Sequential Predication

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Vladimir Markin proposes a certain construction—a generalisation of syllogistic—in which he uses the constant @ with indefinite arity. The atomic formulae are of the following sort: $S_1S_2 \dots S_m @ P_1P_2 \dots P_n$, where $m+n>0$. The standard syllogistic functors are here interpreted as follows: $SaP=$: $S@P$ $SeP=$: $SP@$ $SiP=$: $\sim SP@$ $SoP=$: $\sim S@P$. Markin constructs a system of Fundamental Syllogistic (FS) with constant @ in an axiomatic way. Based on Markin's idea, we propose two constructions, which are formulations of the system of sequential predication built upon the quantifier-less calculus of names. The first one includes the FS system. The second one is enriched with individual variables and, among other things, allows including sequences of individual names in which one has to do with enumerative functors. The counterpart of Hao Wang's algorithm holds in the first system extended with negative terms.

Keywords: sequential predication, generalization of syllogistic, quantifier-less calculus of names, Hao Wang's algorithm

What we mean by sequential predication here is the predication schema of the following kind: $x_1x_2 \dots x_m @ y_1y_2 \dots y_n$ for $m+n \geq 1$ where $x_1, x_2 \dots x_m, y_1, y_2 \dots y_n$ are general or singular terms. The constant @ functions here as an operator with an indefinite number of arguments and is read as "is" ("are"). In the natural language, we can encounter the following realizations of this schema:

(1) for general terms:

(1a) *a man, a dog, a cat, and a horse are mammals.*

(1b) *a dog is a mammal.*

(1c) *a German, a Chinese, and a Nigerian man are (respectively) a European, an Asian, and an African man.*

(2) for singular terms:

(2a) *Socrates, Plato, and Aristotle are philosophers.*

(2b) *Aristotle is the author of the Nicomachean Ethics.*

(2c) *Socrates, Plato, and Aristotle are (respectively) Plato's teacher, Aristotle's teacher, and the author of the Nicomachean Ethics.*

(3) for general or singular terms ($m \neq n$):

(3a) *a philosopher and (at the same time) logician is an analytic philosopher.*

(3b) *a philosopher is an analytic philosopher or a non-analytic philosopher.*

(3c) *Plato's most prominent student and the author of syllogistic is Aristotle.*

(3d) *the author of this distinction is Hume or Kant.*

(4) for general or singular terms ($n=0$):

(4a) *there is a philosopher—meaning—a philosopher (philosophers) exists (exist).*

(4b) *such standpoint is (present/recorded)—meaning—there is (exactly one present/recorded) such standpoint.*

Sentences (1a) and (2a) are examples of occurrence of the so-called “and-enumerative” in the subject group, while in (3b) and (3d) we have to do with the functor of nominal sum. Sentence (1b) is an example of syllogistic general positive sentence and in (2b) the word “is” is used in the sense of identity. Sentence (2c), in turn, represents the property of an ordered n -tuple (here: an ordered triple). Sentence (1c) is an analogous case (for general terms). Sentences (4a) and (4b) are existential sentences by nature, where the reference of respectively general name and singular name occur.

1. Preliminaries

1.1. The Quantifier-Less Calculus of Names

The Quantifier-Less Calculus of Names (QLCN) constructed by means of a suppositional method has the following rules (Borkowski 1985):

R1 $x\epsilon y/x\epsilon x$

R2 $x\epsilon y/\wedge y\epsilon z/x\epsilon z$

R3 $x\epsilon y/\wedge y\epsilon z/y\epsilon x$

There are also rules of omitting and introduction of functors of existence, singularity, weak inclusion, and partial inclusion:

O_{ex} $ex(x)/A\epsilon x$

I_{ex} $x\epsilon y/ex(y)$

O_{sol} $sol(x)/z\epsilon x \rightarrow x\epsilon z$

I_{sol} $z\epsilon x/\wedge u\epsilon x \rightarrow z\epsilon u/sol(x)$

O_c $x\subset y/z\epsilon x \rightarrow z\epsilon y$

I_c $z\epsilon x \rightarrow z\epsilon y/x\subset y$

O_Δ $x_{\Delta}y/A\epsilon x/\wedge A\epsilon y$

I_Δ $z\epsilon x/\wedge z\epsilon y/x_{\Delta}y$

where “A” is a nominal constant which does not recur in lines in the case of applying this rule (more than once) in the proof. The variable “z” does not occur in the assumption of proof.

The elementary expressions with functors are read as follows:¹

$x\epsilon y$ - *x is y*

$ex(x)$ - *x exists*

$sol(x)$ - *one-object-at-most-is x*

$x\subset y$ - *x is included in y*

$x_{\Delta}y$ - *certain x is y*

Moreover, in the system, the rules of omitting and introduction of functors quantifying *all* (π) and *certain* (σ) are adopted. The expressions *every* and *certain*, which are substitutes of quantifiers, are introduced by means of rules:

O π	$\alpha(\pi x)/z\epsilon x \rightarrow \alpha(z)$
I π	$z\epsilon x \rightarrow \alpha(z)/\alpha(\pi x)$
O σ	$\alpha(\sigma x)/A\epsilon x \wedge \alpha(A)$
I σ	$z\epsilon x \wedge \alpha(z)/\alpha(\sigma x)$

