Simulating Particle Swarm Optimization Algorithm to Estimate Likelihood Function of $ARMA(1, 1)$ Model

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Abstract: This paper present a simulation study of an evolutionary algorithms, Particle Swarm Optimization PSO algorithm to optimize likelihood function of $ARMA(1, 1)$ model, where maximizing likelihood function is equivalent to maximizing its logarithm, so the objective function 'obj.fun' is maximizing log-likelihood function. Monte Carlo method adapted for implementing and designing the experiments of this simulation. This study including a comparison among three versions of PSO algorithm "Constriction coefficient $CCPSO$, Inertia weight $IWPSO$, and Fully Informed $FIPSO$ ", the experiments designed by setting different values of model parameters a_1 , b_1 sample size n , moreover the parameters of PSO algorithms. MSE used as test statistic to measure the efficiency PSO to estimate model. The results show the ability of PSO to estimate $ARMA$'s parameters, and the minimum values of MSE getting for $COPSO$.

Key words: Particle Swarm Optimization algorithm, Likelihood function, $ARMA(1, 1)$ Model

1. Introduction

A mathematical optimization problem is one in which some real-valued function is either maximized or minimized relative to a given set of feasible alternatives. Mathematical optimization is a selection of "best available" values of the objective function defined in its domain, this selection of a best value according to some criteria.

That is, we consider optimization problems having a mathematical representation, a real function $f : X \rightarrow Y$, is known as fitness or the objective function, defined on a $X \subseteq \mathcal{R}^d$ domain of the function, and it is the search space of $x \in \mathcal{R}^n$, each of $x_i, i = 1, \dots, d$ defined in particular range $[x_{lb_i}, x_{ub_i}]$, mathematical formulation of a minimization problem is

$$\min_{\vec{x}} f(\vec{x}) \quad (1)$$

$$s.t \ x_{lb_i} \leq x_i \leq x_{ub_i}, \text{ for } i = 1, \dots, d$$

Most of researches prefer to use heuristic algorithms to optimize the solution of the optimization problems because it is easy to implement it and programming, also the availability of there software, in other side, we can see the complexity of the functions that used to describe a systems and there is no analytically solution encouraged the researchers to adopt this type of algorithms. $ARMA$ model is one of models that its complexity increase with increasing it's orders, the number coefficients in the model. Likelihood function used to estimate $ARMA$'s parameters, in fact the complexity of solving likelihood function also effected by increasing the model's order, using likelihood function to estimate model's parameters requires a numerical methods like fixed point method, Newton Raphson method, using these method need to find derivative of the likelihood function, the looping of searching about the solution starting by an initial guess solution this value may be far away from the optimal solution, these methods poor of a global convergence properties, i.e., these methods are iterative methods and almost there solutions are locally

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Fig. 1 An example of function with multi global and local maximum and minimum optimal point.

evaluated, and in case of zero computing derivative may loop indefinitely.

The purpose of this study is simulating one of the soft computing algorithms, known as Particle Swarm Optimization *PSO* algorithm [1], Eberhart and Kendy 1995, to optimize likelihood function of time series model autoregressive moving average *ARMA*(1, 1) model.

The idea of *PSO* is an simulation to a natural behavior of flock of birds, fish.., during their searching about the food, it is known that the best way followed by flock to reach of an ideal sources of the food depend on the information keeping in their memory where each bird has particular memory about the best position, beside that, the connections and exchanging the information between each other support them to find the best position.

There are a lot of studies main in developing and using *PSO* in time series analysis, it was used for model selection, identifying *ARMAX* models, chaotic time series modeling, model order reduction, and for prediction of chaotic system... This study is a continuous to works given in [8],[9] and [10] that using different versions of soft computing algorithm for *ARMA* model estimation and forecasting, the aim of these studies is to show the ability of soft computing algorithms in time series modeling and its

applications.

Rest of this paper given by, section 2 introduces *ARMA* model and likelihood function, section 3 present *PSO* and its versions, section 4 gives a simulation and the results, and the last section is for the conclusions.

