Preliminary Test Difference Estimator When two Prior Values of Population Mean (μ_x) of an Auxiliary Variable (X) are Available

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A preliminary test difference estimator (PTDE) of population mean μ_y of the variable Y of interest is developed in the present paper when two prior values of the population mean μ_x of an auxiliary variable X linearly related to Y are available, i.e. $\mu_x = \mu_1$ or $\mu_x = \mu_1$ ($\mu_1 > \mu_0$). The proposed (PTDE) has been found in general more efficient than the usual option of difference estimators if μ_x in the estimators are replaced by μ_1 or μ_0 , the prior values of μ_x , when an investigator does not desire to resort for double sampling. Numerical findings also support the theoretical results.

Key words: Preliminary test difference estimator, prior values, double sampling, regression estimator

Introduction

An usual difference estimator of finite population mean $\bar{Y}$ of the variable Y of interest is given by

$$\bar{y}_d = \bar{y} + \beta_0 (\mu_x - \bar{x}) \quad (1.1)$$

where $\bar{y}$ and $\bar{x}$ are sample means of Y and X, respectively based on a random sample of size n drawn by simple random sampling without replacement from the population of size N, μ_x is known population mean of X, an auxiliary variable linearly related to Y. An important feature of difference estimator is that it is unbiased. The variance of $\bar{y}_d$ is given by

$$V(\bar{y}_d) = \frac{\sigma^2}{n} (1 - \rho^2) (1 + \delta^2) \quad (1.2)$$

Where $\delta = \frac{\rho}{\sqrt{1 - \rho^2}} (1 - K)$, $K = \frac{\beta_0}{\beta}$ and β is regression coefficient of Y on X in the population and ρ is coefficient of correlation coefficient between Y and X. If $\beta_0 = \beta$, then (1.2) reduces to

$$V(\bar{y}_d) = \frac{\sigma^2}{n} (1 - \rho^2) \quad (1.3)$$

which is the variance of the usual regression estimator up to first order of approximation (Sukhatme et al, 1970). If on the basis of past experience β_0 is good guessed value of β , then the difference estimator and regression estimator both may perform almost equally. When μ_x is not known, a double sampling difference estimator is generally employed (Chochran, 1977). Some authors in the past have dealt with preliminary test estimators in double sampling (see, Han, 1973, Sisodia and Srivastava, 1982, Das & Bez, 1995). However, on the basis of

past experience if an investigator expects that either $\mu_x = \mu_0$ or $\mu_x = \mu_1$ ($\mu_1 > \mu_0$) then difference estimator (1.1) reduces to

$$T_1 = \bar{y} + \beta_0(\mu_0 - \bar{x}), \text{ if } \mu_x \text{ is replaced by } \mu_0 \quad (1.4)$$

$$T_2 = \bar{y} + \beta_0(\mu_1 - \bar{x}), \text{ if } \mu_x \text{ is replaced by } \mu_1 \quad (1.5)$$

For example, in estimation of crop yield, area under the crop can be an auxiliary variable. Area under the crop is being reported by Remote Sensing as well as by Revenue Department of State Government, and both differ in general. These can be considered as two prior values of actual area under the crop.

The estimator T_1 and T_2 are biased and their biases are obtained as

$$B(T_1) = K\rho \frac{\sigma_y}{\sqrt{n}} \Delta_0 \quad (1.6)$$

and

$$B(T_2) = K\rho \frac{\sigma_y}{\sqrt{n}} \Delta_1 \quad (1.7)$$

$$\text{where } \Delta_0 = \frac{\mu_0 - \mu_x}{\sigma_x / \sqrt{n}}, \quad \Delta_1 = \frac{\mu_1 - \mu_x}{\sigma_x / \sqrt{n}}$$

The mean square error (MSE) of T_1 can be obtained as

$$\begin{aligned} \text{MSE}(T_1) &= V(T_1) + [B(T_1)]^2 \\ &= E(T_1^2) - [E(T_1)]^2 + [B(T_1)]^2 \end{aligned} \quad (1.8)$$

On simplification, we get

$$\text{MSE}(T_1) = \frac{\sigma_y^2}{n} (1 - \rho^2) (1 + \delta^2) + \frac{\sigma_y^2}{n} K^2 \rho^2 \Delta_0^2 \quad (1.9)$$

Similarly, the MSE of T_2 is obtained as

$$\text{MSE}(T_2) = \frac{\sigma_y^2}{n} (1 - \rho^2) (1 + \delta^2) + \frac{\sigma_y^2}{n} K^2 \rho^2 \Delta_1^2 \quad (1.10)$$