Formulae of the type $\alpha(\pi a)$ and $\alpha(\sigma a)$ are meaningful atomic formulae in the framework of this language. However, the expressions πa and σa appear as the first ones on the left side of the formula $\alpha(\pi a)$ (and, respectively, $\alpha(\sigma a)$) and are not preceded by any context in the form πb or σb . The variable z present in the formula $\alpha(z)$ appears only once in it.²

We shall introduce the concepts of object and contradictory object by means of definition:

DV	$x\epsilon V \leftrightarrow x\epsilon x$	x is an object
D Λ	$x\epsilon \Lambda \leftrightarrow x\epsilon x \wedge \sim x\epsilon x$	x is a contradictory object,

and the functors of: being an object, negation, strong inclusion, scope identity, singular identity, product, nominal union, and fulfilment:

Dob	$ob(x) \leftrightarrow x\epsilon x$	x is an object
Dn	$x\epsilon ny \leftrightarrow x\epsilon x \wedge \sim x\epsilon y$	x is not y
D $\sqsubset$	$x\sqsubset y \leftrightarrow ex(x) \wedge x\subset y$	x is included in y (in the strong sense)
D $\circ$	$x\circ y \leftrightarrow x\subset y \wedge y\subset x$	x in terms of scope is identical with y
D $=$	$x=y \leftrightarrow x\epsilon y \wedge y\epsilon x$	x is identical with y
D $\cap$	$x\epsilon y \cap z \leftrightarrow x\epsilon y \wedge x\epsilon z$	x is y and z
D $\cup$	$x\epsilon y \cup z \leftrightarrow x\epsilon y \vee x\epsilon z$	x is y or z

The whole schema is constructed upon the Classical Propositional Calculus (CPC), which is also constructed suppositionally. The rule of detachment (for implication) is extended here to include all meaningful expressions of this system.

1.2. The System of Fundamental Syllogistic

Vladimir Markin proposed the system of Fundamental Syllogistic (FS) (Markin 2004).

The glossary. The glossary of the system includes:

- general nominal variables	- S, P, Q, R, M (with or without indexes);
- sequential variables	- s, p, q, r, m (with or without indexes);
- logical functors	- $\sim, \wedge, \vee, \rightarrow, \leftrightarrow$;
- specific constant	- $@$;
- brackets	- $(,)$.

The sequential variables mean sequences of general nominal variables. The atomic formula of this construction looks as follows: $S_1 S_2 \dots S_m @ P_1 P_2 \dots P_n$ where $m+n \geq 1$.

The interpretation. This construction is accompanied by an intended interpretation (*) in a monadic predicate calculus:

$$(S_1 S_2 \dots S_m @ P_1 P_2 \dots P_n)^* = \sim \Sigma x (S_1 x \wedge S_2 x \wedge \dots \wedge S_m x \wedge \sim P_1 x \wedge \sim P_2 x \wedge \dots \wedge \sim P_n x)^3$$

$$(\sim \alpha)^* = \sim \alpha^*$$

$$(\alpha \sim \beta)^* = \alpha^* \sim \beta^* \text{ where } \sim \text{ is any logical conjunctive.}$$

The standard syllogistic functors are here interpreted as follows:

$$D1 \quad SaP \leftrightarrow S @ P \quad [\Pi x (Sx \rightarrow Px)]$$

D2 $SeP \leftrightarrow SP@$ $[\prod x(Sx \rightarrow \sim Px)]$

D3 $SiP \leftrightarrow \sim SP@$ $[\sum x(Sx \wedge Px)]$

D4 $SoP \leftrightarrow \sim S@P$ $[\sum x(Sx \wedge \sim Px)]$

The system. The FS system has specific axioms-schemas:

A1 $Mq@r \wedge s@pM \rightarrow sq@pr$

A2 $S@S$

A3 $sSPp@q \rightarrow sPSp@q$

A4 $s@pSPq \rightarrow s@pPSq$

A5 $SSs@p \rightarrow Ss@p$

A6 $s@pPP \rightarrow s@pP$

A7 $s@p \rightarrow Ss@p$

A8 $s@p \rightarrow s@pP$

A9 $\sim(S@ \wedge @S)$

The only inferential rule here is the Modus Ponens (MP) rule. The system is constructed upon the Classical Propositional Calculus (CPC).

1.3. Hao Wang's Algorithm

Hao Wang gives the algorithm for automatic theorem-proving in the framework of CPC (Wang 1963). In his formulation beside propositional variables ($p, q, r, \dots$) and logical functors, there are also metalinguistic variables ($\alpha, \beta, \gamma, \dots$) which represent formulae, sequential variables ($A, B, C, \dots$) which refer to sequences of formulae (their individual elements are separated by commas) and the metalinguistic symbol of inference ($\vdash$).