2. Autoregressiv Moving Average *ARM*(p, p)

A statistical phenomenon that evolves in time according to probabilistic laws is called a stochastic process. In analysis time series we regarded it as a realization of a stochastic process. A very special case of stochastic process, called a stationary process, is based on the assumption that the process is in a particular state of statistical equilibrium. A stochastic process is said to be strictly stationary if it's properties are unaffected by change of time origin.

The special case of stationary process is an Autoregressive Integrated Moving Average *ARIMA* process, this process given by

$$a(B)y_t = \Delta^d a(z)y_t = b_0 + b(z)e_t \quad (2)$$

where $a(z) = 1 - a_1z + \dots - a_pz^p$ know as autoregressive operator, it is assumed to be stationary, the roots of $a(z)$ lie outside the unit circle.

$b(z) = 1 - b_1z - \dots - b_qz^q$ is the moving average operator it is assumed to invertible the roots of $b(z) = 0$ lie outside of unit circle, z is here refer to the back-shift operator and Δ^d is difference operator, usually $d = 1, 2$ the difference take in order to make series more stable, p, q is the order of *ARMA*.

The important representation of *ARMA* process this one given based on the previous noises where y_t represented as a linear filter:

$$\begin{aligned} y_t &= e_t + \psi_1 e_{t-1} + \psi_2 e_{t-2} + \dots \\ &= e_t + \sum_{j=1}^{\infty} \psi_j e_{t-j} \\ &= \psi(z)e_t \end{aligned}$$

where $\psi(z) = 1 + \psi_1 z + \psi_2 z^2 + \dots$.

So, the stationary ARMA(p, q) process is given by

$$y_t = \frac{b(z)}{a(z)} e_t \quad (3)$$

$\psi_0 = 1$ and e_t is a sequence of uncorrelated shocks.

For this note we interest in special case of ARMA process, ARMA(1, 1), this model given by,

$$(1 - a_1)y_t = (1 - b_1)e_t$$

as a result,

$$y_t = a_1 y_{t-1} + e_t - b_1 e_{t-1} \quad (4)$$

The linear filtering of this model is defined by finding ψ 's as follows:

$$a(z)\psi(z) = b(z)$$

$$[1 - a_1 z][1 + \psi_1 z + \psi_2 z^2 + \dots] = [1 - b_1 z]$$

by equaling powers of the coefficients for both sides

$$\psi_j = a_1^{j-1} (a_1 - b_1)$$

and

$$y_t = a_1^{j-1} (a_1 - b_1) e_{t-j} + e_t$$

2.1 Likelihood Function of ARMA(1, 1)

The likelihood function of n random variables $x_1, x_2, \dots, x_n$ is defined to be the joint density of the n random variables, $\mathbf{P}_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n; \theta)$, which is considered to be a function of θ . In particular if $x_1, x_2, \dots, x_n$ is a random sample from the density $\mathcal{P}(x; \theta)$, then the likelihood function is $\mathbf{p}(x_1), \dots, \mathbf{p}(x_n)$.

Be the likelihood function of the random variables $x_1, \dots, x_n$. Assuming that $e_t \sim i.i.d. N(0, \sigma_e^2)$, the joint probability density of $e = (e_1, e_2, \dots, e_n)^t$ is given by

$$\mathbf{P}(e) = (2\pi\sigma_e^2)^{-\frac{n}{2}} \exp\left(-\frac{1}{2\sigma_e^2} \sum_{t=1}^n e_t^2\right) \quad (5)$$

Let $y_t, t = 1, 2, \dots, n$, be sequence of time series observations generated by stationary ARMA(1, 1) process, with sample size n , the noise term in Eq. (4) can be written as:

$$e_t = y_t - a_1 y_{t-1} + b_1 e_{t-1}$$

For a given initial values (y_*, e_*) , let

$$\mathbf{L} = \mathbf{L}(\theta; x_1, \dots, x_n)$$

Then, log-likelihood function is

$$\mathbf{L}_*(a_1, b_1, \sigma_e) = \frac{-n}{2} \ln 2\pi \sigma_e^2 - \frac{\mathbf{S}_*(a_1, b_1)}{2\sigma_e^2} \quad (6)$$

where, $\mathbf{S}_*(a_1, b_1) = \sum_{t=1}^n e_t^2(a_1, b_1 | y_*, e_*)$

Moreover,

$$\hat{\sigma}_e^2 = \frac{S_*(\hat{a}_1, \hat{b}_1)}{d.f.}$$

where $d.f. = no.$ of terms in S_* - , no. of parameters.

