The Life Predictions in Whole Process Realized with Different Variables and Conventional Materials Constants for Elastic-Plastic Materials Behaviors under Unsymmetrical Cycle Loading

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Abstract: Author researches a lot of the mathematical models and the related conventional material constants in the traditional and the modern mechanics; to adopt two types of variables a and D , for the fatigue-damage-fracture behaviors to elastic-plastic steels contained flaws, to put forward several calculation models, which are the driving force and the life prediction expressions at each stage and in whole process; for the key parameters A_1' and A_2' in two stages, there are functional relation with other conventional material constants σ_f', m_1' and m_2', λ_2' , they are defined as the new calculable comprehensive material constants, and indicate their physical and geometrical meanings. In addition, for conversion methods between two types of variables, relevant calculating example is provided. Thereby, make a linking between the fracture mechanics and the damage mechanics, communicating their relationships. This works for saving man powers and funds on fatigue-damage-fracture testing that will be having practical significance.

Key words: Life in whole process, variables of two types, driving force, elastic-plastic materials, conventional material constants.

1. Introduction

As is well known, for describing material behaviors on the strength and the life problems in the traditional material mechanics, it adopts the stress σ or the strain ε , etc. as variables and uses the yield stress σ_s , and the reduction of area φ , etc. as material constants; in the fracture mechanics, it adopts the crack size a as variable, and uses the fracture toughness K_{Ic} and the critical crack tip open displacement δ_c as material constants; in the damage mechanics, it adopts the variable D as damage variable, and uses the fatigue strength coefficient σ_f' and the fatigue ductility coefficient ε_f' etc. as material constants.

Author thinks, in the mechanics area, the material science area, and the engineering one, where there are also such similar principles as the genetic and clone technology in the life science [1]. If we can use the route diagram of material behaviors [1, 2]; if can be converted each other for their relations between the variables, between the material constants, and between the dimensional units; and if can provide respectively some conversion methods; then, we are also able both to adopt the kind of variable a_1 , a_2 and corresponding conventional materials constants, and to use other kind of variables D_1 , D_2 and ones, such that we can all be calculated for the crack growth life N_1 and N_2 at each stage or damage growth life N_1 and N_2 . Even we can also adopt variable a or variable D in whole process to describe their behaviors and to calculate their lifetime $\Sigma N = N_1 + N_2$. That way, it will be having practical significance for stint man powers and funds in

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fatigue-damage-fracture testing, for promoting developing and applying of related disciplines, especially for the safety design, evaluation, and accident analysis to modern industrial engineering that will all be having practical significance.

2. The Estimations of Life at Each Stage for Elastic-Plastic Materials with Crack

For some elastic-plastic materials with short cracks a_1 , the life's estimations for short cracks growth processes, or the life's estimations for damage growth processes can be imitated by means of method in Refs. [3, 4], that is to realize damage calculations by multiplication parameters $\varepsilon_p \sigma$ with plastic strain and stress at the crack forming stage (the first stage); For long crack a_2 , this paper also adopts the multiplication parameters δ, K_1 with the crack tip open displacement and the stress intensity factor at the crack growth stage (the second stage), thereby achieve severally for their life prediction for crack growth or damage one at each stage.

2.1 The Life's Prediction Estimations at First Stage under Mean Stress $\sigma_m \neq 0$ Condition

The life's curve of short cracks a_1 for elastic-plastic materials at the first stage is just described with curve 5 ($\sigma_m \neq 0$), where it is in the reversed direction coordinate system in calculating figure 1 of materials behaviors. And its life's prediction equation about short crack growth is as following form,

$$N_1 = \int_{a_{01}}^{a_{mc}} \frac{da_1}{A^* \times (\Delta Q_1)^{\frac{m_1 m'_1}{m_1 + m'_1}}} \text{ (cycle)} \quad (1)$$

Here,

$$Q_1 = (\varepsilon \cdot \sigma) \cdot a_1^{1/(\frac{m_1 m'_1}{m_1 + m'_1})}, (\%MPa \cdot \frac{m_1 m'_1}{m_1 + m'_1} \sqrt{m}) \quad (2)$$

$$\Delta Q_1 = (\Delta \varepsilon \cdot \Delta \sigma) \cdot a_1^{1/(\frac{m_1 m'_1}{m_1 + m'_1})} \quad (3)$$

The parameters Q_1 and ΔQ_1 are defined severally as the short-crack stress-strain factor and the stress-strain factor range, that are driving force of short crack growth. Their units are $MPa \cdot \frac{m_1 m'_1}{m_1 + m'_1} \sqrt{m}$, the parameters m_1

and m'_1 are respectively material constant under high cycle and low cycle fatigue. $m_1 = -1/b'_1$, b'_1 is the fatigue strength exponent under high cycle fatigue. $m'_1 = -1/c'_1$, c'_1 is the fatigue ductility exponent under low cycle fatigue.

$$A_1^* = 2[4(\sigma_f' - \sigma_m)(\varepsilon_f' - \varepsilon_m)]^{\frac{m_1 m'_1}{m_1 + m'_1}} \frac{\ln[1/(1-\phi)]}{(N_{1f} - N_{01})} (MPa)^{\frac{m_1 m'_1}{m_1 + m'_1}} / \text{cycle} \quad (4)$$

where, the parameter A_1^* is a calculable comprehensive material constant. Under mean stress $\sigma_m \neq 0$ condition, it can be corrected by the Morrow's method [5], but must be obtained from combining experiment. Author researches and thinks, physical meaning of the A_1^* is a concept of power, it is an energy gave out that is the material to resist external force, just is a maximal increment value given out energy before causing failure in one cycle. Its geometrical meaning is a maximal micro-trapezium area approximating to beeline (Fig. 1), that is a projection of corresponding to curve 5 on the y-axis, also is an intercept between $Q_1 - Q_3$. Its slope of micro-trapezium bevel edge just is corresponding to the exponent $\frac{m_1 m'_1}{m_1 + m'_1}$ of Eq. (4). The calculable comprehensive material constant A_1^* is having function relation with among other parameters in Eq. (4) [6]. σ_f' is a fatigue strength coefficient, ε_f' is a fatigue ductility coefficient. ε_m is a mean strain value, it can ignore ($\varepsilon_m \approx 0$). ϕ is a reduction of area in material mechanics. N_{01} is initial life at first stage, $N_{01} = 0$; N_{1f} is failure life, $N_{1f} = 1$. It should point that fraction $\ln[1/(1-\phi)]/(N_{1f} - N_{01})$ in Eq. (4) equal parameter v_f , if it is used in crack growth rate equation, it is follow form.

$$v_f = \ln[1/(1-\phi)]/(N_{1f} - N_{01}) = \ln[1/(1-\phi)] \cdot (m / \text{cycle}) \quad (5-1)$$

where, its unit of v_f is the "m/cycle". If it is used in damage growth rate equation, its unit of v_f is the "damage-unit-number/cycle", it is follow form.

$$v_f = \ln[1/(1-\phi)]/(N_{1f} - N_{01}) = \ln[1/(1-\phi)] \cdot (\text{damage-unit-number} / \text{cycle}) \quad (5-2)$$

And the parameter v_f in eq. (5) is defined as an effective damage rate correction factor in first stage, its physical meaning is the effective damage rate to cause whole failure of specimen material in a cycle.