It is evident from the expression (1.1), (1.9) and (1.10) that $\text{MSE}(T_1) = \text{MSE}(T_2) = V(\bar{y}_d)$ when $\Delta_0 = 0$ and $\Delta_1 = 0$, respectively, that means $\mu_0 = \mu_1 = \mu_x$. However, it will be worthwhile to test the hypothesis $H_0: \mu_x = \mu_0$ against $H_1: \mu_x = \mu_1$ to decide whether μ_0 or μ_1 should be used in place of μ_x in the usual difference estimator (1.1). The present study is an attempt to develop a preliminary test difference estimator (PTDE) of μ_y after a preliminary test of significance of the hypothesis $H_0: \mu_x = \mu_0$ against the hypothesis $H_1: \mu_x = \mu_1, \mu_1 > \mu_0$. The properties of preliminary test difference estimator is presented in section 3 and 4. Numerical findings for its relative efficiency and conclusion are given in section 5.

Development of Preliminary Test Difference Estimator (PTDE)

Let (X, Y) have a bivariate normal distribution with means μ_x and μ_y and variances $\sigma_x^2 = \sigma_y^2$ of x and y, respectively. A random sample of size n is drawn from this bivariate normal population by simple random sampling without replacement. Let $\bar{x}$ and $\bar{y}$ be the sample means of x and y, respectively. We need to test the hypothesis

$$\left. \begin{array}{l} H_0 : \mu_x = \mu_0 \\ H_1 : \mu_x = \mu_1 \end{array} \right\}, \text{ where } \mu_1 > \mu_0 \quad (2.1)$$

On the basis of sample mean $\bar{X}$, the following test statistics was given by Pandey and Singh (1978) to test the hypothesis (2.1).

When $\bar{X} \leq \frac{\mu_0 + \mu_1}{2}$, then null hypothesis H_0 is accepted and when $\bar{X} > \frac{\mu_0 + \mu_1}{2}$, then alternative hypothesis H_1 is accepted.

On the basis of test of significance of the hypothesis (2.1), the preliminary test difference estimator (PTDE) of μ_y is developed as follows

$$\hat{\mu}_{yd} = \begin{cases} T_1 = \bar{y} + \beta_0 (\mu_0 - \bar{x}), & \text{when } H_0 \text{ is accepted} \\ T_2 = \bar{y} + \beta_0 (\mu_1 - \bar{x}), & \text{when } H_1 \text{ is accepted} \end{cases} \quad (2.2)$$

Expectation and Mean Square Error of PTDE $(\hat{\mu}_{yd})$

Taking the expectations of $(\hat{\mu}_{yd})$

$$\begin{aligned} E(\hat{\mu}_{yd}) &= E[\{\bar{y} + \beta_0 (\mu_0 - \bar{x})\} | H_0] P_r(H_0) + E[\{\bar{y} + \beta_0 (\mu_1 - \bar{x})\} | H_1] P_r(H_1) \\ &= E\left(\bar{y} \middle| \bar{x} \leq \frac{\mu_0 + \mu_1}{2}\right) P_r\left(\bar{x} \leq \frac{\mu_0 + \mu_1}{2}\right) + \beta_0 E\left\{(\mu_0 - \bar{x}) \middle| \bar{x} \leq \frac{\mu_0 + \mu_1}{2}\right\} P_r\left(\bar{x} \leq \frac{\mu_0 + \mu_1}{2}\right) \\ &\quad + E\left(\bar{y} \middle| \bar{x} > \frac{\mu_0 + \mu_1}{2}\right) P_r\left(\bar{x} > \frac{\mu_0 + \mu_1}{2}\right) + \beta_0 E\left\{(\mu_1 - \bar{x}) \middle| \bar{x} > \frac{\mu_0 + \mu_1}{2}\right\} P_r\left(\bar{x} > \frac{\mu_0 + \mu_1}{2}\right) \\ E(\hat{\mu}_{yd}) &= \mu_y + K\rho \frac{\sigma_y}{\sqrt{n}} \Delta_1 + K\rho \frac{\sigma_y}{\sigma_x} [(\mu_0 - \mu_x) - (\mu_1 - \mu_x)] \Phi(a) \\ &= \mu_y + K\rho \frac{\sigma_y}{\sqrt{n}} [\Delta_1 - (\Delta_0 - \Delta_1) \Phi(a)] \end{aligned} \quad (3.1)$$

where $a = \frac{\mu_0 + \mu_1 - 2\mu_x}{2\sigma_x/\sqrt{n}}$ and $\Phi(\cdot)$ is cumulative distribution function of standard normal variate.