3. Particle Swarm Optimization

3.1 Description of PSO Algorithm

Optimize an d -dimensional objective function $f(\bar{x})$, the original PSO algorithm is initialize with a population of compute solutions (called particles) $\{p_1, \dots, p_k\} = P$, randomly located in the solution space. The objective function determines the quality of particle's position, the equality of the solution it represent.

A particle p_i at time step g has a position vector $\bar{x}_i^g$ and an associated velocity vector $\bar{v}_i^g$, every particle remembers the position in which he it has received the best evolution of the objective function. This memory is represented by a vector $p\bar{best}_i$, this vector is updated every iteration particle p_i finds a better position.

The vector $g\bar{best}$ stores the best position any particle in the swarm has visited, so for the minimization problem

$f(\vec{gbest}) \leq f(\vec{pbest}) \quad \forall p_i \in P$, the same. The algorithm updating the particle's velocity and the position iteratively until the stopping criterion valid.

The population of *PSO* is the swarm consist of a number of particles, a particle in the swarm has inter connected with other particles, this inter connected is called the neighborhood topology, where the neighborhood used for a communication structure not for geographical neighborhood. The particles moving in the search space by following a change rule, based on this rule the particles move through the search space at a given iteration t depending on its position at iteration $t-1$ as well as the position of its previous best location, this describe of phase cognitive of *PSO*. The social phase of *PSO* introduces by an interaction rule, where the particles position are not sufficient by the best position in there history, but also on the best position in history of there neighbors.

The particle has three d -dimensional principles vectors,

$\vec{x}_i$: Current position.

$\vec{p}_i$: Previous best position.

$\vec{v}_i$: Velocity used to update the position, velocity used as a step size to modified algorithm's solution which is adjusted through the algorithm's computations.

Equations used for updating the velocities and the positions are,

$$\begin{aligned} \vec{v}_i &\leftarrow \vec{v}_i + \vec{U}(0, \phi_1) \otimes (\vec{p}_i - \vec{x}_i) \\ &\quad + \vec{U}(0, \phi_2) \otimes (\vec{p}_g - \vec{x}_i) \\ \vec{x}_i &\leftarrow \vec{x}_i + \vec{v}_i \end{aligned} \quad (7)$$

$\vec{U}(0, \phi_i)$: Vector of randoms numbers uniformly distributed in $(0, \phi_i)$ generated randomly at each iteration and for each particle.

$\otimes$: Component-wise multiplication, and ϕ_i : Fixed parameters known as acceleration parameters, they determine the magnitude of the random forces in the direction of best personal position $\vec{p}_i$ and neighborhood best $\vec{p}_g$.

3.2 PSO Algorithm Implementation

The real description of *PSO*'s mechanism explaining by the following three basic steps:

(1) Initialization: The positions of the particle in the search space they are initialized uniformly random within the search space. If one choose to initialize the velocities to a vector of zero then $\vec{p}_i$ should be different from $\vec{x}_i$ to enable the particle to start moving, usually $\vec{x}_i$ and $\vec{v}_i$ are initialized randomly while $\vec{p}_i$ is initialized as $\vec{x}_i$ for the first iteration.

The nonzero velocities move the particle through the search space in a randomly chosen direction and magnitude. $pbest_i$ contains the objective function value of the best position of the particle.

(2) Iteration terms and functional evaluation: The function computation is given in general loop where, for each particle, evaluate the desired optimization fitness function in d -dimensional variables.