So its final expansion equation for (1) is as follow form [7-12].

$$N_1 = \int_{a_{01}}^{a_u} \frac{da_1}{2[(\sigma'_f - \sigma_m)(\epsilon'_f - \epsilon_m)]^{\frac{m_1 m'_1}{m_1 + m'_1}} \times v_f (\Delta \epsilon \cdot \Delta \sigma)^{\frac{m_1 + m'_1}{m_1 + m'_1}} \times a_1}, (\text{cycle}) \quad (6-1)$$

or

$$N_{oi} = \frac{\ln a_{oi} - \ln a_{01}}{2[(\sigma'_f - \sigma_m)(\epsilon'_f - \epsilon_m)]^{\frac{m_1 m'_1}{m_1 + m'_1}} (\ln a_{mac} - \ln a_{01}) v_f (\Delta \epsilon \cdot \Delta \sigma)^{\frac{m_1 + m'_1}{m_1 + m'_1}}} \quad (6-2)$$

where, a_{oi} is a initial micro-crack size; a_{mac} is a macro crack size corresponding forming long crack, a_u is a transitional size transited from short crack to long crack, a_{oi} is a medial crack size at the first stage. On the other hand, if use the damage variable D_1 to express it, as is equivalent with above mentioned Eqs. (1) and (6), then life equations of damage growth should be Eqs. (7) and (8) below:

$$N_1 = \int_{D_{01}}^{D_{mac}} \frac{dD_1}{A^*_{11} \times (\Delta Q'_1)^{\frac{m_1 m'_1}{m_1 + m'_1}}}, (\text{cycle}) \quad (7)$$

$$N_{oi} = \frac{\ln D_{oi} - \ln D_{01}}{2[(\sigma'_f - \sigma_m)(\epsilon'_f - \epsilon_m)]^{\frac{m_1 m'_1}{m_1 + m'_1}} (\ln D_{mac} - \ln D_{01}) v_f (\Delta \epsilon \cdot \Delta \sigma)^{\frac{m_1 + m'_1}{m_1 + m'_1}}} \quad (8)$$

where the parameter Q_1 in Eq. (1) should convert corresponding to Q'_1 , the parameters Q'_1 and $\Delta Q'_1$ are defined severally as the damage stress-strain factor and the damage stress-strain factor range, that are also driving forces of damage growth process of materials at first stage, their units of Q'_1 and $\Delta Q'_1$ are $MPa \cdot D^{\frac{1}{m_1 + m'_1}}$. The damage variable D_1 is a non-dimensional value, it is equivalent to short crack a_1 discussed as reference, here must be defined in 1 m equivalent to 1,000 of damage-units, 1 mm equivalent to one of damage-unite, and must put up conversion to the dimensions and the units.

2.2 The Life Prediction Estimations at Second Stage under $\sigma_m \neq 0$ Condition

In Fig.1, the beeline $C_2 C_1$ via logarithm predigesting transacting can be represented for long crack growth behavior of materials at the second stage. For its life at this stage can be also adopted by above mentioned the multiplication parameters $\Delta \delta_i \cdot \Delta K_1$ to

describe,

$$N_2 = \int_{a_{02}}^{a_{2c}} \frac{da_2}{A^*_{22} \times (\Delta Q_2)^{\frac{m_2 + \lambda_2}{m_2 + \lambda_2}}} \quad (9)$$

Here, a_{02} is an initial crack size at second stage; a_{2c} is a critical crack size,

$$Q_2 = \delta_i \cdot K_1 \quad (MPa \cdot m^{3/2}) \quad (10-1)$$

$$Q_{2c} = \delta_c \cdot K_{1c} \quad (10-2)$$

$$\Delta Q_2 = \Delta \delta_i \cdot \Delta K_1 \quad (MPa \cdot m^{3/2}) \quad (11-1)$$

$$Q_{2ic} = \delta_{ic} \cdot K_{1c} \quad (MPa \cdot m^{3/2}) \quad (11-2)$$

where, the Q_2 and ΔQ_2 are defined severally the stress-strain factor and the stress-strain factor range as the long cracks, that are driving forces under monotonous and fatigue loading at second stage, the $\delta_{ic} \cdot K_{1c}$ are critical values under fatigue and monotonous loading, respectively.

For work stress, σ is less than yield stress σ_s ($\sigma < \sigma_s$), the δ_i [13] and K_1 are as follow:

$$\delta_i = \pi a_2 \sigma_s \times (\sigma / \sigma_s)^2 / E \quad (12)$$

$$\Delta K_1 = K_2 = F_2 \Delta \sigma \sqrt{\pi a_2} \quad (13)$$

$$Q_2 = \frac{\sigma^3}{E \sigma_s} (\sqrt{\pi a_2})^3 \quad (14-1)$$

$$\Delta Q_2 = \frac{\Delta \sigma^3}{4 E \sigma_s} (\sqrt{\pi a_2})^3, \quad (MPa \cdot m^{3/2}) \quad (14-2)$$

Here, F_2 is a correction coefficient of concerning crack shape and size. E is modulus of elasticity. And for work stress is equal or greater than yield stress ($\sigma \geq \sigma_s$),

$$\delta_i = 0.5 \pi \times \sigma_s \times a_2 (\sigma / \sigma_s + 1) / E, \quad (MPa \cdot m^{3/2}) \quad (15)$$

$$\Delta Q_2 = [0.5 \times \Delta \sigma \cdot \sigma_s (\sqrt{\pi a_2})^3 (\Delta \sigma / 2 \sigma_s + 1)] / E \quad (16)$$

