A Study on Simple Prediction Method of Heat Load: A Use of Linear Approximation Indicial Response in Basements

Kyung-Soon Park¹, Hiroaki Kitano² and Hisaya Nagai²

1. Department of Building Systems Engineering, Dong-Eui University, Busan 614-714, Korea

2. Department of Architecture, Graduate School of Engineering, Mie University, Tsu 514-8507, Japan

Abstract: This study was conducted to establish a predictable method for a heat load of an underground structure with sufficient accuracy. As the first step, our previous paper reported the measurement results of field experiments on an underground experimental basement under internal heat generation conditions. Also, it presented the results of numerical analyses on the heat and moisture behavior and the influence of internal heat generation of the experimental basement and ground. However, it is practically impossible to utilize the model of simultaneous heat and moisture transfer at the design phase because the prediction by the model of simultaneous heat and moisture transfer requires a long calculation time. In this paper, the authors present the simple load calculation technique, using a linearized approximation indicial response of the inner surface heat flux in a basement to outdoor air temperature change. In addition, the approximation indicial responses for each part of the single-walled concrete drawn using this technique are arranged. The heat load calculation example of application to the basement of the optional size by this technique is shown.

Key words: Underground structure, simultaneous heat and moisture transfer, linearized prediction method, indicial response.

Nomenclature

c Specific heat of concrete/soil containing the moisture (J/kg·K)
 C_γ Heat capacity of moist air (J/m³·K)
 $C'\gamma'$ Moisture capacity of moist air (kg/m³·Pa)
 F_w Gravity (m/s²)
 J_p Rate of precipitation (kg/m²·s)
 J_r Internal moisture generation (kg/m³·s)
 N Inward normal vector
 N_v Rate of change of the room air (times/h)
 P Water vapor partial pressure (Pa)
 P_o Water vapor partial pressure of outdoor air (Pa)
 P_j Water vapor partial pressure of the inner surface of the wall (Pa)
 P_r Water vapor partial pressure of indoor air (Pa)
 P_{sat} Saturated water vapor partial pressure (Pa)
 q_{noc} Heat flux due to long wave net sky radiation (W/m²)
 q_{sol} Heat flux due to absorbed solar radiation (W/m²)
 Q_r Internal heat generation (W/m³)
 R Heat of vaporization water (J/kg)

R_v Universal gas constant for water vapor (Pa·m³/kg·K)
 S_j Area of j^{th} part of the inside wall (m²)
 t Time (s)
 T Temperature (K)
 T_j Surface temperature of j^{th} part of the inside wall (K)
 T_o Outdoor air temperature (K)
 T_r Indoor air temperature (K)
 T_s Temperature at the earth surface (K)
 V Volume of the room (m³)
 A Total heat transfer coefficient (W/m²·K)
 α_c Convective heat transfer coefficient (W/m²·K)
 α_r Radiative heat transfer coefficient (W/m²·K)
 α'_m Vapor transfer coefficient related to the difference of water vapor partial pressure (kg/m²·s·Pa)
 α'_T Moisture transfer coefficient related to temperature (kg/m²·s·K)
 α'_μ Moisture transfer coefficient related to water chemical potential (kg/m²·s (J/kg))
 λ Thermal conductivity (W/m·K)
 λ'_T Total moisture conductivity for temperature gradient (kg/m·s·K)
 λ'_{Tg} Moisture conductivity in the gas phase for the temperature gradient (kg/m·s·K)
 λ'_μ Total moisture conductivity for water chemical potential gradient (kg/m·s (J/kg))
 $\lambda'_{\mu g}$ Moisture conductivity in the gas phase for the water chemical potential gradient (kg/m·s (J/kg))

Corresponding author: Kyung-Soon Park, Ph.D., assistant professor, research fields: numerical analysis, energy simulation, energy saving and green building system. E-mail: pks2180@deu.ac.kr.

μ	Water chemical potential relative to free water (J/kg)
μ_o	Water chemical potential of the outdoor air (J/kg)
μ_s	Water chemical potential at the earth surface (J/kg)
ρ	Density of concrete/soil (kg/m ³)
ρ_w	Density of liquid water (kg/m ³)
Φ	Water content (m ³ /m ³)

1. Introduction

This study has aimed to clarify the heat and moisture behavior and heat load characteristics of underground structure and surrounding ground, and to establish the predictable method for the heat load of the underground structure in enough accuracy.

Many studies have been conducted on the thermal properties and heat load calculations of underground spaces. Ogura and Matsumoto et al. [1, 2] have investigated heat and moisture behavior of underground space and its surrounding ground under natural condition.

Park et al. [3] have conducted field experiments to evaluate the heat and moisture behavior of an underground basement and surrounding ground under an internal heat generation condition and natural meteorological. Park et al. [4] reported the results of prediction for the heat load of an underground structure and the results of evaluation for the heat and moisture behavior by conducting a numerical analysis of simple heat diffusion and simultaneous heat and moisture transfer based on measured value. It was made clear that it is possible to estimate the field of heat and moisture of surrounding ground as well as the heat/moisture behavior in the underground structure under meteorological conditions such as a rainfall, etc.. However, it is practically impossible to utilize the model of simultaneous heat and moisture transfer at the design phase because the prediction by the model of simultaneous heat and moisture transfer requires a long calculation time.