Compare particle fitness evaluation with its $Pbest_i$, if current value $vec P_i$ equal to the current location $\vec{x}_i$ in d -dimensional space :

Identify the particle in the neighborhood with best success so far and assign its index to the variable g . change the velocity and position of particle as Eq. (7) if a criterion is met, exit loop.

(3) Stopping criteria

Stopping criteria is used to terminate the iterative search process some of using stopping criteria

- The algorithm is terminated when maximum numbers of iterations function evaluations has been reached, If the maximum number of iterations is too small the search process may stop before a good results has been found.

- The algorithm is terminated when there is no significant improvement over a number of iterations, this improvement can be measured in different ways. The process may be considered to have terminated if the average change of the particles position are very small or the average velocity of the particles is approximately zero over a number of iterations.

3.2.1 Canonical PSO Algorithm

The scheme of canonical PSO algorithm to the minimization problem given by the following figure.

Result: Getting the best particle after g -generation as an optimal solution of object function

Initialization Create particles for initial population 'Swarm'

Input: lb : lower bound for the search space, ub : upper bound for the search space, $k = Pop.size$: size of the population.

```

for  $i = 1$  to  $Pop.size$  do
    Generate randomly
     $\vec{x}_i \sim \vec{U}(\vec{x}_{lb}, \vec{x}_{ub})$ ,
     $\vec{v}_i \sim (v_{lb_v}, v_{ub_v})$ 
    set  $\vec{p}_i = \vec{x}_i$ ,  $Pbest_i = \infty$ 
end
while stop criterion not met do
    Update best personal positions

    for  $i=1:Pop.size$  do
        if  $f(\vec{x}_i) < Pbest_i$  then
             $\vec{p}_i = \vec{x}_i$ 
             $Pbest_i = f(\vec{x}_i)$ 
        end
        Update best particle in each neighborhood
        if  $f(\vec{x}_i) < Pbest_g$  then
             $g = i$ 
        end
    end
    Update velocities and positions  $\vec{v}_i \leftarrow$ 
     $\vec{v}_i + \vec{U}(0, \phi_1) \otimes (\vec{p}_i - \vec{x}_i) + \vec{U}(0, \phi_2) \otimes$ 
     $(\vec{P}g - \vec{x}_i)$ 
     $\vec{x}_i \leftarrow \vec{x}_i + \vec{v}_i$ 
end
    
```

Fig. 2 Schem of canonical PSO algorithm.

3.3 Neighborhood Topology

Population typologies work as a method to select the vector $lb\vec{est}_i$ that will guide particle i in its search.

$$\forall Par.i, Par.j \in Swarm : Par.i \in N(Par.i)$$

$$\Leftrightarrow dist(Par.i, Par.j) \leq \delta$$

$N(Par.i)$ is a topological neighbor of particle $Par.i$

The most popular types of population's neighborhood typologies used in this work are:

Fully connected topology, (*Gbest* topology) This

type of topology the particle's neighborhood contains all particles in the population, where in the evolutionary process, only the particle that obtains the optimal position is considered in the entire population and *PSO* algorithms with this topology converse very fast but easy to fall into local optimum [4].

Ring topology -(*lbest* topology) The particle's neighborhood includes the particles on both sides of one or a few particles, the exchange of information is relatively slow within the population, as a result the information eventually will slowly spread to the entire population. The following figure give the method of connections between the population particles in the given neighborhood typologies [4].

Fig. 3 Gbestand and Lbest population's neighborhood typologies.

3.4 Versions of Canonical PSO

There are many of versions of PSO founded, this versions given by updating equation of the velocity, versions used in this study are:

3.4.1 Constriction Coefficient PSO Algorithm CCPSO

Clerc and Kennedy, 2002 introduced a constriction factor into the velocity update rule of the original PSO algorithm, the purpose of this factor is to avoid particles velocities to increase toward infinity, a phenomenon present in the original PSO, the updating equations of velocity and positions

$$\vec{v}_i \leftarrow \chi \left(\vec{v}_i + \vec{U}(0, \phi_1) \otimes (\vec{p}_i - \vec{x}_i) + \vec{U}(0, \phi_2) \otimes (\vec{p}_g - \vec{x}_i) \right) \quad (8)$$

$$\vec{x}_i \leftarrow \vec{x}_i + \vec{v}_i$$

$$\phi = \phi_1 + \phi_2 > 4, \text{ and } \chi = \frac{2}{\phi - 2 + \sqrt{\phi^2 - 4\phi}}$$

3.4.2 Inertia Weight PSO Algorithm IWPSO

In 1998, first time Shi and Eberhart [2] presented the concept of Inertia Weight by introducing Constant Inertia Weight. They stated that a large Inertia Weight facilitates a global search while a small Inertia Weight facilitates a local search.