The bias of $\hat{\mu}_{yd}$ is therefore, given by

$$\begin{aligned} B(\hat{\mu}_{yd}) &= E(\hat{\mu}_{yd} - \mu_y) \\ &= K\rho \frac{\sigma_y}{\sqrt{n}} [\Delta_1 - (\Delta_0 - \Delta_1) \Phi(a)] \end{aligned} \quad (3.2)$$

It can be observed from the above expression that if $\mu_x = \mu_0 = \mu_1$ then $\Delta_1 = \Delta_0 = 0$ and $B(\hat{\mu}_{yd}) = 0$. In fact $\hat{\mu}_{yd}$ reduces to $\bar{y}_d$, the difference estimator.

Mean square error of $\hat{\mu}_{yd}$ is derived as follows

$$\text{MSE}(\hat{\mu}_{yd}) = E(\hat{\mu}_{yd}^2) - 2\mu_y E(\hat{\mu}_{yd}) + \mu_y^2 \quad (3.3)$$

Deriving $E(\hat{\mu}_{yd}^2)$ on similar lines as in case of $E(\hat{\mu}_{yd})$, we have

$$\begin{aligned} E(\hat{\mu}_{yd}^2) &= E\left[\bar{y} + \beta_0(\mu_0 - \bar{x})\right]^2 \Big| H_0 \Big] P_r(H_0) + E\left[\bar{y} + \beta_0(\mu_1 - \bar{x})\right]^2 \Big| H_1 \Big] P_r(H_1) \\ &= E\left(\bar{y}^2 \Big| \bar{x} \leq \frac{\mu_0 + \mu_1}{2}\right) P_r\left(\bar{x} \leq \frac{\mu_0 + \mu_1}{2}\right) + \beta_0^2 E\left\{(\mu_0 - \bar{x})^2 \Big| \bar{x} \leq \frac{\mu_0 + \mu_1}{2}\right\} P_r\left(\bar{x} \leq \frac{\mu_0 + \mu_1}{2}\right) \\ &\quad + 2\beta_0 E\left\{\bar{y}(\mu_0 - \bar{x}) \Big| \bar{x} \leq \frac{\mu_0 + \mu_1}{2}\right\} P_r\left(\bar{x} \leq \frac{\mu_0 + \mu_1}{2}\right) + E\left(\bar{y}^2 \Big| \bar{x} > \frac{\mu_0 + \mu_1}{2}\right) P_r\left(\bar{x} > \frac{\mu_0 + \mu_1}{2}\right) \\ &\quad + \beta_0^2 E\left\{(\mu_1 - \bar{x})^2 \Big| \bar{x} > \frac{\mu_0 + \mu_1}{2}\right\} P_r\left(\bar{x} > \frac{\mu_0 + \mu_1}{2}\right) + 2\beta_0 E\left\{\bar{y}(\mu_1 - \bar{x}) \Big| \bar{x} > \frac{\mu_0 + \mu_1}{2}\right\} P_r\left(\bar{x} > \frac{\mu_0 + \mu_1}{2}\right) \\ &= E(\bar{y}^2) - 2\beta_0 E(\bar{x}\bar{y}) + \beta_0^2 \mu_0^2 \int_{-\infty}^{\frac{\mu_0 + \mu_1}{2}} f(\bar{x}) d(\bar{x}) + \beta_0^2 \int_{-\infty}^{\frac{\mu_0 + \mu_1}{2}} \bar{x}^2 f(\bar{x}) d(\bar{x}) - 2\beta_0^2 \mu_0 \int_{-\infty}^{\frac{\mu_0 + \mu_1}{2}} \bar{x} f(\bar{x}) d(\bar{x}) \\ &\quad + 2\beta_0 \mu_0 \mu_y \int_{-\infty}^{\frac{\mu_0 + \mu_1}{2}} f(\bar{x}) d(\bar{x}) + \beta_0^2 \mu_1^2 \int_{\frac{\mu_0 + \mu_1}{2}}^{\infty} f(\bar{x}) d(\bar{x}) + \beta_0^2 \int_{\frac{\mu_0 + \mu_1}{2}}^{\infty} \bar{x}^2 f(\bar{x}) d(\bar{x}) - 2\beta_0^2 \mu_1 \int_{\frac{\mu_0 + \mu_1}{2}}^{\infty} \bar{x} f(\bar{x}) d(\bar{x}) \\ &\quad + 2\beta_0 \mu_1 \mu_y \int_{\frac{\mu_0 + \mu_1}{2}}^{\infty} f(\bar{x}) d(\bar{x}) \\ E(\hat{\mu}_{yd}^2) &= \frac{\sigma_y^2}{n} + \mu_y^2 - 2\beta_0 \left(\rho \frac{\sigma_x \sigma_y}{n} + \mu_x \mu_y \right) + \beta_0^2 \left(\frac{\sigma_x^2}{n} + \mu_x^2 \right) - 2\beta_0^2 \mu_1 \mu_x + 2\beta_0 \mu_1 \mu_y + \beta_0^2 \mu_1^2 \\ &\quad + \left(\beta_0^2 \mu_0^2 - \beta_0^2 \mu_1^2 + 2\beta_0 \mu_1 \mu_y - 2\beta_0 \mu_1 \mu_x \right) \Phi(a) - 2\beta_0^2 (\mu_0 - \mu_1) \left(-\frac{\sigma_x}{\sqrt{n}} \varphi(a) + \mu_x \Phi(a) \right) \end{aligned} \quad (3.4)$$

