Two Concepts of Guide Vane Profile Design for a Low Head Hydraulic Turbine

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Abstract: Two concepts of the guide vanes channels design for a low head hydraulic turbine were investigated using 2D and 3D models. Model 2D was used to generate the geometry of profiles which form a blade channel. After that by means of 3D commercial code (ANSYS/Fluent v.15), the designed cascades were examined. The characteristic parameters of compared guide vanes have been presented. The problem of low head hydraulic turbine design is important from the technical point of view for usually not typical environmental circumstances, in which the hydropower plants are planned.

Key words: Hydraulic turbine, inverse problem, turbomachinery, guide vane design.

1. Introduction

Following the authors' previous paper [1] devoted to the analytical and numerical design of guide vanes cascade, here, the attention will be paid to the details of guide vanes cascade designed for a low head hydraulic turbine. The problem is important from the practical point of view for a design of simple low head hydraulic turbine, which is not the matter of standardization due to the fact of the variety of environmental circumstances in which such installations are planned. From the theoretical point of view, the problem is also interesting, on the contrary to the common practice where aerodynamically well elaborated profiles of classical type are applied. By classical type here is meant the profiles known as NACA or GOETTINGEN series of profiles. Usually, they serve as a good solution for a single profile. In the case of turbine cascade, we need bended profiles to create angular momentum in front of a rotor cascade [2]. The profiles are located in the form of

cascades. In such situation, the channels should be of high efficiency rather than the efficiency of single profile. We will distinguish here two typical classes of profiles. The first class is typical "classical" (C) profiles with thick leading edge. The second class are the profiles with rather thin leading edge (M) directed to the flow upstream perpendicular to the front of cascade, which is in the most common situation. The procedure of design, proposed here, is based on 2D and 3D models. Referring to the publication [1], application of 2D model for the inverse problem will be briefly described. After that, a chosen version of 3D commercial program (ANSYS/Fluent 15™) for the direct problem (for the known geometry) will be briefly described as well.

2. Version of 2D Model for the Inverse Problem

2D model is very suitable for the inverse problem, in which we seek for the geometry of blades. The basic assumption of the model in the presented here version is an infinite number of blades. Therefore, for axisymmetric cascade, one can eliminate from the consideration all derivatives with respect to

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circumferential coordinate, leaving only the change of flow parameters in axial and radial directions [3, 4]. This assumption allows formulating the inverse algorithm being at the same time a weak point of 2D model. That is why we need always to check the results by means of a 3D model as a more realistic model. Additionally, if there is an intuitive feeling about the discrepancy of the outlet flow parameters due to finite and infinite number of blades the conditions for 2D model computation ought to be formulated with anticipated corrections computed by means of 3D results. Such comparison was presented in Ref. [5].

In Fig. 1, the example of guide vane position in a meridional cross section of the designed hydraulic turbine is shown.

The lines which cover the guide vane, gap and rotor domains result from intersection of assumed stream surfaces with a meridional plane section. The arrangement of the flow in form of assumed stream surfaces is one of basic assumptions for 2D model. Let us mark two surfaces with number 0 for the outer border and number 1 for the inner border. In such way, the non-orthogonal system of coordinates: $x^{(1)}$ (denotes the position of a stream surface in range $[0, 1]$),

Fig. 1 Schematic of axial turbine meridional cross-section—marks: domain of DV (guide vane), DG (domain of gap) and DR (domain of rotor).

$x^{(2)}$ (circumferential coordinate) and $x^{(3)}$ (the axial coordinate), is established. The shape of lines in Fig. 1 are described by a function of radius: $r = f(x^{(1)}, x^{(3)})$. The simplest form of function f is as follows:

$$r = f(x^{(1)}, x^{(3)}) = r_{x^{(1)}=0} + (r_{x^{(1)}=1} - r_{x^{(1)}=0})x^{(1)} \quad (1)$$

where, $r_{x^{(1)}=0}$, $r_{x^{(1)}=1}$ – the radii of outer and inner borders of a meridional cross-section (these are generally functions of $x^{(3)}$).

For the cross-section shown in Fig. 1, the considered example contains the following simple functions (cylindrical borders): $r_{x^{(1)}=0} = 0.195$ m and $r_{x^{(1)}=1} = 0.06$ m. The transformation between the Cartesian coordinate system and introduced the non-orthogonal system is given by the relations:

$$\begin{aligned} x &= f(x^{(1)}, x^{(3)}) \cos(x^{(2)}) \\ y &= f(x^{(1)}, x^{(3)}) \sin(x^{(2)}) \\ z &= x^{(3)} \end{aligned} \quad (2)$$

The set of conservation equations written in coordinate system: $x^{(1)}, x^{(2)}, x^{(3)}$ and method of solution for 2D model is presented in Refs. [1, 6], so the details of the method will be omitted in the present text.

