Computational Study of the A_0 Scattered Field Due to an Edge Delamination

Nithurshan Nadarajah¹, Benjamin Steven Vien¹, Wing Kong Chiu¹ and L. R. Francis Rose²

1. Department of Mechanical and Aerospace Engineering, Faculty of Engineering, Monash University, Clayton 3800, Australia

2. Aerospace Department, Defence Science & Technology Organisation, Fishermens Bend 3207, Australia

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Abstract: Delamination detection presents a pertinent problem for SHM (structural health monitoring), as in most cases, there may be no visible signs of the damage on the surface of the structure. This study investigates the scattering of a zeroth-order anti-symmetric (A_0) Lamb wave mode by an edge delamination using the commercial FE (finite element) package ABAQUS. The A_0 Lamb mode is chosen because the corresponding stress distribution is more sensitive to delamination than is the case for symmetric modes. The paper presents results for the scatter field for various angles of incidence, and for varying defect sizes. The regime of small defect size relative to the incident wavelength is of particular interest for SHM as it corresponds to early damage detection. It is shown that, in this regime the scattered field appears to originate from a point source at the origin of the delamination, and the corresponding amplitude is linearly proportional to area of the delamination. These results can be used to guide the use of Lamb waves to detect and quantify edge delamination in plate-like structures.

Key words: Structural health monitoring, Lamb waves, wave scattering, delamination damage.

1. Introduction

Ultrasonic interactions with structural defects have been investigated in many fields for NDT (non-destructive testing). By use of the fundamental properties of damage interaction such as transmission and reflection coefficients, the pulse-echo and pitch-catch methods have been refined to pick out structural defects [1-4]. The drawback with this traditional application of ultrasonic waves for damage detection is that it is often restricted to detecting damages in the immediate vicinity of the acoustic source and hence requires time-consuming scanning for wide-area inspection [5]. The use of Lamb waves is a widely studied alternative, which has the potential to scan entire plates and detect damage that is not in direct line of sight of the source [6]. Studies have shown great promise in the diagnostic ability of Lamb waves in

detecting several types of defect: cracks, corrosion and delamination damage [7].

Lamb waves are dispersive in nature and an infinite number of possible propagating modes can exist depending on the type and frequency of loading. Several considerations influence wave mode selection, such as sensitivity of the mode to a particular type of damage, the ease of generation, the dispersive nature of the mode and resolution [8-10]. The forward problem of detecting the damage using a selected input wave has been extensively studied in literature [11, 12]. The practical application of Lamb waves in SHM (structural health monitoring) also requires the study of the inverse problem, i.e., to estimate the damage size based on measurements of the scattered field.

This paper presents a computational study of the forward problem of A_0 wave scattering by an edge delamination in a plate. For simplicity, the plate is modeled as being isotropic, with the material properties of aluminium, and mid-plane delamination is assumed.

Corresponding author: Nithurshan Nadarajah, Ph.D., research field: structural health monitoring. E-mail: Nithurshan.nadarajah@monash.edu.

The frequency is selected below the A_1/S_0 cut off, so that the only propagating modes are A_0 , S_0 and SH_0 [7]. The A_1 cut-off in aluminium is obtained from DISPERSE [13] as 1.53 MHz-mm. A value of 600 kHz-mm was selected as a trade-off between a sufficiently short wavelength, for enhanced resolution, and avoiding excitation of higher order modes.

The scattered field amplitude was studied in the Rayleigh limit of defect size, i.e., defect size smaller than interrogating wavelength, as a function of the angle of incidence and the delamination size. The study will establish a relationship between the damage size and the SDP (scattered directivity pattern) amplitude and propose a solution to the inverse problem of damage size characterisation using the information from the scatter field. This study also investigates the possibility of using a point source at the delamination origin to model the scattered field.

2. Model and Methodology

The FE (finite element) package ABAQUS was used for the simulations.