Moreover,

$$A_2^* = 2[4(K_{2c} - K_{2m})(\delta_{2c} - \delta_m)]^{\frac{m_2 \lambda_2}{m_2 + \lambda_2}} \times \frac{(a_{pv} - a_{02})}{(N_{pv} - N_{02})}, \quad (17-1)$$

$$(m^{3/2} \cdot MPa)^{\frac{m_2 \lambda_2}{m_2 + \lambda_2}} \times m / \text{cycle}$$

By the way, if for Eq. (17) is adopted by the stress to express, it can be used as following form,

$$A_2^* = \{4[\frac{0.5 \times \sigma'_f \cdot \sigma_s (\sigma / \sigma_s + 1)}{E} (\sqrt{\pi a_{2c}})^3] (1 - \sigma_m / \sigma'_f)\}^{\frac{m_2 \lambda_2}{m_2 + \lambda_2}} \times v_{pv}, \quad (17-2)$$

$$(m^{3/2} \cdot MPa)^{\frac{m_2 \lambda_2}{m_2 + \lambda_2}} \times m / \text{cycle}$$

where, the ϱ_2 and $\Delta\varrho_2$ in Eqs. (14-1) and (14-2) are defined severally as the long cracks stress-strain factor and stress-strain factor range, that are also driving forces of long cracks growth. Their units are this $MPa \cdot m^{3/2}$. Here, the m_2 and λ_2 are respectively materials constants under high cycle or low cycle fatigue. $m_2 = -1/b'_2$, b'_2 is fatigue strength exponent under high cycle fatigue; $\lambda_2 = -1/c'_2$, c'_2 is fatigue ductility exponent under low cycle fatigue. The parameter A_2^* is also a calculable comprehensive material constant at the second stage; it should be obtained from combining experiment. Its physical meaning is also a concept of the power, it is the maximal increment value given out energy before causing specimen fracture in one cycle. Its geometrical meaning is a maximal micro-trapezia area approximating to beeline, also is an intercept between $\sigma_3 - \sigma_4$ on the y-axis, is also a projection of corresponding to curve 6 ($C_1 k C_2$) (Fig. 1). Its slope of micro-trapezium bevel edge just is corresponding to the exponent $\frac{m_2 \lambda_2}{m_2 + \lambda_2}$ of Eq. (17). The comprehensive material constant A_2^* is having function relation with other parameters in (17). K_m is a mean stress intensity factor, δ_m is a mean value of crack tip open displacement, it can also ignore ($\delta_m \approx 0$).

where,

$$v_{pv} = \frac{(a_{pv} - a_{01})}{(N_{pv} - N_{01})} \approx 3 \times 10^{-8} \sim 3 \times 10^{-7} = v^*, (m / cycle) \quad (18)$$

Author researches and thinks, the parameter v_{pv} is a constant of damage rate at the second stage, it is defined to be the virtual rate, its physical meaning is the damage rate contributed at material loaded yield point, that can take same value by [14], $v_{pv} = v^* (m / cycle)$. The short crack a_{pv} is precrack size, a_{01} is initial micro-crack size. N_{pv} is a virtual life. For the sake of safety, the comprehensive constant A_2^* in Eq. (17) should be computed as following form:

$$A_{2eff}^* = \frac{2[(K_{2eff} - K_{2m})(\delta_{2eff} - \delta_m)]^{\frac{m_2 \lambda_2}{m_2 + \lambda_2}} \times v_{pv} \cdot (m^{3/2} \cdot MPa)^{\frac{m_2 \lambda_2}{m_2 + \lambda_2}} \times m / cycle}{m_2 + \lambda_2} \quad (19)$$

where, A_{2eff}^* is effective comprehensive material constant at the second stage, Its physical meaning is maximal increment value to achieve effective power in

one cycle, that is to pay an energy resisted external force, The K_{2eff} is the effective value of stress intensity factor, δ_{eff} is the effective value of the crack tip open displacement, that are all the effective parameters during steady growth course for long crack, it must be obtained from experiment. Suppose that,

$$K_{eff} = \sqrt{K_{1c} \times K_{th}} \quad \text{or} \quad \delta_{eff} = (0.15 \sim 0.55) \delta_c \quad (20)$$

So, for $\sigma < \sigma_s$, its final expansion equations of the Eq. (9) is as follow form,

$$N_{oj} = \frac{\frac{4(m_2 + \lambda_2)}{4m_2 + 4\lambda_2 - 6m_2\lambda_2} \left(\frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} - a_{02} \frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} \right)}{2[(K_{2eff} - K_{2m})(\delta_{2eff} - \delta_m)]^{\frac{m_2 \lambda_2}{m_2 + \lambda_2}} v_{pv} \left[\frac{(\Delta\sigma)^3}{4E\sigma_s} \times \pi^{\frac{3}{2}} \right]^{\frac{m_2 \lambda_2}{m_2 + \lambda_2}}}, (cycle) \quad (21)$$

For $\sigma \geq \sigma_s$,

$$N_{oj} = \frac{\frac{4(m_2 + \lambda_2)}{4m_2 + 4\lambda_2 - 6m_2\lambda_2} \left(\frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} - a_{02} \frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} \right)}{2[(K_{2eff} - K_{2m})(\delta_{2eff} - \delta_m)]^{\frac{m_2 \lambda_2}{m_2 + \lambda_2}} \times v_{pv} \times [0.5 \times \Delta\sigma \times \sigma_s \times \pi^{\frac{3}{2}} (\Delta\sigma / 2\sigma_s + 1) / E]^{\frac{m_2 \lambda_2}{m_2 + \lambda_2}}} \quad (22)$$

Here, a_{oj} is a medial variable at the second stage, a_{eff} is a crack size in effective region. $a_{02} < a_{oj} \leq a_{eff}$. N_{oj} a medial life, $N_{02} < N_{oj} \leq N_{eff}$. Accordingly, for $\sigma < \sigma_s$, the life for effective region at the second stage should become,

$$N_{eff} = \frac{\frac{4(m_2 + \lambda_2)}{4m_2 + 4\lambda_2 - 6m_2\lambda_2} \left(\frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} - a_{02} \frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} \right)}{2[(K_{2eff} - K_{2m})(\delta_{2eff} - \delta_m)]^{\frac{m_2 \lambda_2}{m_2 + \lambda_2}} v_{pv} \times \left[\frac{\Delta\sigma^3}{4E\sigma_s} (\sqrt{\pi})^3 \right]^{\frac{m_2 \lambda_2}{m_2 + \lambda_2}}}, \quad (23)$$