Therefore, the authors suggested the method for heat load prediction by linear approximation under the optional time fluctuation of various meteorological conditions. It also precisely calculates the dynamic

heat load in the basement.

In this paper, the authors present the simple load calculation technique using a linearized approximation indicial response of the inner surface heat flux in a basement to outdoor air temperature change and its validity. In addition, the approximation indicial responses for each part of the single-walled concrete drawn using this method are arranged and the heat load calculation example of application to the basement of the optional size by this method is shown.

2. The Governing Equations

The simultaneous heat and moisture transfer equations are the governing equations for the analysis as a whole [5]. The simultaneous heat and moisture transfer in the concrete and ground can be described by Eqs. (1) and (2), equations derived with the assumption of local equilibrium between the liquid phase and the gas phase of water without the solid phase (ice).

$$\frac{\partial}{\partial t}(cpT) = \nabla(\lambda + r\lambda'_{\mu})\nabla T + \nabla r\lambda'_{\mu}\nabla \mu \quad (1)$$

$$p_w \left(\frac{\partial \phi}{\partial \mu} \right) \frac{\partial \mu}{\partial t} = \nabla \lambda'_{\mu} (\nabla \mu - F_w) \nabla \lambda'_T \nabla T \quad (2)$$

where, T and F_w are the temperature and gravity, respectively, and μ is the water chemical potential relative to the free water. The boundary conditions of heat and moisture transfer at the ground surface level are described by the following equations:

$$-(\lambda + r\lambda'_{\mu}) \left. \frac{\partial T}{\partial n} \right|_s - r\lambda'_{\mu} \left(\left. \frac{\partial \mu}{\partial n} \right|_s - nF_w \right) = \quad (3)$$

$$(\alpha + r\alpha'_T)(T_o - T_s) + r\alpha'_\mu(\mu_o - \mu_s) + q_{sol} + q_{noc}$$

$$-\lambda'_\mu \left(\left. \frac{\partial \mu}{\partial n} \right|_s - nF_w \right) - \lambda'_T \left. \frac{\partial T}{\partial n} \right|_s = \quad (4)$$

$$\alpha'_\mu(\mu_o - \mu_i) + \alpha'_T(T_o - T_i) + J_p$$

where, subscript s is the surface.

The heat and moisture equation for the air in the underground space are described by the following equations:

$$c\gamma V \frac{\partial T_r}{\partial t} = \sum_{j=1}^n S_j \alpha_i (T_j - T_r) \quad (5)$$

$$+ c\gamma N_v V (T_o - T_r) + Q_r$$

$$c'\gamma V \frac{\partial p_r}{\partial t} = \sum_{j=1}^n S_j \alpha'_m (p_j - p_r) + c'\gamma N_v V (p_o - p_r) + J_r \quad (6)$$

The relation between water chemical potential μ and water vapour pressure p is as follows:

$$\mu = R_v T \ln \left(\frac{p}{p_{sat}} \right) \quad (7)$$

where, R_v , p and p_{sat} are the universal gas constant for water vapour partial pressure, respectively.

3. The Linearized Prediction Method under the Optional Time Fluctuation

If the object system is the linear and time-invariant, the optional vibration and response is expressed by Duhamel's theorem of analogy [6]. If the weight function that is the response of the system to the delta function of Dirac of Impulse function is expressed to $h_\phi(t)$, the response $q(t)$ to the optional time fluctuation of meteorological condition $\Theta(t)$ is described by the following equation:

$$q(t) = \int_0^\infty h_\phi(\tau) \Theta(t-\tau) d\tau = \int_0^\infty h_\phi(t-\tau) \Theta(\tau) d\tau \quad (8)$$

As the object in the present study is nonlinear simultaneous heat and moisture transfer system, the weight function is a time-varying system and it is difficult to acquire the solution as linear system. Accordingly, the calculation becomes extremely difficult by using Eq. (8), due to the heat flux on the surface of the inner wall of basement by the optional disturbance. However, if the time-invariance of the weight function (derivatives of indicial response) to the optional disturbance is extremely small and it is possible to handle it as time-invariance approximately. The calculation of the response to optional disturbance becomes possible by using the indicial response that expresses the function of linear approximation.

The response to outdoor air temperature is much larger than those to other meteorological elements, therefore, this paper presents a time-invariance of indicial response of heat flux on the inner wall of

basement to the outdoor air temperature.

