Jan. 2013, Volume 7, No. 1 (Serial No. 62), pp. 68-77
Journal of Civil Engineering and Architecture, ISSN 1934-7359, USA

Numerical Simulation of Geotechnical Problems by Coupled Finite and Infinite Elements

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Abstract: In geotechnical engineering, numerical simulation of problems is of great importance. This work proposes a new formulation of coupled finite-infinite elements which can be used in numerical simulation of geotechnical problems in both static and dynamic conditions. Formulation and various implementation aspects of the proposed coupled finite-infinite elements are carefully discussed. To the authors' knowledge, this approach that considers coupled finite-infinite elements is more efficient in the sense that appropriate and accurate results are obtained by using less elements. The accuracy and efficiency of the proposed approach is considered by comparing the obtained results with analytical and numerical results. In a static case, the problem of circular domain of infinite length is considered. In a dynamic case, one dimensional wave propagation problems arising from the Heaviside step function and impulse functions are considered. In order to get a more complete picture, two dimensional wave propagation in a circular quarter space is considered and the results are presented. Finally, a soil-structure interaction system subjected to seismic excitation is analyzed. In the analysis of soil-structure interaction phenomenon, frames with different number of storeys and soil media with various stiffness characteristics have been taken into consideration. In the analysis, the finite element software ANSYS has been used. For the newly developed infinite element, the programming has been done by the help of the User Programmable Features of the ANSYS software, which enable creating new elements in the ANSYS software.

Key words: Numerical methods, infinite elements, soil-structure interaction, wave propagation.

1. Introduction

In simulating geotechnical problems, the soil medium is usually the most difficult task to be dealt with since the end boundaries of the soil media are at infinity. Due to the nature of these unbounded problems, analytical solutions are scarce and the solution is looked over in numerical simulation of the problems. In numerical simulation of soil media, it is common to take a wide region while the boundaries are truncated such that they do not have an impact on the accuracy of the results.

In the static case of geotechnical problems, it is of interest to find out the effect of a point load upon a half space. This subject has been carefully studied by Bettess [1]. In order to numerically simulate this problem, the finite element region should be widened or usage of appropriate boundary conditions has to be considered.

In the dynamic analysis, the situation is additionally complicated by the inertia terms so that the radiation of the wave should be considered. As given in the work of Pitilakis et al. [2], the dynamic analysis of the unbounded soil is not trivial since finite elements do not satisfy the radiation-towards infinity condition at the boundaries.

In geotechnical engineering, the seismic response of large structures has been an issue of interest for a longer time. In recent publications such as the works of Spyridon et al. [3], Ferraioli et al. [4] etc., the safety of existing massive structures under seismic action has been examined. The response of big structures founded in relatively soft ground in dynamic cases might be influenced by the soil-structure interaction as well as the characteristics of the excitation loads. As a

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result, in dynamic analysis of structures in semi-infinite soil medium, the interaction effects have to be considered [5]. Although a huge number of investigations have been conducted, all methods can be classified into two main categories such as direct and substructure method. The direct method evaluates the response of a structure and its surrounding soil in a single analysis step, while the substructure method considers the problem in several steps [6]. Various types of numerical methods such as FEM (finite element method), BEM (boundary element method) and hybrid techniques are commonly used to model soil-structure interaction effects [7-10].

The use of FEM has advantages in the implementation of its procedures as given in Ref. [11]. The finite element method offers efficient advantages in various aspects of simulating the soil domain such as arbitrary geometries, soil layering, material modeling, etc.. Nevertheless, due to the difficulty in analyzing media of infinite extent, the radiation boundary conditions at infinity have to be considered in order to simulate numerical problems in a more realistic manner [12].

In this work, the direct method of soil-structure interaction problem has been analyzed as an implementation example. Infinite elements, which are an extension of the finite elements offer an alternative for simulating the boundaries in soil modeling. Infinite elements gain in popularity as efficient means of extending the finite element method to cope with unbounded domains. In both cases of static and dynamic analysis, the usage of infinite elements shows promising results and is offered for further problems in the field of geotechnical engineering. Coupling finite and infinite elements maintains the advantages of each method and eliminates their disadvantages. The coupled finite-infinite approach is mainly used to get good solution in the finite element region reducing the area of analysis significantly as stated in the work of Honjo [13].