A rectangular plate was modeled with the dimensions shown in Fig. 1a. The plate contains a semi-circular mid-plane edge delamination, indicated by the shaded area in Fig. 1a. The centre of the delamination is taken as the origin of coordinates, Fig. 1b shows the locations for the loading and measurement points for the output variables analysed. The input signal for the applied load is a 7-cycle Hann-windowed tone burst, with a center frequency of 200 kHz. At this center frequency the wavelength (λ), of the A_0 wave in the given plate is 10 mm, as determined from Ref. [13]. Conventionally, a 5-cycle tone burst is used in Lamb wave applications [14]. The purpose of the increased number of cycles is to reduce the bandwidth and hence the dispersion effects of the generated A_0 mode. The input loading was applied as a vertical point force on the top and bottom surface of the plate at point P, with polar coordinates (R_l, θ^l) , $R_l = 10 \times \lambda$ and θ^l depends on the specific angle of incidence

Fig. 1 Geometrical details of the model setup.

studied. The choice of the 10λ for R_l was decided in order to approximate a plane wave incidence at the location of the delamination. The large radius of curvature of the wave front on incidence with the delamination ensures that we closely model a plane wave despite having a cylindrical source.

The resulting vertical displacement $u_z(x, t)$ is measured at point Q with polar coordinates (R_s, θ^s) , $R_s = 4\lambda$. The choice of the 4λ for R_s was to avoid non propagating edge waves that are generated on reflection from a free surface [15]. The response is recorded for a time interval of 50 μ s, between 50 μ s and 100 μ s of the simulation, which was designed to capture the scattered signal from the defect, while avoiding the incident wave field and reflections from the plate boundaries (other than the edge along $y = 0$).

The development of the scatter field necessitates defining two fields, the total field which includes the scatter both from the free edge and delamination and the baseline field which includes purely the reflection from the edge. The scatter field can be mathematically defined as:

$$u_z^{Scatter}(x, t) = u_z^{Total}(x, t) - u_z^{Baseline}(x, t) \quad (1)$$

The scatter displacement information is used to

work out the modal content of the scatter field and the scatter pattern. Only the out of plane displacement was used in all analysis as this is the dominant direction of particle vibration in A_0 propagation. A 2D-FFT (fast Fourier transform) method was used to identify the modal content of the scatter field [16]. The FFT was done along the line shown in Fig. 1b. The starting point of the FFT had polar coordinates $(4\lambda, 135^\circ)$. The length of the FFT scan length was 140 mm which is twice the length of the input wave train. Other 2D FFT requirements of having evenly spaced temporal and spatial records have been ensured by defining the meshing and time output requirements appropriately.

The scatter pattern was obtained by analyzing the time series at nodes closest to the arc defined by Q. A Hilbert transform was applied on the section of the time series selected and its peak was used to indicate the amplitude. A simple normalization technique was employed to remove the dependence of the scattered wave amplitude on the source location R_I . The incident wave amplitude at a distance of 10λ from the source was determined using the Hilbert transform, as

indicated in Fig. 2. The scattered field amplitudes were divided by this incident wave amplitude to produce results that can be interpreted as the scattered field amplitudes due to an incident plane wave of unit amplitude. These normalized results are presented below.

The specimen was meshed using structured elements while maintaining the aspect ratio very close to 1 in all elements. Convergence tests were done to prove that four elements in the through thickness region provides sufficient accuracy in modeling for the given loading frequency-thickness product. A mesh density of ten elements per wavelength was used in the propagation direction and the requested time step size in the simulation was well below $0.8L/c$, where L is the smallest spacing between any two nodes in the model and c is the fastest propagating wave speed [17].

The applied material properties in our model were those of aluminium listed in Table 1.

The FE model was made of two identical plates of 1.5 mm thickness with the nodes tied together across the mid-plane to generate the baseline data. The nodal

Fig. 2 Schematic of how the normalization factor is obtained.

