Forced Convection of the Bi and Three-Dimensional Flow in a Periodic Channel

Rahima Benchabi¹ and Ahsène Lanani²

1. Mechanical Engineering Department, Faculty of Technology and Engineering, University of Constantine1, Constantine 25000, Algérie

2. Mathematics Department, Faculty of Sciences, University of Constantine1, Constantine 25000, Algérie

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Abstract: The present investigation concerns the numerical study of turbulent flow and heat transfer of the forced convection in a bi and three-dimensional corrugated channel by using periodicity condition. In this study, a model based on finite volume method is used. The treatment of coupling pressure-velocity is assured by the SIMPLE algorithm. Turbulence is simulated via two equation (k-ε) model. The numerical results are obtained for air with a Prandtl number $Pr = 0.7$ and for Reynolds number based on hydraulic diameter of the channel varying between 1,200 and 4,000. The effects of the Reynolds number, the aspect ratio H/S , the corrugation angle and the heat transfer characteristics are discussed.

Key words: Corrugated channel, forced convection, finite volumes, periodicity, turbulence.

Nomenclature

D_h :	Hydraulic diameter (m)
F :	Friction coefficient (-)
H :	Height of the channel (m)
Re :	Reynolds number (-)
u and v :	The velocity components in directions x and y (m/s)
P :	Fluid pressure (N/m ²)
S :	step of the corrugation(m)
T :	Fluid temperature (°C)
V :	Average velocity (m/s)

Greek Letters

ρ :	Density of air (kg/m ³)
θ_1 and θ_2 :	Corrugation angles (degrees)
μ :	Dynamic viscosity (kg/s m)

1. Introduction

The need to develop thermal systems with high performance stimulated research interest in heat transfer by using coefficient (Nusselt number)

augmentation techniques. Among the many possibilities of achieving this goal is to vary the shape of the channel so as to obtain a “corrugated channel”. Such configurations are extensively used in engineering because of their high (performance/volume) ratio and are called “compact exchangers”. The improvement of heat transfer coefficient by such exchangers, which are made of parallel bent plates, is due to the fact that corrugations allow a reduction of boundary layer which is caused by the increase of turbulence in such a layer.

Many researchers have been motivated in recent years to study corrugated surfaces because of the many fields of application: automobile, chemical engineering, air conditioning, etc.. Various numerical and experimental studies have therefore been conducted and the published papers reported the characteristics of the flow of fluid and transfer of heat in corrugated channels. At this stage, it is worth mentioning an important result which emerged from the experimental studies: It was found that after 3-5 cycles, i.e., after a

Corresponding author: Rahima Benchabi, Ph.D., assistant professor, research fields: fluid flow and heat transfer in various exchangers. E-mail: rbenchabi@yahoo.fr.

short distance from the entrance, the dynamic and thermal fields become periodically developed.

Amano [1] studied numerically the flow and heat transfer in a horizontal corrugated channel for both laminar and turbulent regimes. So as to predict the heat transfer and friction coefficients, Faghri and Asako [2] used the finite element method for a range of Reynolds numbers varying between 100 and 1,500. Asako and Nakamura [3], considered a planar bidimensional corrugated channel with rounded off corners and conducted a numerical study based on the finite volume method to predict the fluid flow and heat transfer characteristics for the established laminar regime for an interval of Reynolds numbers varying between 100 and 1000. Xin and Tao [4] simulated, using the finite difference method, a bidimensional sinusoidal channel by assuming the existence of an established laminar flow up to a Reynolds number value of 1,000. Sunder and Trollander [5] considered the laminar bidimensional flow and heat transfer in a corrugated channel by using finite difference approximations. Wang and Vanka [6] determined the characteristics of heat transfer of an unstable flow in undulating periodical passages. Asako et al. [7] considered laminar flow in a corrugated channel having a trapezoidal section. The effects of the Reynolds number and the parameters linked to wavy geometry on heat transfer (Nusselt number) and friction coefficients were studied by Wang and Chen [8]. A lot of experimental studies on the characteristics of mass and heat transfer in corrugated channels were also conducted in recent years. For example, the study of a turbulent flow led by O'Brien and Sparrow [9], in which the authors presented an empirical correlation for the average Nusselt number and pointed out that the friction coefficient is independent of Reynolds number. Sparrow and Comb [10] supplemented the previous observations by analyzing the effect of variation of amplitude and conditions of entrance of fluid on the characteristics of flow and heat transfer in a corrugated channel for Reynolds numbers ranging from 2,000 to

27,000.

Measurements of heat transfer and pressure drop in a corrugated channel were performed by Islamoglu et al. [11] with an angle $\alpha = 30^\circ$ for a turbulent flow of air. The authors proposed an empirical correlation for the Nusselt number.