Where $\varphi(\cdot)$ is ordinate of standard normal variate and $\Phi(a)$ is cumulative distribution function of standard normal variate and $a = \frac{\mu_0 + \mu_1 - 2\mu_x}{2\sigma_x/\sqrt{n}}$

From (3.1), (3.3) and (3.4), the MSE of $(\hat{\mu}_{yd})$ is obtained as

$$\begin{aligned} \text{MSE}(\hat{\mu}_{yd}) &= \frac{\sigma_y^2}{n} + \mu_y^2 - 2\beta_0 \left(\rho \frac{\sigma_x \sigma_y}{n} + \mu_x \mu_y \right) + \beta_0^2 \left(\frac{\sigma_x^2}{n} + \mu_x^2 \right) - 2\beta_0^2 \mu_1 \mu_x + 2\beta_0 \mu_1 \mu_y + \beta_0^2 \mu_1^2 \\ &\quad + \left(\beta_0^2 \mu_0^2 - \beta_0^2 \mu_1^2 + 2\beta_0 \mu_1 \mu_y - 2\beta_0 \mu_1 \mu_x \right) \Phi(a) - 2\beta_0^2 (\mu_0 - \mu_1) \left(-\frac{\sigma_x}{\sqrt{n}} \varphi(a) + \mu_x \Phi(a) \right) \\ &\quad + \mu_y^2 - 2\mu_y \left(\mu_y + K\rho \frac{\sigma_y}{\sqrt{n}} \{ \Delta_1 - (\Delta_0 - \Delta_1) \Phi(a) \} \right) \end{aligned}$$

afAfter little simplification, the above equation reduces to

$$\text{MSE}(\hat{\mu}_{yd}) = \frac{\sigma_y^2}{n} \left[(1 - \rho^2)(1 + \delta^2) + k^2 \rho^2 \Delta_1^2 - k^2 \rho^2 (\Delta_1^2 - \Delta_0^2) \Phi(a) - 2k^2 \rho^2 (\Delta_1 - \Delta_0) \phi(a) \right] \quad (3.4)$$

Comparison of Different Estimators

The preliminary test difference estimator $\hat{\mu}_{yd}$ is an alternative to $\bar{y}_d$, the usual difference estimator, when μ_x is not exactly known but one expects that either $\mu_x = \mu_0$ or $\mu_x = \mu_1$, where μ_0 and μ_1 ($\mu_1 > \mu_0$) are pre-assigned value based on past experience. We shall state and prove the following theorems.

Theorem 4.1

$$\text{If } a = \frac{\mu_0 + \mu_1 - 2\mu_x}{2\sigma_x/\sqrt{n}} \rightarrow \infty, \text{ then } \lim_{a \rightarrow \infty} \text{MSE}(\hat{\mu}_{yd}) = V(\bar{y}_d)$$

Proof.