In this study, a combination of finite and infinite elements is proposed for general analysis of geotechnical problems both in static and dynamic cases. In order to further develop the approach in nonlinear material modeling, the formulation has been developed in time domain. Using the known direct method, the soil domain has been divided into near field modeled by finite elements and far field modeled by infinite elements. The results obtained have been compared with those from literature. The coupled model has been applied to several numerical examples to illustrate the usage of the coupled finite and infinite elements.

2. Infinite Elements

According to their formulation, infinite elements can be categorized into two types, namely displacement descent and coordinate ascent which are also known as mapped infinite elements [14]. The formulation of mapped infinite elements is the same as for the finite elements in addition to the mapping of the domain [15]. The main advantage of the mapped infinite elements is the usage of the conventional Gauss-Legendre abscissae and weights. Infinite elements have first been developed by Bettess [16]. They have further been developed in frequency and time domain by different authors as Zienkiewicz et al. [17], Marques et al. [18], Angelov [19] etc.. In the work of Häggblad et al. [20], static infinite elements have been amended with absorbing properties enabling the infinite elements to be used in dynamic cases. In this work, the development of an infinite element has followed the techniques given in Häggblad et al. [20] considering an infinite element with added absorbing layer. The main difference is in the increased number of nodes of the infinite element and the used mapping functions, whose verification has shown great accuracy of results. The infinite element has been obtained from an eight nodded finite element, as shown in Fig. 1.

The element displacement in u and v direction is interpolated with the usual shape functions N^1, N^2, N^4,

N^5 and N^7:

$$u = [N^1 \quad N^2 \quad 0 \quad N^4 \quad N^5 \quad 0 \quad N^7 \quad 0]\mathbf{u}$$
$$v = [N^1 \quad N^2 \quad 0 \quad N^4 \quad N^5 \quad 0 \quad N^7 \quad 0]\mathbf{v} \qquad (1)$$

In Expression (1), u and v are vectors with nodal point displacements in global coordinates. The shape functions are given in Expression (2) as:

$$\begin{aligned} N^1 &= -(r-1)(-1+s)(s+1+r)/4 \\ N^2 &= (r-1)(1+r)(-1+s)/2 \\ N^4 &= -(r-1)(1+s)(s-1-r)/4 \\ N^5 &= -(r-1)(1+r)(1+s)/2 \\ N^7 &= (-1+s)(1+s)(r-1)/2 \end{aligned} \qquad (2)$$

Based on the isoparametric concept, the infinite element in global coordinate is mapped onto an element in local coordinate system using the expression in Expression (3). In the formulation of the infinite element, only the positive r direction extends to infinity.

$$r = [M^1 \quad M^2 \quad 0 \quad M^4 \quad M^5 \quad 0 \quad M^7 \quad 0]\mathbf{r}$$
$$s = [M^1 \quad M^2 \quad 0 \quad M^4 \quad M^5 \quad 0 \quad M^7 \quad 0]\mathbf{s} \qquad (3)$$

where,

$$\begin{aligned} M^1 &= -\frac{(1-s)rs}{1-r} \\ M^2 &= -\frac{(1-s)(1+r)}{2(1-r)} \\ M^4 &= -\frac{(1+s)rs}{1-r} \\ M^5 &= -\frac{(1+s)(1+r)}{2(1-r)} \\ M^7 &= -\frac{2r(1+s)(1-s)}{(1-r)} \end{aligned} \qquad (4)$$

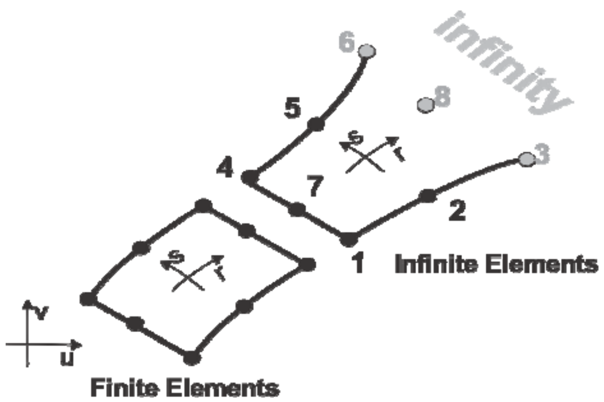

Fig. 1 Coupling of finite and infinite elements.