One of the functions which closes the set of conservation equations is function of a blockage factor: $\tau(x^{(1)}, x^{(3)})$. This takes into account the fact that part of guide vanes and rotor domains are occupied by the thickness blades so that full space is not to disposal to the flow. It appears in the mass conservation equation as a factor reducing the space for the flow field.

Mass conservation equation in 2D model can be derived in the following form:

$$(1 - \tau(x^{(1)}, x^{(3)})) \rho U_{x^{(3)}} \frac{f \left| \frac{\partial f}{\partial x^{(1)}} \right|}{\sqrt{1 + \left(\frac{\partial f}{\partial x^{(3)}} \right)^2}} = m(x^{(1)}) \quad (3)$$

where, ρ —density, $U_{x^{(3)}}$ —meridional component of velocity tangential to streamlines $x^{(1)} = \text{const}$.

One can easily note that if τ increases then it has to be compensated by a higher value of $U_{x^{(3)}}$. Energy conservation equation we can write as follows:

$$\frac{U_{x^{(2)}}^2}{2} + \frac{U_{x^{(3)}}^2}{2} - U_{x^{(2)}} U_R + e + \frac{p}{\rho} + \Pi = E(x^{(1)}) \quad (4)$$

where, $U_{x^{(2)}}$ —tangential component of velocity, U_R —rotor circumferential velocity (for the guide vane domain $U_R = 0$), p —pressure determined by solving momentum conservation equation, e —internal energy changing due to the dissipation, Π —potential energy of gravity.

In 2D model, the dissipation intensity has to be prescribed “a priori” by defining the loss coefficient e.g., in the proposed hereafter function:

$$\zeta(x^{(1)}, x^{(3)}) = (w_0 + w_1 x^{(1)})^{n_l} (w_2 + w_3 \overline{x^{(3)}})^{n_s} \quad (5)$$

where, $w_0, w_1, w_2, w_3, n_l, n_s$ —coefficients, $\overline{x^{(3)}}$ —normalized axial coordinate defined below.

From mass and energy conservation equations, the following conclusion may be drawn: for high values of blockage factor and loss coefficient and pressure obtained by means of momentum equation the energy conservation equation may not lead to the real value of tangential velocity component $U_{x^{(2)}}$ (as a solution of quadratic equation with respect to $U_{x^{(2)}}$).

This means that in such case we have no solution of the problem within the frame of 2D model.

Let us denote the axial coordinates of leading and trailing edges of guide vanes cascade by $x_{le}^{(3)}, x_{tr}^{(3)}$, respectively. Then it can be defined non-dimensional axial coordinate in domain of guide vane as follows:

$$\overline{x^{(3)}} = \frac{x^{(3)} - x_{le}^{(3)}}{x_{tr}^{(3)} - x_{le}^{(3)}} \quad (6)$$

so that we obtain the range of changing this parameter: $0 \leq \overline{x^{(3)}} \leq 1$.

Blockage factor function is proposed as follows:

$$\tau(x^{(1)}, x^{(3)}) = t_1 \overline{x^{(3)}}^{t_2} (1 - \overline{x^{(3)}})^{t_3} (1 - t_4 x^{(1)}) \quad (7)$$

There are four parameters (coefficients) that control a distribution of blade thickness in domain of guide vanes: t_1, t_2, t_3 and t_4 . The shape of the proposed function takes into account the fact that at leading edge $\overline{x^{(3)}} = 0$, at trailing edge $\overline{x^{(3)}} = 1$ and blockage factor $\tau = 0$.

If one imposes an additional condition that maximum thickness τ_{max} of the blade is located at coordinate $\overline{x_{max}^{(3)}}$ then some of the parameters can be tied up according to the relations:

$$t_3 = t_2 \left(\frac{1}{\overline{x_{max}^{(3)}}} - 1 \right)$$

$$t_1 = \tau_{max} \left(\frac{t_2}{t_2 + t_3} \right)^{-t_2} \left(\frac{t_3}{t_2 + t_3} \right)^{-t_3} \quad (8)$$

From these restrictions two parameters remain free, namely t_2, t_4 . Parameter t_4 stays in front of $x^{(1)}$ coordinate. Thus, it controls the thickness change of profile along the radius with respect to the thickness at outer diameter. Parameter t_2 influences the thickness of guide vane profiles close to the trailing edge.