For $a \rightarrow \infty$, the value of $\Phi(a) \rightarrow 1$ and $\phi(a) \rightarrow 0$. This implies that $H_0: \mu_x = \mu_0$ is always accepted with probability almost one and hence μ_0 is almost equal to μ_x indicating thereby that $\Delta_0 \cong 0$ and equation (3.5) results to

$$\lim_{a \rightarrow \infty} \text{MSE}(\hat{\mu}_{yd}) = \frac{\sigma_y^2}{n} (1 - \rho^2)(1 + \delta^2) = V(\bar{y}_d)$$

Theorem 4.2

$$\text{If } a = \frac{\mu_0 + \mu_1 - 2\mu_x}{2\sigma_x/\sqrt{n}} \rightarrow -\infty, \text{ then } \lim_{a \rightarrow -\infty} \text{MSE}(\hat{\mu}_{yd}) = V(\bar{y}_d)$$

Proof:

For $a \rightarrow -\infty$, the value of $\Phi(a) \rightarrow 0$ and $\phi(a) \rightarrow 0$. This implies that $H_1: \mu_x = \mu_1$ is always accepted with probability almost one and hence μ_1 is almost equal to μ_x indicating thereby that $\Delta_1 \cong 0$. Therefore

$$\text{equation (3.4) results to } \lim_{a \rightarrow -\infty} \text{MSE}(\hat{\mu}_{yd}) = \frac{\sigma_y^2}{n} (1 - \rho^2)(1 + \delta^2) = V(\bar{y}_d)$$

The theorems (4.1) and (4.2) exhibit the ideal situations where preliminary test of significance of hypotheses has no meaning and in both the situations PTDE reduces to $\bar{y}_d$ as $\mu_x = \mu_0$ or $\mu_x = \mu_1$ with probability almost one. Therefore, a comparison is made of the PTDE ($\hat{\mu}_{yd}$) with the difference estimator T_1 , and T_2 which an investigator would have preferred to use without preliminary test of significance.

Let $e_1(\hat{\mu}_{yd}, T_1)$ denote the relative efficiency of $\hat{\mu}_{yd}$ over T_1 . This is given by

$$e_1(\hat{\mu}_{yd}, T_1) = \frac{\text{MSE}(T_1)}{\text{MSE}(\hat{\mu}_{yd})}$$

From (1.8) and (3.5), we get

Values of relative efficiency $e_1(\mu_y, T_1)$ expressed in percentage for $k=1.0$ and $\rho=0.9$

Values of relative efficiency $e_1(\mu_y, T_1)$ expressed in percentage for $k=0.9$ and $\rho=0.9$

Values of relative efficiency $e_1(u, T_1)$ expressed in percentage for $k=1.1$ and $\rho=0.7$

Values of relative efficiency $e_1(\mu_y, T_1)$ expressed in percentage for $k=1.0$ and $\rho=0.7$

[illegible]

Values of relative efficiency $e_1(u, T_1)$ expressed in percentage for $k=0.9$ and $\rho=0.7$

[illegible]

Values of relative efficiency $e_2(u_v, T_2)$ expressed in percentage for $k=1.1$ and $\rho=0.9$

[illegible]

Values of relative efficiency $e_2(\mu_v, T_2)$ expressed in percentage for $k=1.0$ and $\rho=0.9$

[illegible]

Values of relative efficiency $e_2(u_i, T_2)$ expressed in percentage for $k=0.9$ and $\rho=0.9$

[illegible]

Table 5.10

Values of relative efficiency $e_2(\mu_y, T_2)$ expressed in percentage for $k=1.1$ and $\rho=0.7$

Δ_0/Δ_1	-4	-3	-2	-1	0	1	2	3	4
-4	100	100	100	101	108	119	126	139	161
-3	-	100	100	102	125	155	168	194	238
-2	-	-	100	103	162	274	290	341	436
-1	-	-	-	100	184	685	714	821	1071
0	-	-	-	-	100	395	910	1425	2107
1	-	-	-	-	-	100	269	538	909
2	-	-	-	-	-	-	100	203	347
3	-	-	-	-	-	-	-	100	171
4	-	-	-	-	-	-	-	-	100

Table 5.11

Values of relative efficiency $e_2(\mu_y, T_2)$ expressed in percentage for $k=1.0$ and $\rho=0.7$

Δ_0/Δ_1	-4	-3	-2	-1	0	1	2	3	4
-4	100	100	100	101	107	117	125	138	160
-3	-	100	100	102	120	148	165	191	234
-2	-	-	100	103	147	239	273	328	422
-1	-	-	-	100	161	459	590	731	975
0	-	-	-	-	100	316	712	1161	1752
1	-	-	-	-	-	100	254	500	840
2	-	-	-	-	-	-	100	199	338
3	-	-	-	-	-	-	-	100	170
4	-	-	-	-	-	-	-	-	100

