bath-Mixing Characteristics." In *Proceedings of Extraction Metallurgy '89*, 577-600.
- [3] <http://www.csiropedia.csiro.au/display/CSIROPedia/SiroSmelt>.
- [4] Haynes, W. M. 2014. *CRC Handbook of Chemistry and Physics*. 95th edition. Florida: CRC Press.
- [5] Isolite Insulating Products Co. Ltd. Material Data Sheet.
- [6] Shinagawa Refractories Australasia Pty Ltd. Material Data Sheet.
- [7] Mikron Instrument Company Inc. "Table of Emissivity of Various Surfaces." http://www-eng.lbl.gov/~dw/projects/DW4229_LHC_detector_analysis/calculations/emissivity2.pdf.
- [8] Morgan Thermal Ceramics. "Kaowool 1260 Ceramic Felt." http://www.morganthermalceramics.com/sites/default/files/datasheets/2_kaowool1260ceramicfeltenglis h.pdf.
- [9] Geiger, G. H., and Poirier, D. R. 1973. *Transport Phenomena in Metallurgy*. Boston: Addison-Wesley Publishing Company.
- [10] Whitaker, S. 1972. "Forced Convection Heat Transfer Correlations for Flow in Pipes, Past Flat Plates, Single Cylinders, Single Spheres, and for Flow in Packed Beds and Tube Bundles." *American Institute of Chemical Engineers* 18: 361-71.
- [11] The New Zealand Institute of Food Science & Technology Inc. "Unit Operations in Food Processing." <http://www.nzifst.org.nz/unitoperations/httrtheory6.htm#natural>.

Appendix A—Thermal Resistance Calculation and Modelling Procedure

A.1 Calculation of Thermal Resistances

The thermal resistance, R_i , involved in Eq. (1) can be calculated as

(i) for a flat material layer in bottom or top lid,

$$R_i = \frac{X_i}{k_i A_i} \quad (\text{A.1})$$

where, X is the material layer thickness, m; k the material thermal conductivity, W/mK; A the heat transfer area, m²; and i the material layer identity;

(ii) for a cylindrical material layer in sidewall,

$$R_i = \frac{1}{2\pi k_i H_i} \ln \left(\frac{r_{i,c}}{r_{i,h}} \right) \quad (\text{A.2})$$

where, H is the height of a furnace sidewall section, m; and $r_{i,h}$ and $r_{i,c}$ the hot face and cold face radii of the i th material layer, respectively, m;

(iii) at furnace sidewall, bottom or top lid shell outer surfaces,

$$R_{sa} = \left[\underbrace{(h_{nc} + h_{fc}) A_{sa}}_{\text{Convection}} + \underbrace{\varepsilon \sigma (T_{sa} + T_a)(T_{sa}^2 + T_a^2) A_{sa}}_{\text{Radiation}} \right]^{-1} \quad (\text{A.3})$$

where, h is the convective heat transfer coefficient, $\text{W/m}^2\text{K}$; ε the emissivity of steel shell; σ the Stefan-Boltzmann constant ($= 5.6703 \times 10^{-8} \text{ W/m}^2\text{K}^4$); subscripts sa , nc , and fc stand for, shell/ambient surface, natural convection and forced convection, respectively; and, as an assumption, h_{fc} was set to zero for the top lid shell outer surface.

(iv) in a flat air gap in bottom or top lid (assuming the hot and cold faces are in parallel),

$$R_i = \left[\underbrace{\frac{k_a A_{hc}}{\delta_i}}_{\text{Conduction}} + \underbrace{\frac{\sigma (T_{s,h} + T_{s,c})(T_{s,h}^2 + T_{s,c}^2) A_{hc}}{\frac{1}{\varepsilon_h} + \frac{1}{\varepsilon_c} - 1}}_{\text{Radiation}} \right]^{-1} \quad (\text{A.4})$$

where, δ is the thickness of air gap, m; A_{hc} the average area between hot and cold faces in air gap, m^2 ; and subscripts a , s , h and c stand for, air, lining surface, hot face and cold face, respectively;

(v) in a cylindrical air gap in sidewall (assuming the hot and cold faces are in parallel),

$$R_i = \left[\underbrace{\frac{2\pi k_a H_k}{\ln\left(\frac{r_h}{r_c}\right)}}_{\text{Conduction}} + \underbrace{\frac{\sigma (T_{s,h} + T_{s,c})(T_{s,h}^2 + T_{s,c}^2) A_{hc}}{\frac{1}{\varepsilon_h} + \frac{A_h}{A_c} \left(\frac{1}{\varepsilon_c} - 1\right)}}_{\text{Radiation}} \right]^{-1} \quad (\text{A.5})$$

where, k stands for a furnace sidewall section identity.

A.2 Calculation of Heat Transfer Coefficients

The convective heat transfer coefficient (h) involved in Eqs. (A.3)-(A.5), was evaluated from the Nusselt number, i.e.,

$$h = \frac{k_a}{L} Nu \quad (\text{A.6})$$

where, h is the heat transfer coefficient ($\text{W/m}^2\text{K}$); k_a the thermal conductivity of air (W/mK); L the characteristic length of an interface (m); and Nu the Nusselt number.

Nu was calculated using the following published empirical dimensionless correlations:

(i) At bottom shell outer surface,

For natural convection heat transfer (at a hot horizontal flat surface facing downward) [9],

$$Nu_{nc} = 0.27(Gr \cdot Pr)^{1/4} \quad (\text{A.7})$$

where, Gr is the Grashof number and Pr the Prandtl number in usual definitions.