Table 1 Material properties of aluminium used in simulation.

Young's modulus/(GPa)	Poisson's ratio	Density/(kg·m ⁻³)
69	0.33	2,700

constraints were then removed for all nodes within the selected delamination zone to generate the total field [15], and hence to obtain the scattered field by subtraction, as indicated in Eq. (1).

Prior to the wave scattering calculations, the FE model's performance was assessed by generating a phase velocity dispersion curve, using the 2D-FFT approach [16], and comparing the result with that obtained from Ref. [13]. It can be seen from Fig. 3 that the two dispersion curves are in very good agreement, with an error below 3%.

The following three problems were investigated:

- (1) Effect of delamination size on scatter pattern;
- (2) Effect of angle of incidence on the scatter pattern;
- (3) Using force doublet to model the scattered field.

3. Results

The first step in analyzing our results was a 2D-FFT to identify the modes contributing towards the scatter patterns. Fig. 4 shows a representative result obtained from this analysis. As expected, given the mid-plane symmetric conditions of our model and the anti-symmetric loading, the only propagating mode was A_0 .

3.1 Effect of Delamination Size on Scatter Pattern

Fig. 5 shows a series of scatter patterns from

Fig. 3 Comparison of analytical phase velocity dispersion curve and to the phase velocity obtained from FE.

Fig. 4 2D-FFT result identifying A_0 as the only propagating Lamb mode.

delaminations of increasing radius a . The angle of incidence θ^i in all cases was maintained at 45° . The normalized scatter pattern shows that the general pattern is maintained for delamination radius up to 2 mm. Beyond this damage size, the forward scatter (i.e., for $90^\circ < \theta^s < 180^\circ$, cf. Fig. 1b) dominates in comparison to the back scatter ($0^\circ < \theta^s < 90^\circ$).

A ratio of the forward and back scatter was studied as a function of the delamination size a to evaluate the range in which the damage can be assumed to generate a scatter field which is similar in nature to a wave field generated by a point source. For the purposes of our calculation, the forward scatter amplitude was obtained at a θ^s value of 180° and the back scatter was obtained at a θ^s value of 0° . In both cases, the R_S value was chosen to be 4λ . The result in Fig. 6 indicates that as the damage size/wavelength ratio increases beyond a value of 0.25 the forward to back scatter ratio has a sharp increase in value. Below this range, the forward to back scatter ratio is almost constant and tends to the value 1.

3.2 Effect of Angle of Incidence on the Scatter Pattern

To investigate the effect of angle of incidence on the scatter field, a delamination of size $a = 2$ mm was chosen and the angle of incidence was varied, with the

Fig. 5 Normalised scatter pattern for increasing delamination radius a .

results shown in Fig. 7. At an angle of incidence of 90° , the scatter pattern is symmetrical about the line of symmetry of the model. For all other angles, there is a slight bias of the scatter pattern in the forward

Fig. 6 Forward to back scatter ratio obtained as a function of damage size (a) : wavelength ratio.

Fig. 7 Effect of angle of incidence on the scatter pattern. Incidence angle θ^i : (a) 90° ; (b) 60° ; (c) 45° and (d) 30° .

direction. Because our interest lies in comparing the scatter patterns, and not so much in the scatter amplitude, the results in each plot have been normalized relative to the maximum amplitude in that plot.

The ratio of the stresses $\sigma_{zx}:\sigma_{zy}$ measured at the delamination location for the various angles of incidence of 90° , 60° , 45° and 30° is 0.05, 14.32, 35.30 and 174.00, respectively.