In Expression (3), r and s are vectors of nodal point displacements in local coordinates where it is to be mentioned that, on the side of infinity (r = 1), no mappings have been assigned to the nodes as it is taken that no displacement is possible at infinity. The number and location of the nodes connecting finite and infinite elements must coincide to guarantee continuity condition between the elements. The main advantage of the proposed infinite elements is that the number of nodes on the infinite element allow coupling with finite elements with eight nodes which are used for displacement sensitive problems. Construction of element matrices is done by using the usual procedures as described in Bathe [21]. The new coordinate interpolation functions are taken into consideration in the Jacobian matrix as described in Bettess [1]. For the absorbing layer of the infinite element, the Lysmer-Kuhlmeyer approach [22] is used. In all cases, a plane strain two dimensional case is studied. For impact of plane waves on element sides, normal and tangential stresses are derived as follows:

$$\begin{bmatrix} \sigma^n \\ \tau \end{bmatrix} = \begin{bmatrix} a\rho c^p & 0 \\ 0 & b\rho c^s \end{bmatrix} \begin{bmatrix} \dot{u}^n \\ \dot{u}^t \end{bmatrix} \qquad (5)$$

where, c^P and c^S indicate compression and shear waves, ρ is the density of soil medium. In order to take into account the directions of the incident waves, coefficients a and b suggested in White et al. [23] are used as multipliers for better numerical results. Transformation from local to global coordinates is done automatically by the software ANSYS [24] such that there is no need of defining transformation matrices. By adding together the parts from each element, the governing incremental equations for equilibrium in dynamic analysis are obtained. Time derivatives are approximated by the Newmark's method and equilibrium iterations are used in each step as given in the theory reference of the ANSYS [24] software. The programming of the infinite element has been done using the programmable features of the ANSYS [24]. For the sake of verification of the presented infinite elements, four unbounded problems taking into account the static and dynamic cases, are shown.

3. Static Case—Infinite Plate with Circular Hole

The aforementioned methodology of coupling finite and infinite elements has been implemented in a two dimensional static case problem. Namely, an infinite plate with a circular hole of radius R = 1.0 m is subjected to a uniformly distributed pressure p = 1 kPa. The obtained results are compared with the analytical ones given in the work of Abdel-Fattah [14]. The domain is simulated numerically as composed of a plane strain 2D quarter space, as shown in Fig. 2.

The material properties of the plate are Young's modulus E = 1 kPa and Poisson's ratio v = 0.25. The problem is numerically solved using 80 finite elements in total. The infinite region is simulated by using 8 infinite elements. The results from the comparison with the analytical solution as given in the work of Abdel-Fattah [14], are given in Fig. 3.

As can be seen from Fig. 3, the analytical and numerical results of the radial displacement seem to be very similar which proves the correctness of the used coupled finite and infinite elements.

4. Dynamic Case—Wave Propagation in 1D and 2D

In order to verify the absorbing properties of the newly programmed infinite elements, a 1-dimensional wave propagation is performed using the soil layer properties as given in Plaxis Validation Manual [25]. In our case the soil layer is horizontal with a total length of 10 m. The soil layer is simulated in two alternative ways. First only finite elements with fixed boundaries and then the same finite elements with absorbing infinite element boundaries are considered. Fig. 4 shows the soil domain with point A in the middle.

The soil domain is discretized by 40 elements. The properties of the soil are given in Table 1.