The algorithm is made up of the following rules:

P1 *If A, B are sequences of atomic formulae, the sequent $A \vdash B$ is a these as long as a certain atomic formula appears on both sides of this sequent.*

P2a *If $\alpha, B \vdash C, D$, then $B \vdash C, \sim \alpha, D$*

P2b *If $A, B \vdash C, \alpha$, then $A, \sim \alpha, C \vdash C$*

P3a *If $A \vdash B, \alpha, C$ and $A \vdash B, \beta, C$, then $A \vdash B, \alpha \wedge \beta, C$*

P3b *If $A, \alpha, \beta, B \vdash C$, then $A, \alpha \wedge \beta, B \vdash C$*

P4a *If $A \vdash B, \alpha, \beta, C$, then $A \vdash B, \alpha \vee \beta, C$*

P4b *If $A, \alpha, B \vdash C$ and $A, \beta, B \vdash C$, then $A, \alpha \vee \beta, B \vdash C$*

P5a *If $A, \alpha \vdash B, \beta, C$, then $A \vdash B, \alpha \rightarrow \beta, C$*

P5b *If $A, \beta, B \vdash C$ and $A, B \vdash C, \alpha$, then $A, \alpha \rightarrow \beta, B \vdash C$*

P6a *If $\alpha, A \vdash B, \beta, C$ and $\beta, A \vdash B, \alpha, C$, then $A \vdash B, \alpha \leftrightarrow \beta, C$*

P6b *If $\alpha, \beta, A, B \vdash C$ and $A, B \vdash \alpha, \beta, C$, then $A, \alpha \leftrightarrow \beta, B \vdash C$*

2. The Idea

A more natural interpretation of the FS system than its interpretation in the classical predicate calculus is its interpretation in nominal calculus.

We shall propose such interpretation in the Quantifier-Less Calculus of Names (QLCN).⁴

We will express it by means of the rules of Omitting and Introduction for operator @:

RO $x_1 x_2 \dots x_m @ y_1 y_2 \dots y_n / x_1 \cap \dots \cap x_m \cap V \subset y_1 \cup \dots \cup y_n \cup \Lambda$ for $m+n \geq 1$

$$\text{RI} \quad x_1 \cap \dots \cap x_m \cap V \subset y_1 \cup \dots \cup y_n \cup \Lambda / x_1 x_2 \dots x_m @ y_1 y_2 \dots y_n \quad \text{for } m+n \geq 1$$

The interpretation corresponds to the interpretation of the FS system in the classical predicate calculus proposed by Markin.

The border cases of RO would be: $x@/x \subset \Lambda$ and $@x/V \subset x$.

The first case in the framework of QLCN is transformed into $\neg x@/\sim ex(x)$.

It would be more interesting to substitute a special case of the rule of omitting @ in the form - $@x/ex(x)$ for the other border case of this rule.

As a result, we shall adopt two-part rules of Omitting and Introduction for this operator:

$$\begin{aligned} \text{O@} \quad & x_1 x_2 \dots x_m @ y_1 y_2 \dots y_n / x_1 \cap \dots \cap x_m \subset y_1 \cup \dots \cup y_n \cup \Lambda \quad \text{for } m \geq 1 \\ & @x/ex(x) \\ \text{I@} \quad & x_1 \cap \dots \cap x_m \subset y_1 \cup \dots \cup y_n \cup \Lambda / x_1 x_2 \dots x_m @ y_1 y_2 \dots y_n \quad \text{for } m \geq 1 \\ & ex(x)/@x \end{aligned}$$

3. The Theory of Sequential Predication in the First Formulation

We shall propose the Theory of Sequential Predication (SPT) in the first formulation.

The glossary. The glossary of the language of this system includes:

- general nominal variables - S, P, Q, R, M (with or without indices);
- nominal constants - A, B, C, D, E (with or without indices);
- sequential variables - s, p, q, r, m (with or without indices);
- nominal variables - x, y, z, u, v (with or without indices), representing general names and sequences (of general names);
- logical functors - $\sim, \wedge, \vee, \rightarrow, \leftrightarrow$;
- specific functors: - ε of the s/nn category;
- π, σ of the n/n category;
- specific operator - $@$;
- brackets - $(,)$.

The specific rules here are the (two-part) rules of Omitting and Introduction for the operator @:

$$\begin{aligned} \text{O@} \quad & x_1 x_2 \dots x_m @ y_1 y_2 \dots y_n / x_1 \cap \dots \cap x_m \subset y_1 \cup \dots \cup y_n \cup \Lambda \quad \text{for } m \geq 1 \\ & @x/ex(x) \\ \text{I@} \quad & x_1 \cap \dots \cap x_m \subset y_1 \cup \dots \cup y_n \cup \Lambda / x_1 x_2 \dots x_m @ y_1 y_2 \dots y_n \quad \text{for } m \geq 1 \\ & ex(x)/@x \end{aligned}$$

The definitions (and at the same time interpretations) of standard syllogistic functors look as follows:

$$\text{Da} \quad SaP \leftrightarrow \pi S \varepsilon P$$

$$\text{Di} \quad SiP \leftrightarrow \sigma S \varepsilon P$$

$$\text{De} \quad SeP \leftrightarrow \sim SiP$$

$$\text{Do} \quad SoP \leftrightarrow \sim SaP$$

The construction is built upon the QLCN.

Selected theses of the system. The following theses belong to the system:⁵

$$\text{T2.1} \quad xay \leftrightarrow x \subset y$$

$$[Da, O\pi, I\subset, O\subset, I\pi]$$

$$\text{T2.2} \quad \sim x \varepsilon \Lambda$$

$$[D\Lambda]$$

T2.3 $x \subset \Lambda \leftrightarrow \sim ex(x)$

Proof.