It was mentioned above that 2D model assumes an infinite number of blades. Therefore, we obtain the kinematic condition upon $x^{(1)} = const$ surface, which allows drawing the line representing the trajectory of fluid particle. This line can be treated as a skeleton line for the profile with the thickness dependent on the blockage factor τ . In the plane view, the sketch is shown in Fig. 2 as presented in Ref. [7].

Fig. 2 The sketch of skeleton line shown in a plane view.

The problem arises how to distribute the blade thickness Δt (as in Fig. 2; in non-dimensional terms is given by τ) along the skeleton line. It is proposed here the simplest form of profile design by one parameter t_p , which divides τ between pressure (index p) and suction (index s) sides of profile in proportion t_p . Then for the suction side we can write:

$$\begin{aligned} x_s &= r \sin(x^{(2)} - 2\pi(1 - t_p)\tau/n_b) \\ y_s &= r \cos(x^{(2)} - 2\pi(1 - t_p)\tau/n_b) \end{aligned} \quad (9a)$$

and for the pressure side:

$$\begin{aligned} x_p &= r \sin(x^{(2)} + 2\pi t_p \tau/n_b) \\ y_p &= r \cos(x^{(2)} + 2\pi t_p \tau/n_b) \end{aligned} \quad (9b)$$

where, n_b —the number of blades.

The described idea is illustrated in Fig. 3 for different values of t_p . The top profile is designed for $t_p = 0.25$, the middle one for $t_p = 0.5$ and the bottom one for $t_p = 0.75$. The skeleton line in all these cases was identical. It is obvious, that different values of t_p lead to different shapes of channel between neighboring profiles.

3. Results of 2D Computation

3.1 Boundary Conditions

At the inlet of the guide vanes the following parameters, uniformly distributed between the inner radius $r_{in} = 0.195$ m and outer radius $r_{out} = 0.195$ m, were assumed: density $\rho = 999.1$ kg/m³, inlet velocity $U_{x^{(3)}} = 2.175$ m/s, pressure $p_{in} = 100000$ Pa.

The method of solution of momentum conservation equation [1, 6] needs a boundary condition for the pressure distribution along $\overline{x^{(3)}}$ coordinate at outer radius.

$$p_{out}(p_{in}, p_{tr}, \overline{x^{(3)}}, n_b) = p_{in} - (p_{in} - p_{tr})\overline{x^{(3)}}^{n_b} \quad (10)$$

where chosen parameters control the pressure drop at the outer radius. For present example, they were assumed as follows: $p_{tr} = 90,000$ Pa and $n_b = 2.14572$.

To illustrate the idea, two different cases were computed using 2D algorithm. In the first case, the following parameters were taken into account to create the guide vanes profile:

Fig. 3 Three types of profiles for different values of parameter t_p : (a) $t_p = 0.25$, (b) $t_p = 0.5$ and (c) $t_p = 0.75$.

$$\tau_{max} = 0.1, \overline{x_{max}^{(3)}} = 0.25, t_1 = 0.308,$$

$$t_2 = 0.5, t_3 = 1.5, t_4 = -0.4 \quad (11a)$$

The above set of parameters creates a “thick” profile at the leading edge (Fig. 4a). In the second case, the set of parameters:

$$\tau_{max} = 0.13, \overline{x_{max}^{(3)}} = 0.6, t_1 = 11.5481,$$

$$t_2 = 4.0, t_3 = 2.66667, t_4 = -0.2 \quad (11b)$$

creates a “thin” leading edge (Fig. 4b).

Fig. 4 The distribution of the blockage function $\tau(x^{(1)}, x^{(3)})$ assumed in two considered cases of guide vane profiles in a 2D model.

3.2 Results of Computation

The results of computation are shown below. Due to the high blockage factor at the leading edge of profile, there is a large area (Fig. 5) at the inlet of channels, in which no solution of 2D model exists.

This case represents a “classical” (C) shape of profiles with maximum thickness shifted to the leading edge. The profile at outer boundary is shown in Fig. 6.

The empty part of the blade because of lack of solution close to leading edge is shown in Fig. 7.

Fig. 5 The skeleton surface with a large gap close to the leading edge due to the lack of solution.

Fig. 6 The shape of blade (the view from inlet side) for the first considered case (C).