As given in Plaxis Reference Manual [8], the P wave velocity is:

$$V_p = \sqrt{(1-\upsilon)E/\rho\left((1+\upsilon)(1-2\upsilon)\right)} \qquad (6)$$

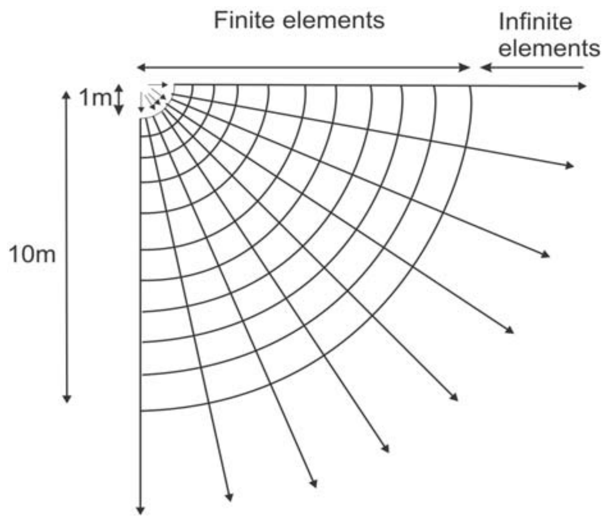

Fig. 2 Mesh of finite and infinite elements.

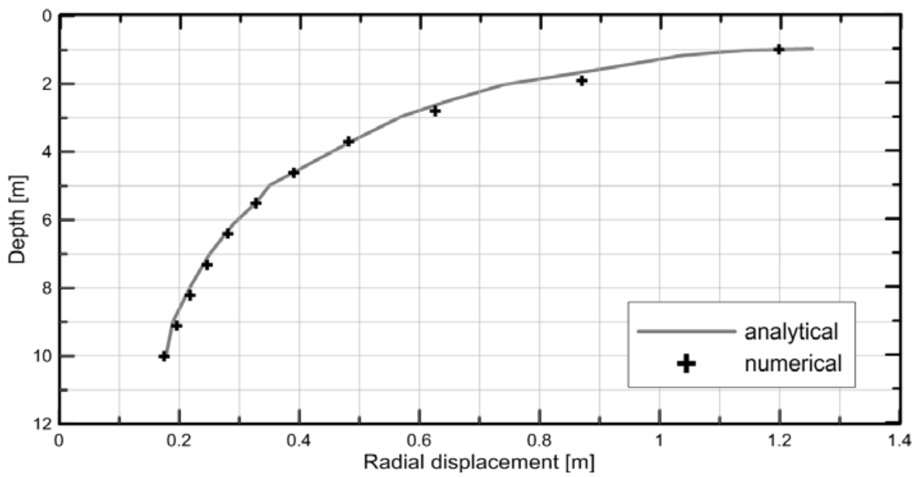

Fig. 3 Radial displacement—analytical v.s. numerical.

Using the values given in Table 1, Vp = 100 m/s is obtained. In order to give a more general picture of the 1D wave propagation, horizontal displacement on the left of the domain is applied as two different types of functions:

(1) Heaviside step function;

(2) Impulse function.

The first type of displacement application is considered to be of the Heaviside step type. In this case, displacement of u_x = 0.001 m is applied at the very beginning of the time. Fig. 5 shows the time history of the traveling P wave at the middle point (point A).

Fig. 5 clearly shows that, in the case of a domain composed of finite elements only, the wave is reflected, while in the case of using the coupled approach of finite and infinite elements, the wave is absorbed in the boundary so that the value of the horizontal displacement oscillates around initially applied displacement value (u_x = 0.001 m). It is to be mentioned that the time needed for the P-wave (Vp = 100 m/s) to reach point A is 5 (m)/100 (m/s) = 0.05 s, which is easily seen in Fig. 5, showing the correctness of the numerical simulation.

In the subsequent case, application of displacement is considered as an impulse function where application of displacement is done only for 0.00166 s and then the displacement is removed. The system is analyzed for 0.3 s in order to see the time-displacement behavior at the middle point of the domain. Fig. 6 shows the obtained displacement time history at the middle point (Point A).

In Fig. 6, it is clearly shown that, in the case of finite elements with fixed boundaries, the impulse is reflected at 0.15 s, while in the case of using infinite element boundaries, the reflection of the wave is reduced considerably.