The effect of two different values of the channel height (5mm and 10mm) with an angle $\alpha = 20^\circ$ on the characteristics of heat transfer and friction coefficient of a corrugated channel, were examined by Islamoglu and Parmaksizoglu [12]. A numerical study was also conducted by Islamoglu and Parmaksizoglu [13], using the finite element method to simulate the heat transfer and friction coefficients in a corrugated periodical bidimensional channel.

2. Mathematical Model

This study concerns the analysis of a flow in a corrugated channel. The geometric configuration is shown in Fig. 1.

The considered fluid (air) is incompressible ($\rho = \text{cste}$), Newtonian and has constant properties (μ and C_p constants). The flow is assumed bi and three-dimensional and steady. The standard k- ϵ model of turbulence is used to simulate momentum and heat transports.

The equations governing the flow and heat transfer can be written in the following tensorial form:

Equation of continuity (conservation of mass):

$$\frac{\partial}{\partial x_i}(\rho u_i) = 0 \quad (1)$$

Fig. 1 The studied configuration.

Equation of momentum conservation (Navier Stokes equations):

$$\frac{\partial}{\partial x_i}(\rho u_i u_j) = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_i} \left[(\mu_t + \mu) \frac{\partial u_j}{\partial x_i} \right] \quad (2)$$

Equation of energy:

$$\frac{\partial}{\partial x_i}(\rho u_i T) = \frac{\partial}{\partial x_i} \left[\left(\frac{\mu_t}{\sigma_t} + \frac{\mu}{\sigma} \right) \frac{\partial T}{\partial x_i} \right] \quad (3)$$

The turbulent kinetic energy equation:

$$\rho u_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i} \left[\left(\mu + \frac{\mu_t}{\sigma_t} \right) \frac{\partial k}{\partial x_i} \right] + G - \rho \varepsilon \quad (4)$$

where $\rho \varepsilon$ is the rate of destruction of k , and G is the rate of generation of k , and is given by:

$$G = \mu_t \left[\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} \right]$$

The rate of dissipation of turbulent kinetic energy equation:

$$\rho u_i \frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_i} \left[\left(\mu + \frac{\mu_t}{\sigma_t} \right) \frac{\partial \varepsilon}{\partial x_i} \right] + \frac{\varepsilon}{k} (C_{\varepsilon_1} G - C_{\varepsilon_2} \rho \varepsilon) \quad (5)$$

where ε is defined by:

$$\varepsilon = \gamma \frac{\overline{\partial u_i' \partial u_i'}}{\partial x_j \partial x_j}$$

The constants which appear in the above equations are given in Table 1.

The general transport equation for fluid flow and heat transfer in the domain of study can be expressed in Cartesian co-ordinates, as follows (Eq.(6)):

$$\underbrace{\frac{\partial}{\partial x} (\rho U \Phi)}_{\text{I}} + \underbrace{\frac{\partial}{\partial y} (\rho V \Phi)}_{\text{II}} = \underbrace{\frac{\partial}{\partial x} \left[\Gamma \left(\frac{\partial \Phi}{\partial x} \right) + \Gamma \left(\frac{\partial \Phi}{\partial y} \right) \right]}_{\text{III}} + S \Phi \quad (6)$$

(I: Term expressing the transport by convection; II: Term expressing the transport by diffusion; III: Term expressing the sources), Φ is the generalized variable which can be any one of the transportable physical quantities (U , V , T , P , k , ε) and Γ is its diffusion coefficient.

Boundary Conditions: The elliptical nature of the

Table 1 The empirical constants of the turbulence model.

Constants	Values
$C_{\varepsilon 1}$	1.44
$C_{\varepsilon 2}$	1.92
σ_{ε}	0.9

equations governing flow and heat transfer requires the knowledge of conditions in all domain boundaries. Values of Φ (Dirichlet condition) or its gradient (Neumann condition) or both (mixed condition) must be specified at each point of the borders of the solution domain.

Condition at inlet and at exit: The experience showed that the flow of fluid becomes periodically developed after a short distance of the entrance (3-5 corrugations). The fluid flow is periodic in x - direction, the Φ values and the quantity $\partial \Phi / \partial x$ are the same at the input and at the output. Given the imposed and applied periodicity condition to the corrugated channel, a mass flow rate was imposed in the present numerical simulation.

Conditions at walls: In the present study, the conditions of impermeability to matter and of no fluid slip are applied to walls, which implicates $U = 0$ and $V = 0$.

Also a constant heat flux q is imposed at the walls of the channel. Finally, wall functions have been applied when solving the turbulence model equations. This special treatment is applied in the cells which are adjacent to the walls and allows an avoiding of numerical difficulties associated with the k - ε model.