For $\sigma \geq \sigma_s$,

$$N_{eff} = \frac{\frac{4(m_2 + \lambda_2)}{4m_2 + 4\lambda_2 - 6m_2\lambda_2} \left(\frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} - a_{02} \frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} \right)}{2[(K_{2eff} - K_{2m})(\delta_{2eff} - \delta_m)]^{\frac{m_2 \lambda_2}{m_2 + \lambda_2}} \times v_{pv} \times [0.5 \times \Delta\sigma \times \sigma_s \times \pi^{\frac{3}{2}} (\Delta\sigma / 2\sigma_s + 1) / E]^{\frac{m_2 \lambda_2}{m_2 + \lambda_2}}} \quad (24)$$

If use above mentioned same method, put up conversion for variable, then the damage growth life equation is deduced as following form:

$$N_2 = \int_{D_{02}}^{D_2} \frac{dD_2}{A_2^* \times (\Delta Q_2)^{\frac{m_2 \lambda_2}{m_2 + \lambda_2}}}, (cycle) \quad (25)$$

$$N_2 = \int_{D_{02}}^{D_{eff}} \frac{dD_2}{A_2^* \times (\Delta Q_2)^{\frac{m_2 \lambda_2}{m_2 + \lambda_2}}}, (cycle) \quad (26)$$

So, for $\sigma < \sigma_s$, above mentioned corresponding equations should be as follows,

$$\Delta Q'_2 = \frac{(\Delta\sigma)^3}{4E\sigma_s} (\sqrt{\pi D_2})^3, [MPa \cdot (damage - unit - number)]^{3/2} \quad (27)$$

For $\sigma \geq \sigma_s$, it is:

$$\Delta Q'_2 = [0.5 \times \Delta\sigma \cdot \sigma_s (\sqrt{\pi D_2})^3 (\Delta\sigma / 2\sigma_s + 1)] / E [MPa \cdot (damage - unit - number)]^{3/2} \quad (28)$$

$$A'_2 = 2[4(K'_{2c} - K'_{2m})(\delta'_{2c} - \delta'_{2m})]^{m_2 + \lambda_2} \times v_{pv}, \\ [(damage - unit - number)^{3/2} \cdot MPa]^{m_2 + \lambda_2} \times damage - unit - number / cycle \quad (29)$$

$$A''_{2eff} = 2[4(K'_{2eff} - K'_{2m})(\delta'_{2eff} - \delta'_{2m})]^{m_2 + \lambda_2} \times v_{pv}, \\ [(damage - unit - number)^{3/2} \cdot MPa]^{m_2 + \lambda_2} \times damage - unit - number / cycle \quad (30)$$

where, $\Delta Q'_2$ is defined to be “the damage stress-strain factor range” at the second stage, that is driving force of material damage growth. The material constant K'_{2c} (K'_{1c}) and K'_{2eff} are defined as the critical damage stress intensity factor and the effective damage stress intensity factor, their units are all this $(MPa)^{\frac{m_2 + \lambda_2}{m_2 + \lambda_2}} \times damage - unit - number / cycle$, that are equivalent to $K_{1c} \cdot K_{eff}$. δ'_{2c} and δ'_{2eff} are defined respectively as the critical damage crack tip open displacement and the effective damage crack tip open displacement, that are equivalent with δ_c and δ_{eff} . The units of other parameters in equations are also invariant. A'_2 is a calculable damage comprehensive material constant at the second stage. A''_{2eff} is a calculable effective damage comprehensive material constant. Therefore their final expansion equations, if adopt critical A'_2 and for $\sigma < \sigma_s$, it is as follow form,

$$N_{oj} = \frac{4(m_2 + \lambda_2)}{4m_2 + 4\lambda_2 - 6m_2\lambda_2} \left(\frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} - D_{02} \right)^{\frac{m_2\lambda_2}{m_2 + \lambda_2}} \times v_{pv} \left[\frac{(\Delta\sigma)^3}{4E\sigma_s} (\sqrt{\pi})^3 \right]^{\frac{m_2\lambda_2}{m_2 + \lambda_2}} \quad (31)$$

If adopt effective A''_{2eff} , it is:

$$N_{eff} = \frac{4(m_2 + \lambda_2)}{4m_2 + 4\lambda_2 - 6m_2\lambda_2} \left(\frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} - D_{02} \right)^{\frac{m_2\lambda_2}{m_2 + \lambda_2}} \times v_{pv} \left[\frac{(\Delta\sigma)^3}{4E\sigma_s} (\sqrt{\pi D_2})^3 \right]^{\frac{m_2\lambda_2}{m_2 + \lambda_2}} \quad (32)$$

For $\sigma \geq \sigma_s$ and if adopt critical A'_2 , its final expansion equation is:

$$N_{oj} = \frac{4(m_2 + \lambda_2)}{4m_2 + 4\lambda_2 - 6m_2\lambda_2} \left(\frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} - D_{02} \right)^{\frac{m_2\lambda_2}{m_2 + \lambda_2}} \times v_{pv} \left[\frac{0.5 \times \Delta\sigma \times \sigma_s \times \pi^2 (\Delta\sigma / 2\sigma_s + 1) / E}{(\Delta\sigma / 2\sigma_s + 1) / E} \right]^{\frac{m_2\lambda_2}{m_2 + \lambda_2}} \quad (33)$$

If not consider influence of δ_{2m} ($\delta_{2m} = 0$), adopt effective A''_{2eff} , it is:

$$N_{eff} = \frac{4(m_2 + \lambda_2)}{4m_2 + 4\lambda_2 - 6m_2\lambda_2} \left(\frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} - D_{02} \right)^{\frac{m_2\lambda_2}{m_2 + \lambda_2}} \times v_{pv} \left[\frac{0.5 \times \Delta\sigma \times \sigma_s \times \pi^2 (\Delta\sigma / 2\sigma_s + 1) / E}{(\Delta\sigma / 2\sigma_s + 1) / E} \right]^{\frac{m_2\lambda_2}{m_2 + \lambda_2}} \quad (34)$$

It must be point that the life units in crack growth process in Eqs. (10), (21) and (22) are the number of cycles; And the life units of damage growth in Eqs. (31)-(34) are also the cycle number.