In the numerical simulation of 2D wave propagation, a quarter-space is taken into consideration. The material parameters are taken as in Table 1. The soil medium is presented as a combination of 450 finite and 15 infinite elements, as shown in Fig. 7. The prescribed displacement of impulse type with duration of 0.0015 s is applied at the upper top part of the domain.

Two cases of analysis are performed considering the domain composed of finite elements and coupled finite—infinite elements. In Fig. 8, wave propagation is simulated considering the effect of infinite element boundaries.

As can be seen from Fig. 8, in the case where infinite elements are used, there is a reduction in reflection of the waves from the boundaries. This implies that, in the coupled finite and infinite elements approach, the wave is absorbed successfully at the boundary. On the other hand, in the case where only

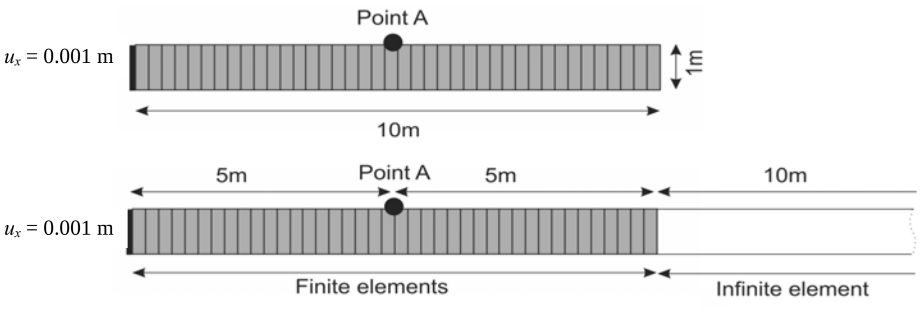

Fig. 4 Soil layer domain.

Table 1 Soil properties.

Young's modulus	E	18,000	kPa
Poisson's ratio	N	0.2	-
Density	ρ	2.04	t/m^3

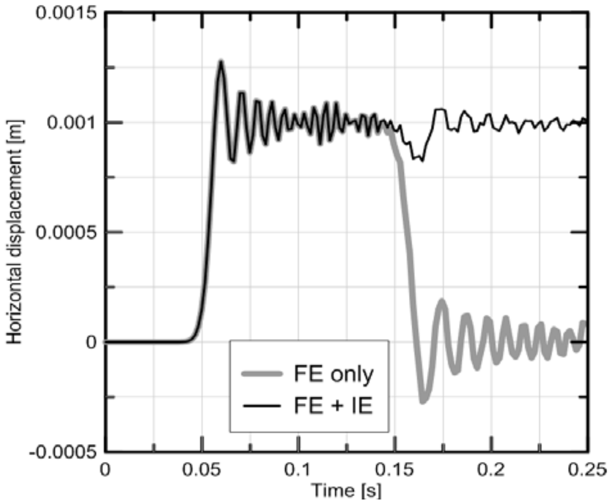

Fig. 5 Time displacement history—Heaviside step function.

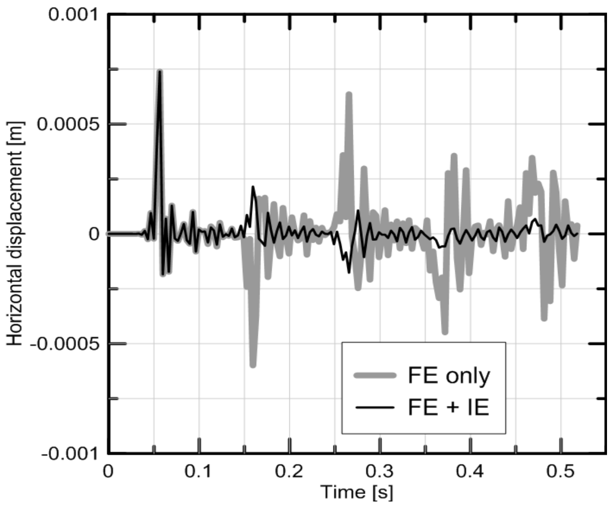

Fig. 6 Time displacement history—impulse function.