Eberhart and Shi proposed a Random Inertia Weight strategy and experimentally found that this strategy increases the convergence of PSO in early iterations of the algorithm. The Linearly Decreasing strategy enhances the efficiency and performance of PSO. It is found experimentally that Inertia Weight from 0.9 to 0.4 provides the excellent results, the updating equations of this version are:

$$\begin{aligned}\vec{v}_i &\leftarrow w\vec{v}_i + \vec{U}(0, \varphi_1)(\vec{p}_i - \vec{x}_i) \\ &\quad + \vec{U}(0, \varphi_2) \otimes (\vec{p}_g - \vec{x}_i) \\ \vec{x}_i &\leftarrow \vec{x}_i + \vec{v}_i\end{aligned}\quad (9)$$

$$w = w_{\max} - (w_{\max} - w_{\min}) \frac{g}{G}$$

g is the generation index representing the current number of evolutionary.

G is a predefined maximum number of generations.

3.5 Fully Informed Particle Swarm 'FIPSO'

Fully Informed Particle Swarm Optimization 'FIPSO', is a variant of PSO algorithm found by Kennedy and Mendes 2002 [5], in canonical version of PSO, the effective sources of influence are in fact only two: self and best neighbor, information from the remaining neighbors is unused. Mendes has revised the way particles interact with their neighbors, whereas in the traditional algorithm each particle is effected by its own previous performance and the single best success found in its neighborhood, FIPSO calls for the particles

to take advantage of the connections with others, his neighbors in the swarm. The implementation of FIPSO with good parameters appears to find better solutions in fewer iterations than the canonical algorithm, but it is still more dependent on the population topology.

$$\vec{v}_i \leftarrow \chi \left(\vec{v}_i + \frac{1}{N_i} \sum_{n=1}^{N_i} \vec{U}(0, \varphi) \otimes (\vec{p}_{ngh} - \vec{x}_i) \right) \quad (10)$$

$$\vec{x}_i \leftarrow \vec{x}_i + \vec{v}_i$$

N_i is the number of neighbors for particle i , ngh_n is i 's n th neighbor.

Input: lb : lower bound for the search space, ub : upper bound for the search space, $k = Pop_size$: size of the population.

Initialization Create particles for initial population 'Swarm' for $i = 1$ to $Pop.size$ do

Generate randomly
 $\vec{x}_i \sim \vec{U}(\vec{x}_{lb}, \vec{x}_{ub})$,
 $\vec{v}_i \sim (v_{lb_v}, v_{ub_v})$
 set $\vec{p}_i = \vec{x}_i$, $Pbest_i = \infty$

end

while stop criterion not met do

Update best personal positions

for $i=1:Pop.size$ do

if $f(\vec{x}_i) < Pbest_i$ then

$\vec{p}_i = \vec{x}_i$

$Pbest_i = f(\vec{x}_i)$

end

Update best particle in each neighborhood

if $f(\vec{x}_i) < Pbest_g$ then

$g = i$

end

end

Update velocities and positions

$$\vec{v}_i = \chi \left(\vec{v}_i + \sum_{n=1}^{N_i} \frac{\vec{U}(0, \phi) \otimes (\vec{p}_{ngh(n)} - \vec{x}_i)}{N_i} \right),$$

$$\vec{x} = \vec{x}_i + \vec{v}_i$$

end

Fig. 4 Schem of FIPSO algorithm.