(α)(1)	$x \subset \Lambda$	[a]
(2)	$ex(x)$	[aip]
(3)	$A \varepsilon x$	[2 $\times$ Oex]
(4)	$A \varepsilon x \rightarrow A \varepsilon \Lambda$	[1 $\times$ O $\subset$]
(5)	$A \varepsilon \Lambda$	[3,4 $\times$ MP]
(6)	$\sim A \varepsilon \Lambda$	[T2.2]
	contr.	[5,6]
(β)(1)	$\sim ex(x)$	[a]
(2)	$\exists \varepsilon x \rightarrow ex(x)$	[Iex]
(3)	$\sim \exists \varepsilon x$	[1,2]
(4)	$\sim \exists \varepsilon x \vee \exists \varepsilon \Lambda$	[3]
(5)	$\exists \varepsilon x \rightarrow \exists \varepsilon \Lambda$	[4]
(6)	$x \subset \Lambda$	[5 $\times$ I $\subset$]

T2.4 $ex(x \cap y) \leftrightarrow \sigma x \varepsilon y$

[Oex, D $\cap$, I σ , O σ , Iex]

T2.5 $S @ \leftrightarrow \sim ex(S)$

Proof.

(α)(1)	$S @$	[a]
(1)	$S \subset \Lambda$	[1 $\times$ O@]
(3)	$\sim ex(S)$	[2, T2.3]
(β)(1)	$\sim ex(S)$	[a]
(2)	$S \subset \Lambda$	[1, T2.3]
(3)	$S @$	[2 $\times$ I@]
T2.6	$@S \leftrightarrow ex(S)$	[O@, I@]

The following theorem occurs:

Theorem 1. The FS system is inferentially included in the SPT system.

In its proof, it is enough to show that the specific definitions and axioms of the first system are the theses of the second system:

T3.1 $SaP \leftrightarrow S@P$ (=D1)

Proof.

(α)(1)	SaP	[a]
(1)	$S \subset P$	[1, T2.1]
(2)	$S \subset P \cup \Lambda$	[2, DV, D Λ]
(3)	$S@P$	[3 $\times$ I@]
(β)(1)	$S@P$	[a]
(2)	$S \subset P \cup \Lambda$	[1 $\times$ O@]
(3)	$S \subset P$	[2, DV, D Λ]
(4)	SaP	[3, T2.1]

T3.2 $SeP \leftrightarrow SP@$ (=D2)

Proof.

(α)(1)	SeP	[a]
(2)	$\sim SP@$	[aip]
(3)	$S \cap P \subset \Lambda \rightarrow SP@$	[I@]
(4)	$\sim S \cap P \subset \Lambda$	[2,3]
(5)	$ex(S \cap P)$	[4,T2.3]
(6)	σSeP	[6,T2.4]
(7)	SiP	[7,Di]
(8)	$\sim SiP$	[1,De]
	contr.	[7,8]
(β)(1)	$SP@$	[a]
(2)	$S \cap P \subset \Lambda$	[1 $\times$ O@]
(3)	$\sim ex(S \cap P)$	[2,T2.3]
(4)	$\sim \sigma SeP$	[3,T2.4]
(5)	$\sim SiP$	[4,Di]
(6)	SeP	[5,De]
T3.3	$SiP \leftrightarrow \sim SP@$ (=D3)	[De,T3.2]
T3.4	$SoP \leftrightarrow \sim S@P$ (=D4)	[Do,T3.1]
T4.1	$Mq@r \wedge s@pM \rightarrow sq@pr$ (=A1)	

Proof.

(1)	$Mq@r$	[a]
(2)	$s@pM$	[a]
(3)	$M \cap q \subset r$	[1 $\times$ O@]
(4)	$s \subset p \cup M$	[2 $\times$ O@]
(5a)	$x \in s \cap q$	[ada1]
(5b)	$x \in p \cup r \vee \sim x \in p \cup r$	[CPC]
(5ca)	$\sim x \in p \cup r$	[ada1a]
(5cb)	$\sim x \in p \wedge \sim x \in r$	[5ca,D $\cup$]
(5cc)	$x \in s$	[5a,D $\cap$]
(5cd)	$x \in p \vee x \in M$	[4 $\times$ O $\subset$,5cc]
(5ce)	$x \in M$	[5cb,5cd]
(5cf)	$x \in M \cap q$	[5a,5ce,D $\cap$]
(5cg)	$x \in r$	[3 $\times$ O $\subset$,3cf]
(5ch)	$\sim x \in r$	[5cb]
	contr.	[5cg,5ch]
(5c)	$x \in p \cup r$	[5b,5ca $\rightarrow$ contr.]
(5)	$x \in s \cap q \rightarrow x \in p \cup r$	[5a $\rightarrow$ 5c]
(6)	$x \in s \cap q \rightarrow x \in p \cup r \cup \Lambda$	[5,D Λ ,D $\cup$]
(7)	$s \cap q \subset p \cup r \cup \Lambda$	[6 $\times$ I $\subset$]
(8)	$sq@pr$	[7 $\times$ I@]