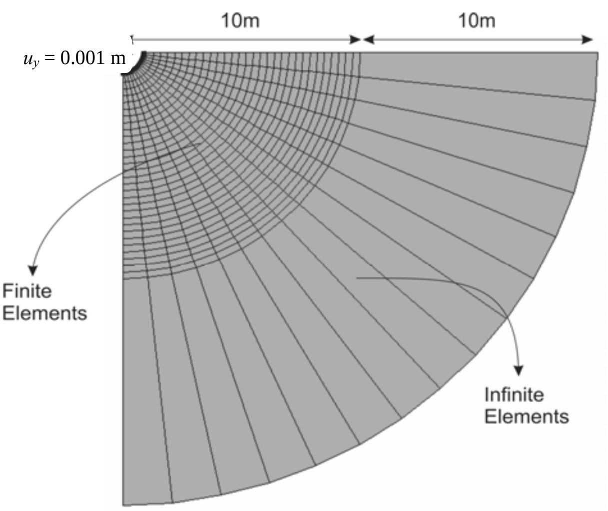

Fig. 7 Domain of finite and infinite elements.

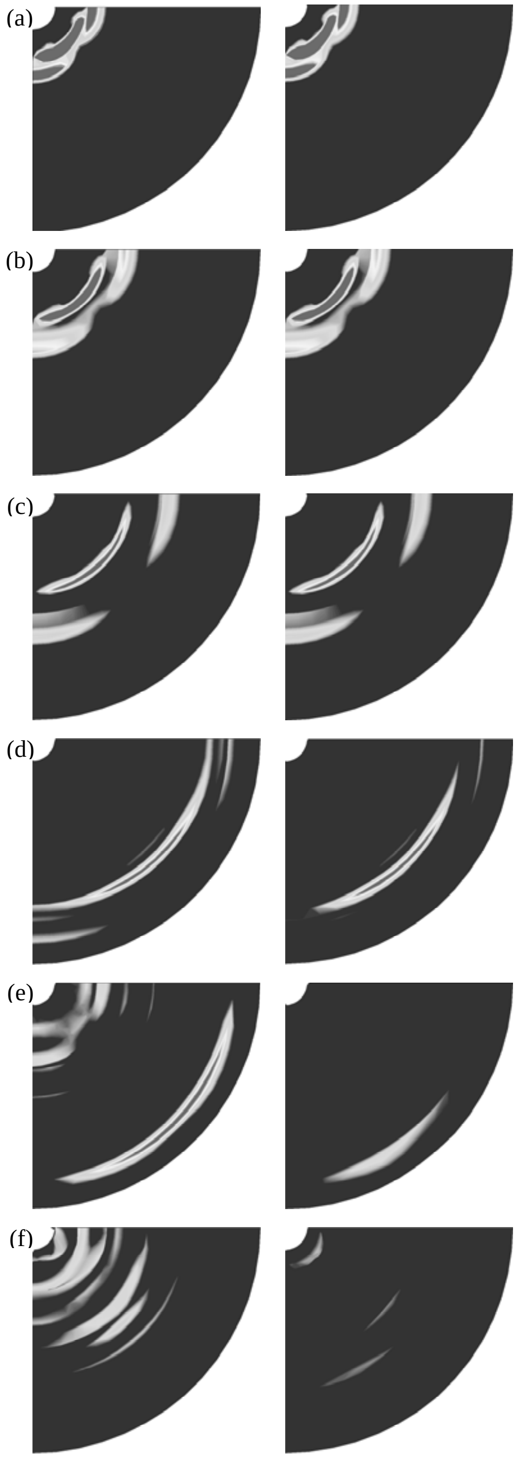

Fig. 8 Wave propagation at time: (a) t = 0.04 s; (b) t = 0.06 s; (c) t = 0.15 s; (d) t = 0.18 s; (e) t = 0.21 s; (f) t = 0.24 s (left-finite elements only, right-finite and infinite elements).

finite elements are considered, the wave reflection is evident in the results. As expected in the case of propagating waves, the absorption by the infinite elements is such that the reflection of the wave back to the finite element domain is reduced considerably.