4. Experimental Section

The data generated by ARMA(1, 1), for $e_t \sim N(0, 1)$

$$y_t = a_1 y_{t-1} + e_t - b_1 e_{t-1}$$

The numerical computation of conditional log-likelihood function given by the following steps

For sample size equal to n , and given a_1, b_1 generate time series measurements $y_t, t = 1, \dots, n$ as Eq. (4), where $e_t, t = 1, \dots, n$ generated randomly as $N(0, 1)$.

Set initial value of $e_* = 0$, and $y_* = \bar{y}$, sample mean.

Let (a, b) be a selected candidate solutions in the search space, then for $t = 1, \dots, n$ calculate e 's recursively as Eq. (4)

$$e_t = y_t - \hat{a}_1 y_{t-1} + \hat{b}_1 e_{t-1}$$

Compute the sum squares function for every $(\hat{a}_1, \hat{b}_1)$.

Compute log-likelihood function.

Set $(\hat{a}_1, \hat{b}_1)$ to be the optimal solution of Eq. (6) values of model parameters corresponding to a maximum value of log-likelihood function.

Remark Conditional likelihood l_* involves the data only through the conditional sum of squares function. It follows that the contours of l_* for any fixed value of σ_e in the space of (a_1, b_1, σ_e) are counters of S_* , that maximum likelihood estimators are the same as the least squares estimates.

Experiments setting

Experiments designed by different setting of Pop.size, initial velocity, type of terminate criterion, type of neighborhood topology, sample size, model parameters,

Setting ARMA(1,1) parameters: The experiments simulated by using different values of objective function parameters

Sample size, $n = 25, 50, 100$.

Model Parameters (a_1, b_1) , $(0.1, -0.2)$, $(0.3, 0.2)$, $(.7, .3)$.

Setting PSO algorithm parameters: The basic PSO algorithm has a small number of parameters that need to be fixed:

Fig. 5 Simulating data generated by ARMA(1, 1).

Pop.size : Population size is often set empirically on the basis of the dimensional and perceived difficulty of the problem, best range of population size is 20-50, here $Pop.size = 20$.

Acceleration coefficients: setting $(\phi_1 = \phi_2 = 2.05)$.

In general: The initial population generated randomly for a_1, b_1 in $(-1, 1)$.

Initial velocity generated randomly in the same range of the positions or set it equal to zero may be not the right choose, the results show putting big values of velocities leads to search out of problem search space, for this study the velocities take the initial generated randomly for $i = 1$ to $Pop.size$ in the range $(-0.08, .08)$, and number of iterations set equal to 100.

The test statistic used Mean Squares Errors MSE

$$MSE(\theta) = \text{var}(\hat{\theta}) + \text{bias}(\theta)^2$$

MSE computed for the best estimator found by PSO's algorithms, each experiment replicated for 10 times.

The results given a table. 1 and Fig. 6 for measuring MSE of estimator getting by versions of PSO's algorithm respect to sample size, Fig. 7, Fig. 8 show the value of obj.fun respect to PSO's iterations for one run and mean of 10 runs. We can see from the values of MSE the ability of PSO's algorithms to estimate ARMA's parameters, the values of MSE decreasing with increasing sample size, COPSO algorithm gives

the smallest values of MSE . Smallest MSE 's values found for $(a_1, b_1) = (.7, .3)$ and $n = 100$. $CCPSO$ and $IWPSO$ used to foster the convergence ability by controlling the velocities, $FIPSO$ used to share information more quickly in the swarm in order to achieve the optimal solution in fewer iterations, comparing the results of this study with results given in [8], [9] and [10], show that PSO is easy to implement, and efficient for computing the optimization solution with few iteration where after few iterations the values of obj.fun being stable, results of this study will get better by adding the steps of mutation and crossover to the computing steps this will be in the future study.

We can use this estimators to show the prediction of time series data. After applying versions of PSO algorithm to estimate parameters, a simple experiment designed to predict observations generated by

$ARMA(1,1)$ time series model for sample size $n = 100$, predict value of the observations given in Fig. 9 respect to time t for different versions of PSO algorithm using in this study.