3. The Life Prediction Estimations in Whole Process

Should point that the estimations of life in whole process can use two of methods:

(1) Method 1, select corresponding equation and variable a_1 (D_1) and a_2 (D_2) by at each stage respectively to compute its life, then, add the life together in two stages.

If adopt crack size a as variable, and if ignore influence of the ε_m and δ_m , for $\sigma < \sigma_s$, the calculation expression in whole process is as following form.

$$\Sigma N = N_1 + N_2 = \frac{\ln a_{01} - \ln a_{01}}{2[4(\sigma'_f - \sigma'_m) \varepsilon'_f]^{m_1 + m'_1} \times v_f (\ln a_{mac} - \ln a_{01})(\Delta \varepsilon \cdot \Delta \sigma)^{m_1 + m'_1}} + \frac{4(m_2 + \lambda_2)}{4m_2 + 4\lambda_2 - 6m_2\lambda_2} \left(\frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} - D_{02} \right)^{\frac{m_2\lambda_2}{m_2 + \lambda_2}} \times v_{pv} \left[\frac{(\Delta\sigma)^3}{4E\sigma_s} (\sqrt{\pi})^3 \right]^{\frac{m_2\lambda_2}{m_2 + \lambda_2}} \quad (35)$$

For $\sigma \geq \sigma_s$, it should be:

$$\Sigma N = N_1 + N_2 = \frac{\ln a_{01} - \ln a_{01}}{2[4(\sigma'_f - \sigma'_m) \varepsilon'_f]^{m_1 + m'_1} \times v_f (\ln a_{mac} - \ln a_{01})(\Delta \varepsilon \cdot \Delta \sigma)^{m_1 + m'_1}} + \frac{4(m_2 + \lambda_2)}{4m_2 + 4\lambda_2 - 6m_2\lambda_2} \left(\frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} - D_{02} \right)^{\frac{m_2\lambda_2}{m_2 + \lambda_2}} \times v_{pv} \left[\frac{0.5 \times \Delta\sigma \times \sigma_s \times \pi^2 (\Delta\sigma / 2\sigma_s + 1) / E}{(\Delta\sigma / 2\sigma_s + 1) / E} \right]^{\frac{m_2\lambda_2}{m_2 + \lambda_2}} \quad (36)$$

If adopt the damage variable D , for $\sigma < \sigma_s$, it is as following form,

$$\Sigma N = N_1 + N_2 = \frac{\ln D_{01} - \ln D_{01}}{\frac{m_1 m_1'}{2[4(\sigma_f' - \sigma_m) \varepsilon_f']^{m_1 + m_1'} \times v_f (\ln D_{mac} - \ln D_{01}) (\Delta \varepsilon \cdot \Delta \sigma)^{m_1 + m_1'}}} + \frac{4(m_2 + \lambda_2)}{4m_2 + 4\lambda_2 - 6m_2\lambda_2} \left(\frac{D_{0j}}{D_{0j}} \frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} - D_{02} \frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} \right) + \frac{m_2\lambda_2}{2[4(K'_{2c} - K_{2m}) \delta_{2eff}']^{m_2 + \lambda_2} \times v_{pv} \times [\frac{\Delta \sigma^3}{4E\sigma_s} (\sqrt{\pi})^3]^{m_2 + \lambda_2}} \quad (37)$$

For $\sigma \geq \sigma_s$, it is:

$$\Sigma N = N_1 + N_2 = \frac{\ln D_{01} - \ln D_{01}}{\frac{m_1 m_1'}{2[4(\sigma_f' - \sigma_m) \varepsilon_f']^{m_1 + m_1'} \times v_f (\ln D_{mac} - \ln D_{01}) (\Delta \varepsilon \cdot \Delta \sigma)^{m_1 + m_1'}}} + \frac{4(m_2 + \lambda_2)}{4m_2 + 4\lambda_2 - 6m_2\lambda_2} \left(\frac{D_{0j}}{D_{0j}} \frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} - D_{02} \frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} \right) + \frac{m_2\lambda_2}{2[4(K'_{2c} - K_{2m}) \delta_{2eff}']^{m_2 + \lambda_2} \times v_{pv} \times [0.5 \times \Delta \sigma \times \sigma_s \times \pi^{\frac{3}{2}} (\Delta \sigma / 2\sigma_s + 1) / E]^{m_2 + \lambda_2}} \quad (38)$$

(2) Method 2, Adopt corresponding equations in two stages, use same damage variable a or D in whole process, by means of computing program complete the calculations and the description of the whole curves. That simplified reversed curves 5 and 6 ($C_2 k C_1 k' B_1 D$) ($\sigma_m \neq 0$) in reversed direction coordinate system in Fig. 1 just are describing the changing law of the life. Their equations as following forms:

$$\Sigma N = \int_{a_{01}}^{a_{tr}} \frac{da}{A^* \times (\Delta Q_1)^{m_1 - m_1'}} + \int_{a_{tr}}^{a_{eff}} \frac{da}{A^* \times (\Delta Q_2)^{m_2 + \lambda_2}} \quad (39)$$

$$\Sigma N = \int_{D_{01}}^{D_{tr}} \frac{dD}{A^* \times (\Delta Q_1)^{m_1 - m_1'}} + \int_{D_{tr}}^{D_{eff}} \frac{dD}{A^* \times (\Delta Q_2)^{m_2 + \lambda_2}} \quad (40)$$

where, a_{tr} and D_{tr} are respectively transition crack size and transition damage value from short crack to long crack. Thus, we have achieved both able to adopt the variable a_1 and a_2 or the variables D_1 and D_2 to calculate severally the life for their curves by two stages; and can also adopt same a or D by means of computing program to describe its curves in whole process.