Fig. 7 The blade with a large gap due to the lack of solution close to the leading edge for the first considered case (C).

The second case (M) represents another type of profile, in which the maximum thickness is shifted to the middle of channel with parameters as follows:

$$\tau_{max} = 0.13, \overline{x_{max}^{(3)}} = 0.6.$$

The parameter t_p was assumed as 0.2. The blade of single guide vane along the above presented skeleton surface is shown in Fig. 9.

The inlet part of the channel is directed to the upstream perpendicularly to the front of cascade. The solution of 2D model exists in the whole domain of guide vane. Fig. 8 shows the skeleton surface obtained

Fig. 8 The skeleton surface with existing solution in the whole flow domain for the second considered case (M).

Fig. 9 The guide vane with maximum thickness shifted towards the middle of channel for the second considered case (M).

from 2D computation. The skeleton lines are placed upon streamlines ($x^{(1)} = \text{const}$). The inlet to the guide vane is from right side of guide vane domain.

Fig. 10 shows the outer profile of the guide vane.

For both cases of designed profiles parameters at the outlet like outlet angle, pressure and velocity are shown below.

The mean value of outlet velocity in the case named as “classical” is higher than in the case of “modern” profiles. The mean velocities are:

- for the first case ‘classical’: $V_{out} = 4.8228$ m/s;
- for the second case ‘modern’: $V_{out} = 4.7045$ m/s.

In both cases, the dissipation effect was determined according to Eq. (5) with the following set of parameters:

$$w_0 = 0.3, w_1 = -0.15, w_2 = 0.0, w_3 = 1.5$$

Fig. 10 The outer profile for $t_p = 0.2$ for the second considered case (M).

Fig. 11 Distribution of outlet angle along the radius for two presented types of profiles.

Fig. 12 Distribution of outlet pressure along the radius for two presented types of profiles.

Fig. 13 Distribution of outlet velocity along the radius for two presented types of profiles.

$$w_0 = 3.0, w_0 = 2.0 \quad (12)$$

Fig. 14 shows the shape of function $\zeta(x^{(1)}, x^{(3)})$.

It increases of internal energy by $\Delta e = 0.41906$ m²/s².

Fig. 14 The shape of function $\zeta(x^{(1)}, x^{(3)})$ assumed to calculation by means of 2D model.

In both cases, the inlet kinetic energy was identical so it produced the same value of dissipation.

For mean values of outlet velocity, the loss coefficient can be defined as the ratio of internal energy increase to kinetic energy of an isentropic expansion $V_{is}^2/2$ as it was introduced in Ref. [8].

$$\zeta \stackrel{\text{def}}{=} \frac{\frac{V_{is}^2}{2} - \frac{V_{out}^2}{2}}{\frac{V_{is}^2}{2}} = \frac{2\Delta e}{V_{is}^2} \quad (13)$$

The value of loss coefficient differs for the both cases. For the first case we have: $\zeta_{class} = 0.03478$ and for the second case we have: $\zeta_{Mod} = 0.03649$. The higher value of loss coefficient in the second case results from lower velocity at outlet. The reference kinetic energy is then lower, what causes the higher value of loss coefficient. It has to be noted that 2D model is not suitable for the estimation dissipation effects. Moreover, a 2D model assumes infinite number of blades which is not realistic. For these reasons, the verification by means of a 3D model computation ought to be carried out.

4. Results of 3D Computation

4.1 Mesh and Boundary Conditions

For a 3D computation, it was assumed (in the first approximation) 14 blades as the realistic value, according to authors' previous considerations. This number was then diminished to 12 for (M) case according to the conclusion drawn from the

computation results.

Computational meshes were created by means of NUMECA/AutoGrid 5TM. The number of cells generated for (C) case of guide vanes was ~6.5 mln. The number of cells generated for (M) case of guide vanes was ~5.8 mln. The whole mesh was created using hexahedral cells. The example of geometry and mesh for (M) case of vanes is presented in Fig. 15.

The boundary conditions used for computations were as follows:

- Mass flow rate: 235 kg/s, density: 999.1 kg/m³, normal inflow;
- Turbulence model: $k-\varepsilon$ RNG non-equilibrium with boundary conditions at inlet and outlet: turbulence intensity: 3% and hydraulic diameter: 0.135 m.

In the case of "Classical" blade, it was necessary to stick the empty part of blade at leading edge. This was done by means of purely geometrical manner.