For forced convection heat transfer (at a horizontal flat surface) [10],

$$Nu_{fc} = 0.664 Re^{1/2} Pr^{1/3}, (Re < 2 \times 10^5) \quad (\text{A.8})$$

$$Nu_{fc} = 0.036Re^{0.8}Pr^{0.43}(\mu_a/\mu_{sa})^{1/4}, (Re \geq 2 \times 10^5) \quad (A.9)$$

where, Re is the Reynolds number, which is defined as:

$$Re = \frac{\rho u L}{\mu} \quad (A.10)$$

where, u is the air velocity (m/s); ρ the air density (kg/m³); μ the air dynamic viscosity (Pa·s); and L the characteristic length for the furnace bottom shell outer surface (m), which is evaluated as the edge length of a square with the same area as the furnace bottom shell outer surface.

(ii) At sidewall shell outer surface,

For natural convection heat transfer (at a vertical cylindrical surface) [11],

$$Nu_{nc} = 0.53(Gr \cdot Pr)^{1/4}, (1 \times 10^4 < Gr < 1 \times 10^9) \quad (A.11)$$

$$Nu_{nc} = 0.12(Gr \cdot Pr)^{1/3}, (1 \times 10^9 \leq Gr < 1 \times 10^{12}) \quad (A.12)$$

For forced convection heat transfer (at a vertical cylindrical surface) [10],

$$Nu_{fc} = (0.4Re^{1/2} + 0.06Re^{2/3})Pr^{0.4}(\mu_a/\mu_{sa})^{1/4} \quad (A.13)$$

where, Re is defined for cylindrical wall shell surface as

$$Re = \frac{\rho u D}{\mu} \quad (A.14)$$

where, D is the furnace outer diameter (m).

(iii) At top lid shell outer surface,

For natural convection heat transfer (at a hot horizontal flat surface facing upward) [9],

$$Nu_{nc} = 0.54(Gr \cdot Pr)^{1/4}, (1 \times 10^5 < Gr < 2 \times 10^7) \quad (A.15)$$

$$Nu_{nc} = 0.14(Gr \cdot Pr)^{1/3}, (2 \times 10^7 \leq Gr < 3 \times 10^{10}) \quad (A.16)$$

Air properties used for calculating the parameters in Eqs. (A.7)-(A.9), (A.11)-(A.13), (A.15) and (A.16) are evaluated at the average temperature of air film at shell surface defined as

$$T_{film} = 0.5(T_{sa} + T_a) \quad (A.17)$$

A.3 Modelling Procedure

An iterative method was adopted to use Eq. (1) to calculate heat transfer rate (Q) through furnace bottom, top lid and three sections of sidewall, respectively. The modelling procedure is:

Estimate a series of the hot face and cold face temperatures of each material layer ($T_{s,h}$ and $T_{s,c}$) in bottom, sidewall or top lid and the temperature at the shell outer surface of bottom, sidewall or top lid (T_{sa});

Use Eqs. (A.1)-(A.5) to calculate thermal resistances (R_i) of various material layers (including air gaps and air film at the shell outer surface), which comprise the furnace bottom, sidewall or top lid;

Use Eq. (1) to calculate heat transfer rate (Q), from which $T_{s,h}$ and $T_{s,c}$ of each material layer as well as T_{sa} are predicted;

Compare the predicted values of $T_{s,h}$, $T_{s,c}$ and T_{sa} with those estimated values in step 1) correspondingly until the absolute

temperature differences all fall below 1 °C. Otherwise, repeat steps 1) through 4) until the fore-mentioned criterion is met.

In the present work, the model simulations were performed by means of Microsoft Excel spreadsheet calculations on a personal computer.

Appendix B—Equations for Calculating Material Thermophysical Properties

The following equations were used for calculating material thermophysical properties as functions of temperature (T in K):

Thermal conductivity of insulation layer (kaowool) [5]:

$$k = -0.527 + 1.080 \times 10^{-3}T - 3.750 \times 10^{-7}T^2, \quad (\text{W/mK}) \quad (\text{B.1})$$

Thermal conductivity of castable refractory layer (alumina-silica) [6]:

$$k = 0.295 + 2.000 \times 10^{-4}T, \quad (\text{W/mK}) \quad (\text{B.2})$$

Thermal conductivity of brick layer (magnesia-chrome) [6]:

$$k = 9.858 - 1.382 \times 10^{-2}T + 1.006 \times 10^{-5}T^2 - 2.444 \times 10^{-9}T^3, \quad (\text{W/mK}) \quad (\text{B.3})$$

Thermal conductivity of air [4]:

$$k = 3.180 \times 10^{-3} + 8.616 \times 10^{-5}T - 3.203 \times 10^{-8}T^2 + 6.214 \times 10^{-12}T^3, \quad (\text{W/mK}) \quad (\text{B.4})$$

Heat capacity of air [4]:

$$Cp = 1031 - 2.010 \times 10^{-2}T + 3.985 \times 10^{-4}T^2 - 3.080 \times 10^{-10}T^3, \quad (\text{J/kgK}) \quad (\text{B.5})$$

Kinematic viscosity of air [4]:

$$\nu = (-7.287 + 5.204 \times 10^{-2}T + 8.281 \times 10^{-5}T^2 - 9.647 \times 10^{-9}T^3) \times 10^{-6}, \quad (\text{m}^2/\text{s}) \quad (\text{B.6})$$

Density of air:

$$\rho = 352.91/T, \quad (\text{kg/m}^3) \quad (\text{B.7})$$

Thermal expansivity of air:

$$\beta = 1/T, \quad (1/\text{K}) \quad (\text{B.8})$$