4. Computing Example

A pressure vessel is made with elastic-plastic steel 16MnR, its strength limit of material $\sigma_b = 573$ MPa, yield limit $\sigma_s = 361$ MPa, reduction of area is $\varphi = 0.51$, modulus of elasticity $E = 200,000$ MPa; Cyclic

strength coefficient $K' = 1,165$ MPa, strain-hardening exponent $n' = 0.187$; Fatigue strength coefficient $\sigma_f' = 947.1$ MPa, fatigue strength exponent $b_f' = 0.111$, $m_1 = 9.009$; Fatigue ductility coefficient $\varepsilon_f' = 0.464$, fatigue ductility exponent $c_f' = -0.5395$, $m_1' = 1.8536$ at short-crack growth stage. Threshold level $\Delta K_{th} = 8.6 \text{ MPa}\sqrt{m}$, critical stress intensity factor $K_{2c} = K_{1c} = 92.7 \text{ MPa}\sqrt{m}$, critical damage stress intensity factor $K'_{2c} = 92.7 \text{ MPa}\sqrt{D_2}$ of equivalent with the $K_{1c}(K_{2c})$. Its working stress $\sigma_{max} = 280 \text{ MPa}$, $\sigma_{min} = 0$ in pressure vessel, local stress 840 MPa at stress concentration point (stress concentration coefficient $K_t = 3$). Suppose that for long crack shape has been simplified via treatment become a through-crack, the correction coefficient F_2 of crack shapes and sizes equal 1 ($F_2 = 1$). Other computing data are all in Table 1.

Table 1 Computing data.

$K_{1c}, \text{MPa}\sqrt{m}$	$K_{2c}, \text{MPa}\sqrt{m}$	$K_{th}, \text{MPa}\sqrt{m}$	v_{pv}	
92.7	28.23	8.6	2×10^{-7}	
m_2	δ_c, m	λ_2	F_2	Kt
3.91	0.00018	2.9	1.0	3

Try to calculate respectively:

- (1) The life N_1 from short crack $a_{01} = 0.00036$ m growth to long crack transitional size $a_{tr} = a_{mac} = 0.001$ m at first stage (or damage life);
- (2) The life N_2 from transitional size $a_{tr} = a_{mac} = 0.001$ m to long crack size $a_{02} = 0.0076455$ m at the second stage (or damage growth life);
- (3) The life ΣN in whole process.

- Calculations for relevant parameters

- 1) Calculations for stress-strain by Ref. [15]

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'} \right)^{\frac{1}{n'}} = \frac{280}{200000} + \left(\frac{280}{1165} \right)^{\frac{1}{0.187}} = 1.4^{-3} + 4.886^4 = 1.8886^3$$

$$\text{Strain range } \Delta \varepsilon = \varepsilon_{max} - \varepsilon_{min} = 1.8886 \times 10^{-3}$$

Computing of mean stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{280 + 0}{2} = 140 \text{ MPa}$$

Computing of mean strain

$$\varepsilon_m = \frac{\varepsilon_{max} + 0}{2} = \frac{1.8886 \times 10^{-3} + 0}{2} = 9.443 \times 10^{-4}$$

2) Calculation for stress intensity factor range at second stage by Eq. (13)

$$K_2 = K_1 = F_2 \times \sigma \times \sqrt{\pi a_2} = 1 \times 280 \sqrt{3.1416 \times 0.001} = 15.69 (MPa\sqrt{m})$$

$$\Delta K_2 = K_{2\max} - K_{2\min} = 15.69 - 0 = 15.69 (MPa\sqrt{m})$$

Computing of mean stress intensity factor

$$K_{2m} = \frac{K_{2\max} + K_{2\min}}{2} = \frac{15.69 + 0}{2} = 7.845 (MPa\sqrt{m})$$

3) Computing for crack tip open displacement range

$\Delta\delta_i$ and mean δ_m at second stage by Eq. (15)

$$\Delta\delta_i = \frac{0.5\pi \times \sigma_s \times a_2 \times \left(\frac{\sigma_{\max} - \sigma_{\min}}{\sigma_s} + 1 \right)}{E} = \frac{0.5\pi \times \sigma_s \times a \times \left(\frac{280 - 0}{2 \times 361} + 1 \right)}{E} = \frac{0.5 \times 3.1416 \times 361 \times 0.001 \times \left(\frac{280}{2 \times 361} + 1 \right)}{200000} = 3.935 \times 10^{-6} \text{ m}$$

$$\delta_m = \frac{0.5\pi \times \sigma_s \times a \times \left(\frac{\sigma_{\max} + \sigma_{\min}}{\sigma_s} + 1 \right)}{E} = \frac{0.5 \times 3.1416 \times 361 \times 0.001 \times \left(\frac{280 + 0}{2 \times 361} + 1 \right)}{200000} = 3.935 \times 10^{-6} \text{ m}$$

4) According to Eq. (10), derive the critical stress-strain factor Q_{2c} and its critical size a_{2c} for 16MnR, $\sigma'_f = 947.1 \text{ MPa}$, $E = 200,000 \text{ MPa}$, so its critical Q_{2c} and a_{2c} are as below:

$$Q_{2c} = K_{1c} \times \delta_c = (\sigma_{fc} \sqrt{\pi a_{2c}}) \times (\pi \times a_{2c} \times \sigma_f / E) = 947.1 \sqrt{3.1416 \times a_{2c}} \times (\pi \times a_{2c} \times \sigma_f / E) = 92.7 \times 0.00018 = 0.016686 \text{ MPa} \cdot \text{m}^{3/2}$$

$$a_{2c} = \left(\frac{Q_{2c} \times E}{\sigma_{fc}^2} \right)^{2/3} / \pi = \left(\frac{0.016686 \times 200000}{(947.1)^2} \right)^{2/3} / 3.1416 = 7.6455 \times 10^{-3} \text{ m}$$

• Estimation for life at the first stage

By example mentioned above, take stress concentration coefficient $K_t = 3$, $\varphi = 0.51$.

According to Eqs. (4) and (5), calculate comprehensive material constant,

$$v_f = \ln[1/(1-\varphi)] = \ln[1/(1-0.51)] = 0.7133, (\text{damage} - \text{unit} - \text{number/cycle})$$

$$A_1^* = 2[4(\sigma'_f - \sigma_m)(\varepsilon'_f - \varepsilon_m)]^{\frac{-m_1 m_2}{m_1 + m_2}} \times v_f = 2[4(947.1 - 140) \times (0.464 - 9.443 \times 10^{-4})]^{\frac{-9.009 \times 1.8536}{9.009 + 1.8536}} \times 1 \times 10^{-3} \times 0.7133 = 1.92^{-8}, (MPa) \frac{-m_1 m_2}{m_1 + m_2} \times m / \text{cycle}$$