Computations were carried out by means of ANSYS/Fluent 15TM with the assumptions as follows: steady flow, second order discretization scheme, double precision.

4.2 Estimation of Increased Internal Energy by Means of the Turbulence Kinetic Energy

A 3D computation provides distribution of turbulence kinetic energy ε in a domain of cascade. In order to estimate the increase of the internal energy due to turbulence let us examine the relation which combines ε with the rate of internal energy increase.

$$\varepsilon \stackrel{\text{def}}{=} \frac{de}{dt} \quad (14)$$

Fig. 15 Example of the geometry and mesh for the (M) case of 12 blades.

Table 1 The results of 3D computation.

Quantity	Symbol	Unit	14 blades (C)	14 blades (M)	12 blades (M)
Axial vel. at inlet	V_a	m/s	2.1922	2.1930	2.1939
Radial vel. at inlet	V_r	m/s	-0.0955	-0.0992	-0.0809
Tangential vel. at inlet	V_t	m/s	0.0000	0.0000	0.0000
Mean vel. at inlet	V	m/s	2.1950	2.1963	2.1970
Pressure at inlet	p	Pa	8801	10822	8018
Turb. kinetic energy at inlet	ε	m ² /s ³	0.3789	0.3800	0.3779
Axial vel. at outlet	V_a	m/s	2.3036	2.3089	2.3675
Radial vel. at outlet	V_r	m/s	-0.1177	-0.0892	-0.0527
Tangential vel. at outlet	V_t	m/s	-2.5959	-2.9588	-2.5330
Mean vel. at outlet	V	m/s	3.5037	3.7814	3.5099
Pressure at outlet	p	Pa	4371	5188	3506
Turb. kinetic energy at outlet	ε	m ² /s ³	6.0751	7.3600	6.8112
Mean turb. kinetic energy	$\bar{\varepsilon}$	m ² /s ³	3.2270	3.8700	3.5946
Turb. kinetic energy difference	$\Delta\varepsilon$	m ² /s ³	0.7149	0.8562	0.7849
Mean axial vel.	$\overline{V_{axial}}$	m/s	2.2479	2.2510	2.2807
Isentropic vel.	V_{is}	m/s	3.6995	4.0128	3.7227
Loss coefficient	ζ	-	0.1045	0.1063	0.1133
Ax. vel./Tang.vel. ratio	V_a/V_t	-	-0.8874	-0.7803	-0.9347
Outlet angle	$\text{ArcTan}(V_a/V_t)$	deg	-41.5857	-37.9662	-43.0657
Mean pressure drop	$\overline{\Delta p}$	Pa	4430	5634	4512
Pressure drop ratio	-	-	1.0000	1.2719	1.0185

Integrating the above formula, the increase of internal energy Δe (m²/s²) is estimated as:

$$\begin{aligned}\Delta e &= \int_{e_1}^{e_2} de = e_2 - e_1 = \int_{t_1}^{t_2} \varepsilon dt = \\ &= \bar{\varepsilon} \int_{t_1}^{t_2} dt = \bar{\varepsilon}(t_2 - t_1) = \bar{\varepsilon}\Delta t\end{aligned}\quad (15)$$

The mean value of $\bar{\varepsilon}$ can be estimated having the numerical values from a 3D computation.

The time interval Δt is connected with the trajectory of representative fluid element.

$$\Delta t \cong \frac{\Delta l_{axial}}{\overline{V_{axial}}}\quad (16)$$

As a representative length Δl_{axial} we can choose the axial length of the rotor at mean diameter and mean value of axial velocity $\overline{V_{axial}}$. The foregoing formulates the estimation of an internal energy increase due to turbulent dissipation, what is the prevailing factor of dissipation.

4.3 Summary of 3D Computation

The results of 3D computation are shown in Table 1.

The parameters are ordered according to the number of blades.

As the reference parameter the pressure drop was chosen to compare blade (C) and blade (M). One can see that in order to achieve the same pressure drop in the channel (M), it is necessary to assume less number of blades. This diminishes the friction losses and cost of the investment.

5. Comparison and Conclusions

(1) The presented approach based on the 2D and 3D models is effective in the design of blades for guide vanes cascade of a low head hydraulic (Kaplan) turbine;

(2) The proposal of two concepts of blade (channel) design shows the possibility of finding the solution which has desirable technical benefit;

(3) Assuming the pressure drop along the guide vane as a criterion, the (M) profile seems to be more reasonable solution than profile (C). The fewer number of blades makes the same pressure drop.

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