5. Conclusion

In general, PSO is an intelligent algorithm has no overlapping and mutation calculation, easy to implement and compute. The results show ability of PSO 's algorithms to estimate $ARMA$ parameters. For the future work, extending this study to cover estimation of like $ARMA(p, q)$, $GRACH$ models, and others models, using PSO as tool to model selections and prediction, improving results of this study by hybridizing PSO algorithm with gradient algorithms or other soft computing algorithms and applying the results on a real data like financial data, industrial data, ...

Table 1 MSE of parameters estimation for different sample size and PSO versions for model parameters estimation.

n	a_1, b_1	CCPSO		IWPSO		FIPSO	
		(a_1)	(b_1)	(a_1)	(b_1)	(a_1)	(b_1)
25	.1, -.2	.3052	.3804	.4458	.7585	.3425	.4637
50		.1249	.2159	.1820	0.356	1.1578	.4191
100		.1040	.093	.1171	.1205	.1056	.2705
25	.3, .23	.1086	.4894	.1562	.4956	.1295	.4224
50		.0318	.4063	.0724	.4850	.972	.3769
100		.0182	.3059	.0194	.3392	.0315	.3000
25	.7, .3	.0891	.6581	0.060	8.4925	.0585	.4910
50		.0083	.4842	.0117	.4902	.0092	.4759
100		.0028	.4556	.0029	.4678	.0027	.4651

Fig. 6 MSE respect to sample size(n) for $CCPSO$, $IWPSO$, $FIPSO$.

Fig. 7 Average of Obj.fun respect to number of generations for different sample size(n), *PSO*' versions (*COPSO*, *IWPSO* and *FIPSO* respectively), for 10 replicates

Fig. 8 Obj.fun respect to number of generations for different sample size(n), *PSO*' versions (*COPSO*, *IWPSO* and *FIPSO* respectively) , and 1 replicate

Fig. 9 Prediction of generated observation y_t based on PSO 's estimators, where figures given respectively for $CCPSO$, $IFPSO$, $FIPSO$, for $a_1 = .7$; $b_1 = .3$ and $n = 100$

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References

- [1] R.Eberhart & J. Kennedy. (1995). A New optimizer using particle swarm theory, IEEE-Sixth international Symposium on Micro Machine and Human Science, 19-27.
- [2] D.P.Rini & S.M.Shamsuddin & S.S.Yuhaniz. (2011). Particle swarm optimization: technique, system and challenges. International journal of computer applications 14, 331-351.
- [3] Z.Zhan & J.Zhang & Y.Li, H.S.Chung. (2009). Adaptive Particle swarm optimization, IEEE-Transactions on system, man, and Cyberntic-part B 39, 1362-1381
- [4] A.J.R. Medina & G.T. Pulido & J.G.R. Torres. (2009). A comparative study of neighborhood typologies for particle swarm optimizer, IJCCI2009, 152-159.
- [5] J.Kennedy & R. Mendes. (2006), Neighborhood topologies in fully informed and best-of-neighborhood particle swarms. IEEE, Vol.36,Issue.4, 515-519.
- [6] W.W.S.Wei. (1994). Time series analysis, Univariate and multivariate methods, Addison-Wesley publishing company, Inc.
- [7] G.E.B.Box & G.M. Jenkins. (1976). Time series analysis: Forecasting and control, Holden-Day, San Francisco
- [8] B.A.h. Al-sarray & R.D.Al-dabbagh. (2011). Variants of hybrid genetic algorithms for optimizing likelihood ARMA model function and many of problems, InTechOpen, Published on: 2011-04-26.
- [9] B.A.h. Al-sarray & R.D. Al-dabbagh. (2008). Hybrid canonical genetic algorithm and steepest descent algorithm for optimizing likelihood estimators of $ARMA(1,1)$ model. ICADIWT2008, Ostrava, Czech Republic.
- [10] B.A. Attea & B.A.h. Al-sarray. (2006). An Evolution Strategy for Likelihood Estimators of $ARMA(1,1)$ Model. CSIT2006, Amman, Jordan.