3.3 Use of Force Doublets to Model the Scatter of A_0 Incidence

Force doublets were used to model the scatter field from a delamination of size $a = 2$ mm. Principle of

superposition is used in determining the equivalent point source. The baseline stress distribution at the delamination is used to determine the tractions that need to be cancelled to result in the stress free surface of the delamination face using the following equation:

$$T_i = \sigma_{ij}n_j \quad (2)$$

According to the coordinate system defined in Fig. 1, the z vector, $[0,0,1]$ is normal to the delamination surface. Thus, the only stress components that need to be evaluated at the delamination to generate the scatter using traction doublet are σ_{zx} , σ_{zy} and σ_{zz} . For the coordinate system established in Fig. 1, there is no σ_{zz} stress component for the propagating A_0 mode at the mid-plate position of the delamination. Thus, it is expected that the shear components will have the largest contribution to the scatter field. This measured traction is then applied as complementary force doublet pairs at both faces of the delamination at its origin and then the resulting field is compared with the field obtained from baseline subtraction.

Force doublets were employed to model the scatter field generated by all studied angles of incidence. The scatter pattern generated by the force doublets were normalized by the absolute maximum amplitude. By visual comparison of the results in Fig. 8, we notice that the scatter pattern generated by the force doublets show no bias in the forward scatter region which is unlike the case in scatter pattern from baseline subtraction.

4. Discussion

4.1 Effect of Delamination Size on Scatter Pattern

The results shown in Fig. 6 indicate how the forward and backscatter changes as the delamination size is changed. The convergence of the forward and back scatter to the same value indicates that the wave interaction up to a ratio of delamination radius (a) to wavelength of 0.25 ($d_{\text{radius}}/\lambda_{A0} = 0.25$) conforms to expectations for the Rayleigh scattering limit. For comparison of the effect of delamination size on the scatter amplitude the back scatter at (R_s, θ^s) , 4λ and 0°

Fig. 8 Comparison of the scatter pattern generated by force doublets to at various angle of attack to the scatter pattern obtained by baseline subtraction. (Angle of incidence θ^i : (a) 90° ; (b) 60° ; (c) 45° and (d) 30°).

Fig. 9 Trend in back scatter amplitude as a function of (delamination radius: wavelength) 2 .

is used. Fig. 9 compared this back scatter amplitude as a function of (delamination radius (a)/ wavelength) 2 . This is seen to have an almost linear trend. Our analysis shows that the line of best fit had an R^2 value of 0.9999. This shows that the strength of the traction doublet must be scaled as a ratio of delamination radius 2 to mimic the scatter pattern of a specific delamination size within the limit shown in Fig. 6.

This linear trend in relation to a^2 indicates that the scatter strength is a function of the delamination area. The quasi-static nature of the stress field associated

with the A_0 mode opening of the delamination can best explain this trend [18].

4.2 Effect of Angle of Incidence on Scatter Pattern

The changes in scatter pattern shown at various angles of incidence can be attributed to the ratio of the shear stress acting at the delamination location. This variation in the relative amplitude of the two shear stress component can explain the changes in scatter pattern. This is verified during our application of force doublets, as changing the relative amplitudes of the traction in various directions closely models the scatter pattern for that given angle of attack.

4.3 Use of Force Doublets to Model the Scatter of A_0 Incidence

The force doublets to model the scatter pattern were determined using the principle of super position. Our studies show that this method of approximating scatter field applies below the limit $a/\lambda = 0.25$. This is because in principle we are trying to use our force doublet to cancel off the stresses at the delamination location of the baseline structure. As the delamination size gets larger the stress distribution is not uniform across the delamination face and a single force is no longer able to correctly cancel the stresses.

5. Conclusions

It has been shown that the scattering amplitude of the A_0 Lamb wave diffracted by a delamination is linearly proportional to the area of the delamination for small damage size. Furthermore, in this range, it has been shown that the scattered field can be closely modeled by an appropriate distribution of point forces acting at the origin of the delamination. The correct strength for these forces can be deduced from the traction across the delamination face in the baseline structure, in accordance with the principle of superposition. These results provide a useful basis for the inverse problem of determining the delamination size from measurements of the scattered field.

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