T4.2	$S@S$	(=A2)	[I $\subset$, T2.1, T3.1]
T4.3	$sSPp@q \rightarrow sPSp@q$	(=A3)	[O@, D $\cap$, I@]
T4.4	$s@pSPq \rightarrow s@pPSq$	(=A4)	[O@, D $\cap$, I@]
T4.5	$SSs@p \rightarrow Ss@p$	(=A5)	[O@, D $\cap$, I@]
T4.6	$s@pPP \rightarrow s@pP$	(=A6)	[O@, D $\cap$, I@]
T4.7	$s@p \rightarrow Ss@p$	(=A7)	[O@, O $\subset$, D $\cap$, I $\subset$, I@]
T4.8	$s@p \rightarrow s@pP$	(=A8)	[O@, O $\subset$, D $\cup$, I $\subset$, I@]
T4.9	$\sim(S@ \wedge @S)$	(=A9)	

Proof.

(1)	$S@ \wedge @S$	[aip]
(2)	$S@$	[1]
(3)	$@S$	[1]
(4)	$\sim ex(S)$	[2, T2.5]
(5)	$ex(S)$	[3 $\times$ O@]
	contr.	[4,5]

This ends the proof of this theorem.

4. The Theory of Sequential Predication in the Second Formulation

The theory of sequential predication in the second formulation (SPT*) differs from the first formulation in terms of language and specific rules. The language of this formulation is enriched with individual nominal variables— a, b, c, d, e (with or without indices).

The nominal variables (x, y, z, u, v) stand here for both general and individual variables. The specific rules of Omitting and Introduction of Sequences look as follows:

OS	$x_1x_2...x_n@y_1y_2...y_n/x_1 \subset y_1 \wedge x_2 \subset y_2 \wedge ... \wedge x_n \subset y_n$	for $n \geq 1$
	$x_1x_2...x_m@y/x_1 \subset y \wedge x_2 \subset y \wedge ... \wedge x_m \subset y$	for $m \geq 1$
	$x@y_1y_2...y_n/x \subset y_1 \vee x \subset y_2 \vee ... \vee x \subset y_n$	$x@ / x \subset \Lambda$ for $n \geq 1$
	$@y_1y_2...y_n/ex(y_1 \cup y_2 \cup ... \cup y_n)$	for $n \geq 1$
IS	$x_1 \subset y \wedge x_2 \subset y \wedge ... \wedge x_m \subset y/x_1x_2...x_m@y$	for $m \geq 1$
	$x \subset y_1 \vee x \subset y_2 \vee ... \vee x \subset y_n \vee x \subset \Lambda/x@y_1y_2...y_n$	for $n \geq 1$
	$x_1 \cap x_2 \cap ... \cap x_m \subset y_1 \cup y_2 \cup ... \cup y_n \cup \Lambda/x_1x_2...x_m@y_1y_2...y_n$	for $m \geq 1$
	$ex(y_1 \cup y_2 \cup ... \cup y_n)/@y_1y_2...y_n$	for $n \geq 1$

A specific rule is also:

RI $a \subset x / a \in x$

The secondary rules are:

OS2	$x_1x_2...x_m@y/x_1 \cap x_2 \cap ... \cap x_m \subset y$	for $m \geq 1$	[OS, QLCN]
IS2	$x_1 \cap x_2 \cap ... \cap x_m \subset y/x_1x_2...x_m@y$	for $m \geq 1$	[IS]
	$x_1 \subset y_1 \wedge x_2 \subset y_2 \wedge ... \wedge x_n \subset y_n/x_1x_2...x_n@y_1y_2...y_n$	for $n \geq 1$	

Proof.

(1)	$x_1 \subset y_1 \wedge x_2 \subset y_2 \wedge ... \wedge x_n \subset y_n$	[a]
(2)	$x_1 \subset y_1 \cup y_2 \cup ... \cup y_n \wedge x_2 \subset y_1 \cup y_2 \cup ... \cup y_n \wedge ... \wedge x_n \subset y_1 \cup y_2 \cup ... \cup y_n$	[1, O $\subset$, D $\cup$, I $\subset$]
(3)	$x_1 \cap x_2 \cap ... \cap x_m \subset y_1 \cup y_2 \cup ... \cup y_n$	[2, O $\subset$, D $\cap$, I $\subset$]

$$(4) \quad x_1x_2...x_n@y_1y_2...y_n \quad [3 \times IS]$$

$$x_1 \cap x_2 \cap ... \cap x_m \subset y_1 \cup y_2 \cup ... \cup y_n \cup \Lambda / x_1x_2...x_m@y_1y_2...y_n \quad \text{for } m \geq 1 \quad [O \subset, D \cup, D \Delta, I \subset, IS]$$

We shall prove the theorem:

Theorem 2. The SPT system is inferentially included in the SPT* system.

