Dividing Process in Base-10 Number System — Reversed Dividing Process for 1/X

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Received: March 26, 2015 / Accepted: April 25, 2015 / Published: July 25, 2015.

Abstracts: A new set of steps for dividing process, the Reversed Dividing Process, is introduced, in which the position based number representation digits for the value of $1/X$ are reversely output one by one compared with digit output order of the normal dividing process.

Key Words: Dividing Process, Natural Number, Integer, Primes, Composite Numbers, and Repeating Fractions.

The normal steps of the dividing process for $1/X$, e.g. $X=7$, is show as below.

Dividing Process for 1/7 in base-10 Number System									
Quotient digits	0.	1	4	2	8	5	7	1	and
the Dividend $d(0)=1 \times 10$	1	0							
- $q(1) \times 7$		7							
$d(1)=3$ multiplied by 10		3	0						
- $q(2) \times 7$		2	8						
$d(2)=2$ multiplied by 10			2	0					
- $q(3) \times 7$			1	4					
$d(3)=6$ multiplied by 10				6	0				
- $q(4) \times 7$				5	8				
$d(4)=4$ multiplied by 10					4	0			
- $q(5) \times 7$					3	5			
$d(5)=5$ multiplied by 10						5	0		
- $q(6) \times 7$						4	9		
$d(6)=1=d(0)$ and the a loop starts							1	0	

The above steps can be described as below in terms of equation.

$$d(i)=d(i-1)*10-q(i-1)*X \quad (01)$$

while $d(i)$ and $q(i)$, $i=1$ to X , are values for 'd' and 'q' in i -th step of the dividing process respectively, and $d(0)=1$, $q(0)=0$. Actually, the $d(0)$ could be any natural number less than X for different quotient value $d(0)/X$.

There are two value types of the quotients, i.e. repeating fractions and non repeating fractions or normal decimals. Normal decimals can be obtained if X is a number with divisors of only 2 and /or 5, which compose the base 10. And the quotient values are numbers of the below forms.

$$1/[(2^{**}m)*(5^{**}n)]=5^{**}(m-n)/10^{**}m \text{ (if } m>n)$$

and

$$2^{**}(n-m)/10^{**}n \text{ (if } n>m) \quad (02)$$

Of which the quotient values can be deduced by the above dividing process only with 0 or null as the repeating part.

And a repeating fraction will be appeared when X contains primes other than 2 and/or 5. That means when X contains any prime other than 2 and/or 5, the value for $1/X$, which can be reached by the above steps of the dividing process, is a repeating fraction, in which the repeating part is repeatedly followed in the number representation of the quotient value in a base positioned number representation system, e.g. base-10 number representation system.

From the above dividing process, the below equation is noticed for the whole dividing process in base-10 positioned number system.

$$10^{**}y=Q*X+D \quad (03)$$

while

$$D=d(y)<X$$

and

$$Q=q(1)*10^{**}(y-1)+q(2)*10^{**}(y-2)+\dots+q(y-1)*10+q(y).$$

For the number $1/7$, $X=7$, $D=1$, and the whole dividing process can be split into the below steps to get the value for $1/X$.

$$10^{**}0=0*7+1, d(0)=D=1 \text{ and } q(0)=\text{null} \quad (i)$$

$$10^{**}1=1*7+3, d(1)=3 \text{ and } q(1)=1 \quad (\text{ii})$$

$$10^{**}2=14*7+2, d(2)=2 \text{ and } q(2)=4 \quad (\text{iii})$$

$$10^{**}3=142*7+6, d(3)=6 \text{ and } q(3)=2 \quad (\text{iv})$$

$$10^{**}4=1428*7+4, d(4)=4 \text{ and } q(4)=8 \quad (\text{v})$$

$$10^{**}5=14285*7+5, d(5)=5 \text{ and } q(5)=5 \quad (\text{vi})$$

and

$10^{**}6=142857*7+1, d(6)=1 \text{ and } q(6)=7$, and here is the start of the next loop of the repeating part.