By Eq. (6), estimate the life N_{1tr} from short crack $a_{01} = 0.00036 \text{ m}$ to transitional size $a_{tr} = a_{mac} = 0.001 \text{ m}$ at the first stage,

$$N_{1tr} = \frac{\ln a_{01} - \ln a_{01}}{\frac{-m_1 m_2}{m_1 + m_2} \times v_f (\ln a_{mac} - \ln a_{01}) (K_t \Delta \varepsilon \cdot \Delta \sigma)^{\frac{m_1 m_2}{m_1 + m_2}}} = \frac{\ln 0.001 - 0.00036}{1.92 \times 10^{-8} (1.8886^{-3} \times 280)^{\frac{9.009 \times 1.8536}{9.009 + 1.8536}}} = \frac{1.0216}{1.92 \times 10^{-8} (3 \times 0.528808)^{1.5374}} = 26183852 (\text{cycle})$$

If adopt damage variable D_1 to calculate damage life at the first stage, then by Eq. (8), it is:

$$N_{1tr} = \frac{\ln D_{01} - \ln D_{01}}{\frac{-m_1 m_2}{m_1 + m_2} \times v_f (\ln D_{mac} - \ln D_{01}) (K_t \Delta \varepsilon \cdot \Delta \sigma)^{\frac{m_1 m_2}{m_1 + m_2}}} = \frac{\ln 0.001 - \ln 0.00036}{1.92 \times 10^{-8} (3 \times 1.8886 \times 10^{-3} \times 280)^{\frac{9.009 \times 1.8536}{9.009 + 1.8536}}} = \frac{1.0216}{1.92 \times 10^{-8} (1.586)^{1.5374}} = 26183852 (\text{cycle})$$

• Estimation for life at the second stage

In Table 1, take $v_{pv} = 2 \times 10^{-7} (m / \text{cycle})$, calculating for comprehensive material constant by Eq. (17-1),

$$A_2^* = 2[4(K_{2c} - K_{2m})(\delta_{2c} - \delta_m)]^{\frac{-m_2 \delta_2}{m_2 + \delta_2}} \times v_{pv} = 2[4(92.7 - 7.845)(0.00018 - 3.935 \times 10^{-6})]^{\frac{3.91 \times 2.9}{3.91 + 2.9}} \times 2 \times 10^{-7} = 4.358 \times 10^{-5}, (MPa\sqrt{m} \cdot m)^{\frac{m_2 \delta_2}{m_2 + \delta_2}} \cdot m / \text{cycle}$$

According to Eq. (22), estimate its growth life at the second stage from long crack $a_{02} = 0.001 \text{ m}$ to critical size $a_{2jc} = 7.6455 \times 10^{-3} \text{ m}$,

$$N_{2oj} = \frac{\frac{4(m_2 + \lambda_2)}{4m_2 + 4\lambda_2 - 6m_2\lambda_2} \left(\frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} - \frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} \right)}{2[4(K_{2c} - K_{2m})(\delta_{2c} - \delta_m)]^{\frac{-m_2 \delta_2}{m_2 + \delta_2}} \times v_{pv} \times [K_t 0.5 \Delta \sigma \times \sigma_s \times \pi^{\frac{3}{2}} (\Delta \sigma / 2 \sigma_s + 1) / E]^{\frac{m_2 \delta_2}{m_2 + \lambda_2}}} = \frac{4(3.91 + 2.9)}{4 \times 3.91 + 4 \times 2.9 - 6 \times 3.91 \times 2.9} \times \frac{3.91 \times 2.9}{3.91 + 2.9} \times \frac{2[4(92.7 - 7.845)(0.00018 - 3.935 \times 10^{-6})]^{\frac{3.91 \times 2.9}{3.91 + 2.9}} \times 2 \times 10^{-7}}{\left(\frac{0.0076455}{2(3.91 + 2.9)} - 0.001 \right)^{\frac{2 \times 3.91 + 2 \times 2.9 - 3 \times 3.91 \times 2.9}{2(3.91 + 2.9)}}} \times \frac{3.91 \times 2.9}{\{3 \times 0.5 \times 280 \times 361 \times \pi^{\frac{3}{2}} [280 / (2 \times 361) + 1] / 200000\}^{\frac{3.91 \times 2.9}{3.91 + 2.9}}} = \frac{-0.66775(0.0076455)^{-1.4975} - 0.001^{-1.4975}}{4.358 \times 10^{-5} \times \{3 \times 1.9528195\}^{1.665}} = \frac{19768}{8.2727 \times 10^{-4}} = 23895463, (\text{cycle})$$

If adopting damage variable D_2 to calculate damage life, according to Eq. (33), it should be:

$$N_{2oj} = \frac{\frac{4(m_2 + \lambda_2)}{4m_2 + 4\lambda_2 - 6m_2\lambda_2} \left(\frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} - \frac{2m_2 + 2\lambda_2 - 3m_2\lambda_2}{2(m_2 + \lambda_2)} \right)}{2[4(K'_{2c} - K'_{2m})(\delta'_{2c} - \delta'_m)]^{\frac{-m_2 \delta_2}{m_2 + \lambda_2}} \times v_{pv} \times [K_t 0.5 \Delta \sigma \times \sigma_s \times \pi^{\frac{3}{2}} (\Delta \sigma / 2 \sigma_s + 1) / E]^{\frac{m_2 \delta_2}{m_2 + \lambda_2}}} = \frac{4(3.91 + 2.9)}{4 \times 3.91 + 4 \times 2.9 - 6 \times 3.91 \times 2.9} \times \frac{3.91 \times 2.9}{3.91 + 2.9} \times \frac{2[4(92.7 - 7.845)(0.00018 - 3.935 \times 10^{-6})]^{\frac{3.91 \times 2.9}{3.91 + 2.9}} \times 2 \times 10^{-7}}{\left(\frac{0.0076455}{2(3.91 + 2.9)} - 0.001 \right)^{\frac{2 \times 3.91 + 2 \times 2.9 - 3 \times 3.91 \times 2.9}{2(3.91 + 2.9)}}} \times \frac{3.91 \times 2.9}{\{3 \times 0.5 \times 280 \times 361 \times \pi^{\frac{3}{2}} [280 / (2 \times 361) + 1] / 200000\}^{\frac{3.91 \times 2.9}{3.91 + 2.9}}} = \frac{-0.66775(0.0076455)^{-1.4975} - 0.001^{-1.4975}}{4.358 \times 10^{-5} \times \{3 \times 1.9528195\}^{1.665}} = \frac{19768}{8.2727 \times 10^{-4}} = 23895463, (\text{cycle})$$

- Estimation for lifetime in whole process

Life in whole process is:

$$\Sigma N = N_1 + N_2 = 26183852 + 23895463 = 50079315 (\text{cycle})$$

It should be point here making calculating process, it is only for providing some calculation methods, is only for to verify consistency between Eqs. (17-1) and (17-2), where the lifetime is calculated by critical value K_{lc} and δ_c . For the sake of safety, it must be predicted according eqn. (24) by the effective values K_{2eff} and δ_{2eff} , and have to consider by the safety coefficient of 20 times to decrease calculated lifetime value ΣN .