In the proof of this theorem it is enough to show that the rules O@ and I@ are secondary rules of the SPT* system. This is really the case:

$$O@1 \quad x_1x_2...x_m@y_1y_2...y_n / x_1 \cap x_2 \cap ... \cap x_m \subset y_1 \cup y_2 \cup ... \cup y_n \cup \Lambda \quad \text{for } m \geq 1$$

Proof.

$$(1) \quad x_1x_2...x_m@y_1y_2...y_n \quad [a]$$

$$(2) \quad x_1 \cap x_2 \cap ... \cap x_m \subset y_1y_2...y_n \quad [1 \times OS2]$$

$$(3) \quad x_1 \cap x_2 \cap ... \cap x_m \subset y_1 \cup y_2 \cup ... \cup y_n \quad [2 \times OS, D \cup, QLCN]$$

$$(4) \quad x_1 \cap x_2 \cap ... \cap x_m \subset y_1 \cup y_2 \cup ... \cup y_n \cup \Lambda \quad [3 \times O \subset, D \Delta, QLCN, I \subset]$$

$$O@2 \quad @x/ex(x) \quad [OS]$$

$$I@1 \quad x_1 \cap x_2 \cap ... \cap x_m \subset y_1 \cup y_2 \cup ... \cup y_n \cup \Lambda / x_1x_2...x_m@y_1y_2...y_n \quad \text{for } m \geq 1 \quad [IS]$$

$$I@2 \quad ex(x)/@x \quad [IS]$$

The secondary rules O@1 and O@2 and I@1 and I@2 are respectively parts of the rules O@ and I@. Thus, the rules O@ and I@ are also secondary rules. This ends the proof of this theorem. The specific theses of the SPT* system include:

$$T5.1 \quad a \varepsilon a$$

Proof.

$$(1) \quad x \varepsilon a \rightarrow x \varepsilon a \quad [QLCN]$$

$$(2) \quad a \subset a \quad [1 \times I \subset]$$

$$(3) \quad a \varepsilon a \quad [2 \times RI]$$

$$T5.2a \quad a_1a_2...a_n@y_1y_2...y_n \rightarrow a_1 \varepsilon y_1 \wedge a_2 \varepsilon y_2 \wedge ... \wedge a_n \varepsilon y_n \quad [OS, RI, QLCN]$$

$$T5.2b \quad a_1a_2...a_n@b_1b_2...b_n \rightarrow a_1=b_1 \wedge a_2=b_2 \wedge ... \wedge a_n=b_n$$

Proof.

$$(1) \quad a_1a_2...a_n@b_1b_2...b_n \quad [a]$$

$$(2) \quad a_1 \varepsilon b_1 \wedge a_2 \varepsilon b_2 \wedge ... \wedge a_n \varepsilon b_n \quad [1, T5.2a]$$

$$(3) \quad b_1 \varepsilon b_1 \wedge b_2 \varepsilon b_2 \wedge ... \wedge b_n \varepsilon b_n \quad [T5.1]$$

$$(4) \quad b_1 \varepsilon a_1 \wedge b_2 \varepsilon a_2 \wedge ... \wedge b_n \varepsilon a_n \quad [2, 3, R3]$$

$$(5) \quad a_1=b_1 \wedge a_2=b_2 \wedge ... \wedge a_n=b_n \quad [3, 4, D=]$$

$$T5.3 \quad a_1a_2...a_m@y \rightarrow a_1 \varepsilon y \wedge a_2 \varepsilon y \wedge ... \wedge a_m \varepsilon y \quad \text{for } m \geq 1 \quad [OS, RI]$$

$$T5.4a \quad a@y_1y_2...y_n \rightarrow a \varepsilon y_1 \vee a \varepsilon y_2 \vee ... \vee a \varepsilon y_n \quad [OS, RI]$$

$$T5.4b \quad a@b_1b_2...b_n \rightarrow a=b_1 \vee a=b_2 \vee ... \vee a=b_n \quad [OS, T5.1, R3, D=]$$

The these T5.2b is the equivalent of the formula which expresses the property of an ordered n -tuple. The theses T5.3 and T5.4a (and its special case T5.4b), on the other hand, express the properties of functors of n -numerative⁶ conjunction and nominal union.

5. Negative Terms and Hao Wang's Algorithm

We shall extend the nominal variables of the SPT system to encompass negative terms. Similarly, we shall extend sequential variables: They will include negative terms (made from general or individual names) as well.

The system extended in such a way shall be marked as SPTN.

For reasons of simplicity and to enhance the clarity of the proofs the negative equivalents of the terms x , y , z ... shall be marked as $\underline{x}$, $\underline{y}$, $\underline{z}$... respectively.

We will show that here, too, the formal equivalent of Hao Wang's algorithm (HW*) is the case in the framework of the SPTN system.

We shall assign the symbols of the HW* formulation to the symbols present in HW:

α, β, γ	A, B, C	$\vdash$	$\wedge$	$\vee$
x, y, z	s, p, q	@	$\cap$	$\cup$

The equivalent of the implicational formula $\alpha \rightarrow \beta$ ($\sim \alpha \vee \beta$) will be $\underline{x} \cup y$.

By analogy, the equivalent of the equivalence formula $\alpha \leftrightarrow \beta$ (implication in both sides) will be $(\underline{x} \cup y) \cap (\underline{y} \cup x)$.

P1* If sequences s and p have got a common term, then $s @ p$.

Proof. Let $s = s_1 x s_2$ i $p = p_1 x p_2$. The formula $s_1 \cap x \cap s_2 \subset p_1 \cup x \cup s_2$ is a these of the SPT system. From it, in turn, pursuant to I@, we get $s_1 x s_2 @ p_1 x p_2$, and so, eventually, $s @ p$.