In this study, the numerical analysis is considered with the intention of determining the Nusselt numbers and friction coefficient for the flow accompanied by heat transfer in a corrugated channel. The Reynolds number Re is based on the hydraulic diameter and is given by:

$$Re = \frac{\rho U D_h}{\mu} \quad (7)$$

The friction coefficient f is computed by using the formula:

$$f = \frac{-\frac{dp}{dx} D_h}{\frac{1}{2} \rho V^2} \quad (8)$$

where, $\partial \Phi / \partial x$ is the pressure gradient, ρ is the density and V is the average fluid velocity.

3. Numerical Procedure

The above equations are discretized with the aid of

the finite volumes numerical method. The velocity pressure coupling has been achieved using the SIMPLE algorithm (Patankar [14]). The resulting algebraic equation of transport can be written in the following general form for a chosen control volume, centered at P:

$$A_P \Phi_P = A_E \Phi_E + A_W \Phi_W + A_N \Phi_N + A_S \Phi_S + S_\Phi \quad (9)$$

The discretisation for all nodes results in a set of equations for each flow parameter. For the resolution of this system of equations, the line by line semi-iterative method has been used in conjunction with the T.D.M.A algorithm (Tri- Diagonal Matrix Algorithm).

The solution of the system is semi-iterative and convergence is attained when the maximum of the sum of absolute values of residuals for all control volumes normalized by division by the inlet value (mass or momentum), is less than 10^{-5} .

In order to choose the computational grid, in this study, different mesh sizes were considered. The test of independence of mesh size showed that the mesh size consisting of 99,270 cells gives satisfactory numerical solutions for the bi-dimensional and 227,392 for the three-dimensional case.

4. Results and Discussion

The numerical results of the Nusselt number as a function of the Reynolds number for both corrugated and smooth channels are represented in Fig. 2. It is seen that for the corrugated channel the Nusselt number is greatly influenced by the Reynolds number, one can notice that the more the Reynolds number increases and the more the Nusselt number increases and that heat exchange is enhanced by the corrugations, since the Nusselt number is higher than that of the smooth channel. It can also be noticed that our numerical results are in good concordance with the experimental data obtained by Islamoglu et al. [15].

The numerical results concerning the coefficients of heat transfer as a function of the Reynolds number for various corrugation angles are presented in Fig. 3. One

notices an increase in the heat transfer coefficient with the increase in the corrugation angle and especially for high values of the Reynolds number. These results are in agreement with the experimental data of Islamoglu et al. [15].

The graphical representation of the friction coefficient as a function of the Reynolds number is given by Fig. 4. It is seen that the friction coefficient decreases with the increase in the Reynolds number. It is also noticed that there is a good agreement between these results and the measurements by Islamoglu et al. [15].

Fig. 2 Variation of the average Nusselt number as a function of Re.

Fig. 3 Effect of corrugation angle on heat transfer coefficient.

Fig. 4 Friction factor for corrugation angles 20° and 60° as a function of the Reynolds number.

Fig. 5 shows the influence of the corrugation angle on friction coefficient. The latter depends on the Reynolds number and increases as the corrugation angle increases.

The velocity contours for different aspect ratios are shown in Fig. 6 for a Reynolds number $Re = 4,000$. A decrease in H/S or in the height causes an increase in velocity. The contours of velocity in different planes $x = \text{constant}$ and $z = \text{constant}$ are shown in Fig. 7. At the entrance, the maximum velocity value is located in the top wall, the same phenomenon is observed at the output (periodicity conditions satisfied) and is shifted towards the bottom walls in the other planes.

The pressure contours in different planes $x = \text{constant}$ and $z = \text{constant}$ are shown in Fig. 8. At the

Fig. 5 Influence of the corrugation angle on friction coefficient as a function of Re .

Fig. 6 Variation of velocity contour for different aspect ratio H/S (0.25, 0.33, 0.5).

entrance, the pressure is relatively low along the top wall and high along the bottom wall, this result is consistent with the velocity field, the same phenomenon is observed at the output.

Fig. 7 Velocity Contours in different planes $x = cte$ with $Re = 4,000$.

Fig. 8 Pressure Contours in different planes $x = cte$ with $Re = 4,000$.

5. Conclusions

In this study, the flow and heat transfer in a periodic channel have been studied numerically. The flow is considered for bi and three-dimensional air in turbulent regime. The effect of the Reynolds number and the geometric parameters, namely the corrugation angle and the aspect ratio on the friction coefficient and the Nusselt number were studied. The results show that the Nusselt numbers are very high as compared to those of the smooth channel. This is due to the fact that the corrugated channel causes good mixing and thus leads to a great improvement of heat transfer. They also show that an increase in the Reynolds number leads to a decrease in the friction coefficient. It can also be noticed that these results are in good concordance with the experimental data.

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