5. Coupled Soil-Structure System Response

In order to show the influence of the soil boundaries on the structure, a comparison of boundary cases in the soil structure interaction problem are performed. In this direct time-domain method, the soil medium is modelled by two dimensional quadrilaterals using the finite element method. Similar soil-structure interaction problems have been studied in the works of other authors [26-29]. In order to provide a complete insight, the soil side boundary is first simulated as a fixed support applied to the truncated soil domain composed of finite elements. Then the same soil medium is bounded by viscous boundaries which are present in commercial softwares such as ANSYS [24]. Finally, the soil composed of less finite elements is surrounded by the newly programmed infinite elements. The frame structural elements are idealized as two dimensional elastic beam elements having three degrees of freedom at each node, translations in the nodal x and y directions and rotation about the nodal z axis. The behaviour of the frame structure is supposed to be elastic and has been modelled by using two parameters, the modulus of elasticity $E = 3.15 \times 10^7$ kPa and Poisson's ration n = 0.2. The bay length of the frame is taken to be 4.0 m, while the storey height is 3.0 m. The section of beams is 40 cm × 50 cm while that of the column is 50 cm × 50 cm. A mass of 11 t is assigned to each node to simulate the real structural behaviour (a total of 44 t per floor). Three different frames are taken into consideration. For all RC frames, the beam and column sections, the floor masses and the number of bays are kept constant in all cases. The only parameter that has been altered is the number of storeys. The structures are modeled as one-storey, three-storey and five-storey RC frames.

The soil medium is presented as a two dimensional model composed of four layers resting on bedrock. In Table 2, the soil layers properties are tabulated in a way that the bottom layers are characterized by better soil characteristics.

The soil is assumed to represent a linear-elastic material and is discretized by using eight nodded plane strain elements. The dynamic analysis has been performed by transient analysis using the step by step method. The proportional viscous damping matrix is taken to be proportional to mass and stiffness matrix (Rayleigh damping). The Rayleigh damping factors, alpha and beta are calculated such that the critical damping is 5% for the first two modes (α = 1.2907, β = 0.001405).

Finite element modelling of the coupled soil-structure system is performed by use of the software ANSYS [24], as shown in Fig. 9. The effect of soil-structure interaction is carried out by using the acceleration time history of the El Centro earthquake with a scaled peak ground acceleration of 0.25 g. The

Table 2 Soil properties.

Soil medium	Number of layers	Thickness (m)	Unit weight (kN/m^3)	Shear velocity (m/s)
Hard	1	3	16	330
	2	7	17	420
	3	6	17.5	510
	4	14	18	690
Moderate	1	3	16	160
	2	7	17	210
	3	6	17.5	250
	4	14	18	340
Soft	1	3	16	90
	2	7	17	100
	3	6	17.5	120
	4	14	18	160

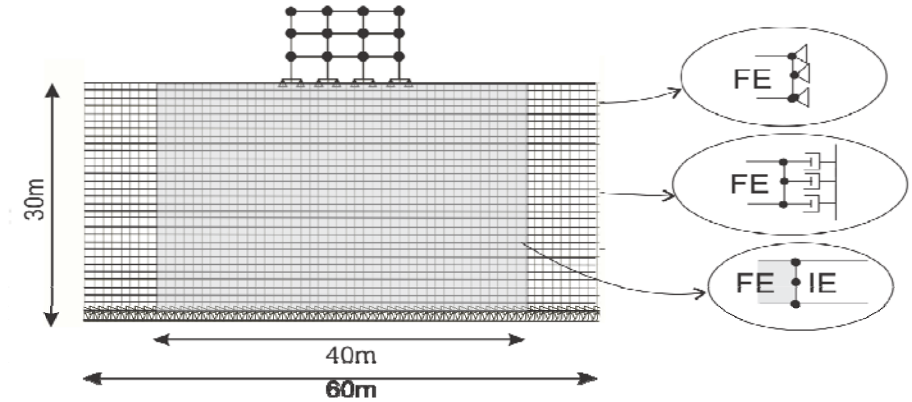

Fig. 9 Coupled soil structure system of a three storey system.

moment transfer capability between the column and the footing is created by using a constraint equation where the rotation of the beam is transferred as force couples to the plane element.