The interesting thing happens if integer solutions are considered for the equation (03). To ensure integer Q and D for integer y of the equation (03), the unit digit of $QX+D$ must be 0 in order to be dividable for 10, i.e. the unit digit of the sum of the product of the unit digits of the Q and X and the unit digit D must be 0. And the equation (03) can be rewritten as below.

$$10^{**}(y-1)=Q(1)*X+[q(0)*X+D]/10 \quad (4-i)$$

$$\text{Let } D(1)=[q(0)*X+D]/10,$$

and

$$10^{**}(y-2)=Q(2)*X+[q(1)*X+D(1)]/10 \quad (4-ii)$$

$$\text{Let } D(2)=[q(1)*X+D(1)]/10, \text{ and so on}$$

.....

$$\text{Let } D(n-1)=[q(n-2)*X+D(n-2)]/10, \text{ and}$$

$$10^{**}(y-n)=Q(n)*X+[q(n-1)*X+D(n-1)]/10 \quad (04)$$

and $D(n)=[q(n-1)*X+D(n-1)]/10$, while $n \leq y$, and until $D(n)=D$ to close a loop of the repeating part.

For the number $1/7$, as per equation (04), the following steps can be observed.

$$D=D(0)=1, X=7, \text{ and}$$

$$10^{**}Y(1)=Q(1)*7+(7*7+1)/10, \text{ i.e. } q(0)=7, D(1)=5 \quad (\text{i})$$

$$10^{**}Y(2)=Q(2)*7+(5*7+5)/10, \text{ i.e. } q(1)=5, D(2)=4 \quad (\text{ii})$$

$$10^{**}Y(3)=Q(3)*7+(8*7+4)/10, \text{ i.e. } q(2)=8, D(3)=6 \quad (\text{iii})$$

$$10^{**}Y(4)=Q(4)*7+(2*7+6)/10, \text{ i.e. } q(3)=2, D(4)=2 \quad (\text{iv})$$

$$10^{**}Y(5)=Q(5)*7+(4*7+2)/10, \text{ i.e. } q(4)=4, D(5)=3 \quad (\text{v})$$

and

$$10^{**}Y(6)=Q(6)*7+(1*7+3)/10, \text{ i.e. } q(5)=1, D(6)=1 \quad (\text{vi})$$

Ending a loop of the repeating part with $D(6)=D(0)=1$.

From the above steps, it is observed that the order of $q(n)$ in equation (04) is reversed with that of the $q(i)$ in equation (01) to form the Q in equation (03), which elaborating the Reversed Dividing Process.

To end the paper with any integer X, X can be a composite with the primes 2 and/or 5 and others. Then X can be rewritten as below.

$$X=X(p)*2^{**}m*5^{**}n \quad (05)$$

while $X(p)$ is a prime or a composite of primes other than 2 and/or 5, in which both of the above dividing processes can be applied for obtaining the value of $1/X(p)$.

Applying equation (02), then

$$10^{**}n/X=2^{**}(n-m)/X(p), \text{ while } m < n \quad (6-i)$$

$$10^{**}m/X=5^{**}(m-n)/X(p), \text{ while } m > n \quad (6-ii)$$

or

$$10^{**}y/X=C+D/X(p) \quad (06)$$

while C is the integer part of the equations (6-i) or (6-ii) if

$$2^{**}(n-m) > X(p) \quad (6-iii)$$

or

$$5^{**}(m-n) > X(p) \quad (6-iv)$$

or null if others, for which the Reversed Dividing Process can directly be applied by letting D equals $2^{**}(n-m)$ or $5^{**}(m-n)$.

Or let C satisfying the below equation to ensure $D < X(p)$,

$$X=C*X(p)+D \quad (6-v)$$

and then practice the Reversed Dividing Process for $D/X(p)$ in order to reach the value for $10^{**}y/X$.

End/Mar 24, 2015