P2a* $xs @ pq / s @ p \underline{x} q$

Proof.

(1)	$xs @ pq$	[a]
(2)	$x \cap s \subset p \cup q$	[1×O@]
(3)	$\underline{x} \cap s \subset \underline{x}$	[QLCN]
(4)	$(x \cap s) \cup (\underline{x} \cap s) \subset p \cup q \cup \underline{x}$	[2,3,QLCN]
(5)	$s \subset p \cup \underline{x} \cup q$	[4,QLCN]
(6)	$s @ p \underline{x} q$	[5×I@]

P2b* $sp @ qx / s \underline{x} p @ q$

Proof.

(1)	$sp @ qx$	[a]
(2)	$s \cap p \subset q \cup x$	[1×O@]
(3)	$s \cap \underline{x} \cap p \subset q$	[2,Dn,QLCN]
(4)	$s \underline{x} p @ q$	[3×I@]

P3a* $s @ p x q / s @ p y q / s @ p x \cap y q$

Proof.

(1)	$s @ p x q$	[a]
(2)	$s @ p y q$	[a]
(3)	$s \subset p \cup x \cup q$	[1×O@]
(4)	$s \subset p \cup y \cup q$	[2×O@]
(5)	$s \subset (p \cup x \cup q) \cap (p \cup y \cup q)$	[3,4,QLCN]
(6)	$s \subset (p \cup q \cup x) \cap (p \cup q \cup y)$	[5]
(7a)	$z \varepsilon (p \cup q \cup x) \cap ((p \cup q) \cup y)$	[ada1]
(7b)	$(z \varepsilon p \vee z \varepsilon q \vee z \varepsilon x) \wedge (z \varepsilon p \vee z \varepsilon q \vee z \varepsilon y)$	[7a,Dn,Du]
(7c)	$z \varepsilon p \vee z \varepsilon p \cap q \vee z \varepsilon p \cap y \vee z \varepsilon q \cap p \vee z \varepsilon q \cap y \vee z \varepsilon x \cap p \vee z \varepsilon x \cap q \vee z \varepsilon x \cap y$	[7b,Dn]
(7d)	$z \varepsilon p \vee z \varepsilon x \cap y \vee z \varepsilon q$	[7c]

	(7e) $z \varepsilon p \cup (x \cap y) \cup q$	[7d, D $\cup$]
	(7) $z \varepsilon (p \cup q \cup x) \cap ((p \cup q) \cup y) \rightarrow z \varepsilon p \cup (x \cap y) \cup q$	[7a $\rightarrow$ 7e]
	(8) $(p \cup q \cup x) \cap ((p \cup q) \cup y) \subseteq p \cup (x \cap y) \cup q$	[7 $\times$ I $\subseteq$]
	(9) $s \subseteq p \cup (x \cap y) \cup q$	[6,8,QLCN]
	(10) $s @ p x \cap y q$	[9 $\times$ I]
P3b*	$s x y p @ q / s x \cap y p @ q$	
	<i>Proof.</i>	
	(1) $s x y p @ q$	[a]
	(2) $s \cap x \cap y \cap p \subseteq q$	[1 $\times$ O@]
	(3) $s \cap (x \cap y) \cap p \subseteq q$	[2]
	(4) $s x \cap y p @ q$	[3 $\times$ I@]
P4a*	$s @ p x y q / s @ p x \cup y q$	
	<i>Proof.</i>	
	(1) $s @ p x y q$	[a]
	(2) $s \subseteq p \cup x \cup y \cup q$	[1 $\times$ O@]
	(3) $s \subseteq p \cup (x \cup y) \cup q$	[2]
	(4) $s @ p x \cup y q$	[3 $\times$ I@]
P4b*	$s x p @ q \wedge s y p @ q / s x \cup y p @ q$	
	<i>Proof.</i>	
	(1) $s x p @ q$	[a]
	(2) $s y p @ q$	[a]
	(3) $s \cap x \cap p \subseteq q$	[1 $\times$ O@]
	(4) $s \cap y \cap p \subseteq q$	[2 $\times$ O@]
	(5) $s \cap (x \cup y) \cap p \subseteq q$	[3,4,QLCN]
	(6) $s x \cup y p @ q$	[5 $\times$ I@]
P5a*	$s x @ p y q / s @ p \underline{x} \cup y q$	
	<i>Proof.</i>	
	(1) $s x @ p y q$	[a]
	(2) $s \cap x \subseteq p \cup y \cup q$	[1 $\times$ O@]
	(3) $s \cap \underline{x} \subseteq \underline{x}$	[QLCN]
	(4) $s \subseteq p \cup (\underline{x} \cup y) \cup q$	[2,3,QLCN]
	(5) $s @ p \underline{x} \cup y q$	[4 $\times$ I@]
P5b*	$s y p @ q \wedge s p @ q x / s \underline{x} \cup y p @ q$	
	<i>Proof.</i>	
	(1) $s y p @ q$	[a]
	(2) $s p @ q x$	[a]
	(3) $s \cap y \cap p \subseteq q$	[1 $\times$ O@]
	(4) $s \cap p \subseteq q \cup x$	[2 $\times$ O@]
	(5) $s \cap \underline{x} \cap p \subseteq q \cap \underline{x}$	[4,QLCN]
	(6) $s \cap \underline{x} \cap p \subseteq q$	[5,QLCN]