In Fig. 9, the coupled system of the soil-structure system is shown. The side boundaries are presented as fixed, viscous and infinite element boundaries. In the case of infinite element boundary, the soil domain is discretized by less elements (two thirds) compared with the analysis of fixed and viscous boundaries.

In Table 3, the difference in the structural response is given.

According to the acceleration values in Table 3, the maximum acceleration at the top of the structure is considerably increased when using fixed boundaries. On the other hand, when using viscous and infinite

Table 3 Variation of structural response quantities.

No. of storey	Soil medium	Boundary	Max. acceleration at top of Str. (m/s²)	Acceleration amplification at top of structure	Max. displacement at top of Str. (cm)	Max. moment at top of Str. (kNm)
1	Hard	Fixed	11.2	4.57	0.447	152.1
		Viscous	5.72	2.33	0.220	58.7
		Infinite el.	4.17	1.70	0.171	48.6
	Moderate	Fixed	13.5	5.51	0.624	223.2
		Viscous	5.13	2.09	0.319	83.5
		Infinite el.	4.91	2.00	0.191	67.8
	Soft	Fixed	11.1	4.53	1.11	222.2
		Viscous	4.61	1.88	0.527	85.3
		Infinite el.	3.29	1.34	0.257	63.2
3	Hard	Fixed	8.95	3.65	1.87	155.1
		Viscous	8.68	3.54	1.93	145.5
		Infinite el.	6.08	2.48	1.45	115.1
	Moderate	Fixed	10.5	4.28	3.45	182.2
		Viscous	7.88	3.21	2.96	118.1
		Infinite el.	5.55	2.26	2.09	99.9
	Soft	Fixed	10.3	4.20	8.22	175.1
		Viscous	7.12	2.91	3.65	108.3
		Infinite el.	4.50	1.83	2.93	92.3
5	Hard	Fixed	9.74	3.98	5.56	153.1
		Viscous	9.15	3.73	4.78	145.3
		Infinite el.	7.83	3.19	4.22	126.3
	Moderate	Fixed	8.51	3.47	6.48	158.3
		Viscous	8.04	3.28	5.86	149.1
		Infinite el.	6.39	2.61	4.08	114.1
	Soft	Fixed	8.80	3.59	11.1	131.2
		Viscous	5.85	2.39	7.58	81.9
		Infinite el.	4.78	1.95	4.83	56.2

element boundaries, the results of acceleration, displacement and moment at the top frame elements show similar values. The main difference is that, when using the coupled finite-infinite elements, the number of finite elements is decreased considerably, saving extra work and time. When comparing the soil stiffness, it is clearly seen that, in the case of soft soil the difference in structural moment values between the fixed and the infinite element boundaries is nearly two times. This fact reveals that, in the case of massive structure founded on soft soils, the interaction effects are expressed greatly. The number of stories affects the results in such a way that the higher storeys exhibit a bigger displacement (which is also expected) that should be considered in the element analysis, separately. To sum up, the usage of the proposed infinite elements in soil-structure interaction problems decreases the number of finite elements without affecting the correctness of the final results. Thus, the usage of coupled finite-infinite elements is advised particularly in complex geometries of soil media.

6. Conclusions

In this work, a coupled computational method of finite and infinite elements is presented and applied in selected geotechnical problems. For the numerical simulation of geotechnical problems, the local region of interest has been modeled by finite elements, which enable simulation of more complex geometries. On the other hand, the surrounding field of the domain has been considered by using infinite elements. By using the coupled finite-infinite elements approach, the number of elements and nodes has been reduced without affecting the accuracy of the results in the near field.

The infinite elements with added absorbing characteristics which have been proposed in this study, provide a very general and easy method to implement frame of infinite elements. Furthermore the mapping functions and number of nodes proposed in this study increase the accuracy since eight nodded finite elements can be coupled with boundaries. Although the limitation of the proposed approach is that the infinite elements do not represent the real solution at the far field, the finite elements in the coupled approach give an acceptable accuracy for geotechnical problems in static and dynamic cases.

Acknowledgments

The author wishes to express his gratitude to the German Academic Exchange Service (DAAD) for their support given in the frame of this Ph.D. work.

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