5. Discussions

(1) About some viewpoints among disciplines intersecting relations and about relationship between varied mathematical models: In the traditional material mechanics and the continuum damage mechanics, researched for the strength and service life, they are as homogeneous and continuous media to materials; in macro-, micro- and nano-mechanics, they are as local problems researched to those defects (cracks). But because of the defects sizes, locations and undergone loading conditions are different, studying problems among the branch disciplines are both having difference, and having similar links. In this paper, many of the calculating models, calculating parameters, variables, dimensional units, consist of coordinate system in route figure, and relationships between curves are all based on the ideas mentioned above, thereby put forward the theoretical analysis, description methods and calculation models for them.

(2) About calculable viewpoint for the new material constant A_2^* . In present paper mention to the calculable comprehensive material constants A_2^* , though that must be to obtain from experiment, author discovers, it is also calculable one, where except for above mentioned method of Eq. (4), also use the yield stress $\sigma_s(\sigma_y)$ and elastic modulus E in the traditional material mechanics, the material constants $\sigma_f', m_2, \lambda_2$ in fatigue-damage-fracture mechanics. Such as:

$$A_2^* = (4Q_{2c})^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \times (a_{2fc} - a_{02}) / (N_{2fc} - N_{02}) \quad (41)$$

For $\sigma < \sigma_s$ and under unsymmetrical cycle, the comprehensive constant A_2^* is expressed to use as following forming:

$$A_2^* = \{4[\frac{\sigma_s^2}{E}(\sqrt{\pi a_{2c}})^3](1 - \sigma_m / \sigma_{fc})\}^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v_{pv} \quad (42)$$

For $\sigma \geq \sigma_s$, it can also adopt Eq. (17-2) to calculate the A_2^* value, here we can check their consistency by means of above example, e.g., A_2^* is as below form:

$$\begin{aligned} A_2^* &= \{4[\frac{0.5 \times \sigma_{fc}' \cdot \sigma_s(\sigma / \sigma_s + 1)}{E}(\sqrt{\pi a_{2c}})^3](1 - \sigma_m / \sigma_{fc})\}^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v_{pv} \\ &= \{4[\frac{0.5 \times 947.1 \times 361(947.1/361 + 1)}{200000}(\sqrt{\pi 0.007655})^3](1 - 140/947.1)\}^{\frac{3.91 \times 2.9}{3.91+2.9}} \times 2 \times 10^{-7} \\ &= \{4[3.09725 \times 3.72944 \times 10^{-3}](1 - 140/947.1)\}^{-1.665} \\ &\times 2 \times 10^{-7} = 4.3652 \times 10^{-5}, [m^{3/2} \cdot MPa]^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \times m / cycle \end{aligned}$$

The calculation result is consistent with above mentioned A_2^* (17-1) in the example, which are as follow,

$$\begin{aligned} A_2^* &= 2[4(K_{2c} - K_{2m})(\delta_{2c} - \delta_m)]^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v_{pv} \\ &= 2[4(92.7 - 7.85)(0.00018 - 3.935^{-6})]^{\frac{3.91 \times 2.9}{3.91+2.9}} \times 2 \times 10^{-7} \\ &= 2[0.058476, 0.059756]^{-1.665} \times 2 \times 10^{-7} = 4.358^{-5}, [m^{3/2} \cdot MPa]^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \times m / cycle \end{aligned}$$

6. Conclusions

(1) About relations between the material properties and the mathematical models: Consider double characters of elastic-plastic materials—both having elastic properties, and plastic one, therefore, the mathematical expressions described for their behavior should also be both to contain the stress component, and the strain component, e.g., for the model “ $\Delta Q_1 = (\Delta \varepsilon \cdot \Delta \sigma) \cdot a_1^{1/(m_1+m_1')}$ ”, it is as the driving force for short-crack growth; and for the model “ $\Delta Q_2 = \Delta \delta_i \cdot \Delta K_I$ ” it is as driving force for the long-crack growth, their inside parameters consisting are all to have corresponding consistency with the material properties.

(2) About the relations between the mathematical models and damage growth processes: under identical loading, the differences between the mathematical models to describe the materials behaviors are due to make change of the stiffness of materials, in that they

are difference of degrees of undergone damages at varied stages. Therefore, the models of driving forces are become from the $Q_1(Q_1)$ at crack forming stage to the $Q_2(Q_2)$ at crack growth stage; the exponents in equation also are become from the $\frac{m_1 \cdot m'_1}{m_1 + m'_1}$ to the $\frac{m_2 \cdot \lambda_2}{m_2 + \lambda_2}$; the comprehensive materials constants are become from the A_1^* to the A_2^* ; the slopes of the curves at each stages also are also changed.

(3) About their physical and geometrical meanings of parameters A_1^* and A_2^* : The parameters A_1^* at the first stage and A_2^* at the second stage, they are calculable comprehensive materials constants to have functional relations with other parameters. Their physical meanings of the A_1^* and A_2^* are a concept of power, are an energy gave out from material's resistance, just are a maximal increment value paying energy before causing failure in one cycle. Their geometrical meanings are a maximal micro-trapezia area approximating to beeline, also are an intercepts on the y-axis.

(4) About conversion regulations on variables, dimensions and units: inside mathematical models to convert crack variable a into damage variable D , it must define "1 mm-crack-length" equivalent to 'one-damage-unit', "1 m-crack-length" equivalent to '1,000-damage-unit', this is a key for making linking between the damage mechanics and the fracture mechanics.

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develop the fatigue-damage-fracture mechanics subjects, gain huge benefits for accident analysis, safety design and operation for which are mechanical equipment in engineering fields. Particularly should explain that author cannot have so many of discovery and provide above the calculable mathematical models and the combined Fig. 1, if have no their research results and supports.

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