(7)	$s \cap (\underline{x} \cup y) \cap p \subset q$	[3,6,QLCN]
(8)	$s \underline{x} \cup y p @ q$	[7×I@]
P6a*	$xs @ pyq \wedge ys @ pxq / s @ p(\underline{x} \cup y) \cap (\underline{y} \cup x) q$	
	<i>Proof.</i>	
(1)	$xs @ pyq$	[a]
(2)	$ys @ pxq$	[a]
(3)	$x \cap s \subset p \cup y \cup q$	[1×O@]
(4)	$y \cap s \subset p \cup x \cup q$	[2×O@]
(5)	$\underline{x} \cap s \subset \underline{x}$	[QLCN]
(6)	$\underline{y} \cap s \subset \underline{y}$	[QLCN]
(7)	$x \cap s \subset p \cup y \cup q \cup \underline{x}$	[3,D∪]
(8)	$\underline{x} \cap s \subset p \cup y \cup q \cup \underline{x}$	[5,D∪]
(9)	$s \subset p \cup y \cup q \cup \underline{x}$	[7,8]
(10)	$y \cap s \subset p \cup x \cup q \cup \underline{y}$	[4,D∪]
(11)	$\underline{y} \cap s \subset p \cup x \cup q \cup \underline{y}$	[6,D∪]
(12)	$s \subset p \cup x \cup q \cup \underline{y}$	[10,11]
(13)	$s \subset (p \cup y \cup q \cup \underline{x}) \cap (p \cup x \cup q \cup \underline{y})$	[9,12]
(14a)	$z \in s$	[ada1]
(14b)	$z \in p \cup y \cup q \cup \underline{x} \wedge z \in p \cup x \cup q \cup \underline{y}$	[13,O _c ,14a]
(14c)	$(z \in p \cup q \vee z \in \underline{x} \cup y) \wedge (z \in p \cup q \vee z \in x \cup \underline{y})$	[14b]
(14d)	$z \in p \cup q \vee z \in (p \cup q) \cap (x \cup \underline{y}) \vee z \in (p \cup q) \cap (\underline{x} \cup y) \vee z \in (\underline{x} \cup y) \cap (x \cup \underline{y})$	[14c]
(14e)	$z \in p \cup q \vee z \in (\underline{x} \cup y) \cap (x \cup \underline{y})$	[14d]
(14f)	$z \in p \cup ((\underline{x} \cup y) \cap (\underline{y} \cup x)) \cup q$	[14e]
(14)	$z \in s \rightarrow z \in p \cup ((\underline{x} \cup y) \cap (\underline{y} \cup x)) \cup q$	[14a→14f]
(15)	$s \subset p \cup ((\underline{x} \cup y) \cap (\underline{y} \cup x)) \cup q$	[14×I _c]
(16)	$s @ p(\underline{x} \cup y) \cap (\underline{y} \cup x) q$	[15×I@]
P6b*	$xy sp @ q \wedge sp @ xyq / s(\underline{x} \cup y) \cap (\underline{y} \cup x) p @ q$	
	<i>Proof.</i>	
(1)	$xy sp @ q$	[a]
(2)	$sp @ xyq$	[a]
(3)	$x \cap y \cap s \cap p \subset q$	[1×O@]
(4)	$s \cap p \subset x \cup y \cup q$	[2×O@]
(5)	$s \cap (x \cap y) \cap p \subset q$	[3]
(6)	$s \cap (\underline{x} \cap \underline{y}) \cap p \subset q$	[4]
(7)	$s \cap ((x \cap y) \cup (\underline{x} \cap \underline{y})) \cap p \subset q$	[5,6]
(8)	$s \cap (\underline{x} \cup y) \cap (\underline{y} \cup x) \cap p \subset q$	[7,QLCN]
(9)	$s(\underline{x} \cup y) \cap (\underline{y} \cup x) p @ q$	[8×I@]

And so the SPTN system follows the rules of the HW* algorithm, which is a formal equivalent of Hao Wang's algorithm.

Notes

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1. By means of the connector “-,” we indicate the fact that a given phrase is regarded here as logically unanalysable.
 2. See (Borkowski 1985, 26). We omit here counterparts of these rules for name-generating functors of the n/n category. See also (Wojciechowski 2000).
 3. In the expression on the left side of this identity there are—marked in the same way—monadic predicates (of the s/n category), which are the equivalents of general names occurring on its left side. This convention shall also be used while interpreting standard syllogistic functors.
 4. This paper was presented at the XX conference on *Application of Logic to Philosophy and Foundations of Mathematics*, Szklarska Poręba, Poland, 4-8 May 2015, organized by Institute of Mathematics, Silesian University, Institute of Mathematics, Opole University, and Department of Logic and Methodology of Science, Wrocław University.
 5. The proofs are conducted by means of the suppositional method. The expressions “a,” “ada,” “aip,” and “contr.,” which appear in the proof lines are respectively the abbreviations of the expressions: “assumption,” “additional assumption,” “assumption of indirect proof,” and “contradiction.” “CPC” demonstrates reference to the classical propositional calculus.
 6. As regards *and*-enumerative and *or*-enumerative functors see (Wojciechowski 2011).

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