Table 5.12

Values of relative efficiency $e_2(\mu_y, T_2)$ expressed in percentage for $k=0.9$ and $\rho=0.7$

Δ_0/Δ_1	-4	-3	-2	-1	0	1	2	3	4
-4	100	100	100	101	106	115	123	137	158
-3	-	100	100	102	116	141	160	187	230
-2	-	-	100	103	135	207	252	310	402
-1	-	-	-	100	144	327	477	631	863
0	-	-	-	-	100	255	549	918	1407
1	-	-	-	-	-	100	237	455	757
2	-	-	-	-	-	-	100	195	327
3	-	-	-	-	-	-	-	100	168
4	-	-	-	-	-	-	-	-	100

* Not feasible as MSE (μ_y) comes out to be negative.

- Not feasible as $\Delta_1 < \Delta_0$ because $\mu_1 > \mu_0$

It can be observed from the results of these Tables 5.1 to 5.6 that the PTDE $\hat{\mu}_{y_d}$ is more efficient than T_1 and gain in precision is quite substantial for different combinations of values of Δ_0 , and Δ_1 , at different values of k , and ρ . The gain in precision is more appreciable for the combinations of the values of Δ_1 (-3 to 4) and Δ_0 (-4 to 0) at different values of k and ρ . The relative efficiency generally increases as the value of Δ_1 increases from -3 to 0 but after $\Delta_1 = 0$ it starts decreasing at all values of ρ & k . On the other hand, the efficiency decreases in general as Δ_0 increases from -4 to 4 for given value of Δ_1 at all values of ρ and k except in few

cases, for example the relative efficiency has jumped tremendously at $\Delta_0 = -3$ & $\Delta_1 = 0$;

$\Delta_0 = -2$ $\Delta_1 = 1$; $\Delta_0 = -1$ $\Delta_1 = -2$ and $\Delta_0 = 4$ $\Delta_1 = 3$. Even in case of deviation of $K = \frac{\beta_0}{\beta}$ from one, the relative efficiency generally remained almost stable. It may also be observed from the results of the tables that the relative efficiency has decreased for low value of ρ ($\rho = 0.7$) but there is still appreciable gain in efficiency. Thus, if we have reasonable prior values μ_0 and μ_1 of μ_x , and β_0 of β , then PTDE $\hat{\mu}_{yd}$ is always efficient than T_1 .

The values of the relative efficiencies $e_2(\hat{\mu}_{yd}, T_2)$ are given in Tables 5.7 to 5.12. The results of these Tables clearly indicate that $\hat{\mu}_{yd}$ is always more efficient than T_2 for different combinations of Δ_0 (-4 to 3) and Δ_1 (-1 to 4) at all values of k and ρ . The gain in efficiency is however quite substantial for different combinations of values of Δ_0 (-3 to 2) and Δ_1 (0 to 4) for all the values of ρ and k . For given value of Δ_1 the efficiency increases as Δ_0 increases up to 0 and thereafter starts decreasing for all values of ρ and k . For given value of Δ_0 there is general tendency of upward trend in efficiency when Δ_1 increases for all values of k and $\rho = 0.7$ but for $\rho = 0.9$ no specific trend has been observed except for some given values of Δ_0 . Behaviour of the efficiency e_2 with respect to k is similar to e_1 .

In view of the above numerical and theoretical findings, it is obvious that if good prior values μ_0 and μ_1 of μ_x and β_0 of β are available on the basis of past experience, an investigator can clarify his suspicion whether $\mu_x = \mu_0$ or $\mu_x = \mu_1$ by performing the preliminary test of significance of the hypothesis given in (2.1), and in such situations PTDE $\hat{\mu}_{yd}$ is most preferred over T_1 and T_2 . It may also be noted from the results of Table 5.1 to 5.12 that PTDE $\hat{\mu}_{yd}$ is most efficient than T_1 as compared to T_2 . However one can argue that why the closeness of β_0 to β cannot be tested. In fact, this problem has already been tackled by developing preliminary test regression estimator after testing the hypothesis $H_0: \beta = \beta_0$ against $H_1: \beta \neq \beta_0$ (see, Grimes & Sukhatme, 1980). Testing of hypothesis (2.1) and the preceding one together and subsequent development of preliminary test estimator could be a future line of research work.

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