4. Outline of the Numerical Analysis

4.1 Analytical Model

Analytical model is the experimental basement which is laid under completely. The experimental basement was constructed in 1997 and located in Tsu, a city in central Japan. The floor plan and sectional plan of the experimental basement are shown in Figs. 1 and 2. Once the basement was installed underground, the roof was at a depth of 1.0 m below the ground level. To clarify the influences of moisture from the ground, the wall of the basement was constructed with reinforced concrete untreated by any form of waterproofing or moisture prevention processing. The thicknesses of the ceiling/side wall and floor were 0.25 m and 0.3 m, respectively. Blinding concrete and crushed stone were additionally paved below the floor level to a thickness of 0.3 m, and refer to previous paper for other data [3].

Fig. 3 shows the schematic diagram of the analyzed system. The validity of nonlinear numerical analyses to evaluate the heat and moisture behaviour and predict the heat load of the underground structure was confirmed by Park et al. [4]. However, to minimize the tedium and complexity of nonlinear numerical analyses by the three-dimensional coordinate system, the numerical analysis of the basement in this study was treated as a two-dimensional coordinate system.

4.2 Meteorological Conditions

The expanded AMeDAS weather data of Tokyo that has precipitation, outdoor air temperature, outdoor relative humidity and solar radiation recorded hourly was used for the numerical analysis [7]. The annual mean temperature of Tokyo was 16.6 °C, and annual solar radiation and precipitation were 75.0 MWh/m² and 1,550 mm, respectively.

4.3 The Physical Properties of Materials

Table 1 shows the heat and moisture transfer

Fig. 1 Floor plan of the basement (unit: mm).

Fig. 2 Sectional plan of the basement (unit: mm).

Fig. 3 Schematic diagram of the analyzed system.

coefficients and other physical properties of materials. The existing measurement by Hendenblad [8] and presumed values by Ogura [2] was applied as the actual physical property values of concrete. These values were used as there were no actual physical

property values which could be applied from an unsaturated field to a saturated field. The characteristic of surrounding ground of experimental basement was a plain field sand, therefore physical property values measured by Jury [9] as shown in Figs. 4 and 5 and were used for the analysis. Figs. 4 and 5 show the thermal and moisture physical property values of the concrete and soil used for this analysis.

Table 1 Heat and moisture transfer coefficients and other properties.

Heat capacity of air $C\gamma$ (J/m ³ ·K)	1,256.0
Moisture capacity of air $C'\gamma'$ (kg/m ³ ·Pa)	7.5×10^{-6}
Heat transfer coefficient	Indoor air α_i 9.30 Outdoor air α_o 23.26
Moisture transfer coefficient (kg/m ² ·s·Pa)	Indoor air α'_{mi} 2.85×10^{-8} Outdoor air α'_{mo} 1.14×10^{-7}
Solar absorption ratio and emissivity	0.83 (-)/0.9 (-)

Subscript i : indoor air; o : outdoor air.

Fig. 4 Equilibrium relation between moisture content and water chemical potential for concrete and soil.

Fig. 5 Thermal conductivity of concrete and soil.

4.4 Boundary Conditions and Calculation Conditions

The annual mean outdoor air temperature of 15.2 °C is applied at the bottom of the ground (G.L.-16.5 m) for the first kind of boundary condition. Also to prevent emanation in the calculation, a water chemical potential of -1 J/kg is applied on the underground water table as the specified value for moisture. Besides that, the third kind of boundary condition is applied at the ground surface, while that at the periphery of the analysis region is assumed for the second kind of boundary condition (heat flux = 0, moisture flux = 0). Indoor air temperature and relative humidity of basement were assumed to be specified during the year by 22 °C and 50%, respectively.

The numerical analysis of the basement and its surrounding ground is treated as a two-dimensional coordinate system and performed by the finite difference method, with assumed homogeneity of the ground soil/concrete (Fig. 3). To acquire an accurate calculation result, the length and time steps are divided into very small increments. The increments of vertical and horizontal direction of the ground were 0.025-2.0 m and 0.01-1.0 m, respectively. The wall of basement was divided into 0.01-0.05 m. The number of increment of $\Delta x \times \Delta y$ is 24×47 . The time step Δt for the calculation is 15 s while during 15 min after rainfall the time step is 5 s to prevent emanation in the calculation.

The indicial response of the heat flux at the inner surface of wall to outdoor air temperature change was calculated from the difference between the heat flux at the reference outdoor air temperature and the heat flux at the step change from it.

As the response of moisture flux at the inner surface of wall to outdoor air temperature change was extremely small, the influence of moisture flux variation can be disregarded.

Therefore, if the moisture flux at the inner surface of the wall to outdoor air temperature changes under the standard meteorological conditions is acquired, the

latent heat loads of the basement can be predicted with sufficient accuracy.

5. Time-Variance of Sensible Heat Flux under the Outdoor Air Temperature

Fig. 6 shows the indicial responses of mean heat flux on inner wall of basement in Tokyo, in cases where the change time of outdoor air temperature differs (0:00 a.m. on March 1, 0:00 a.m. on June 1, 0:00 a.m. on September 1 and 0:00 a.m. on December 1). The time-variance of the indicial response is small, as the figure shows, it can be said that it is possible to approximate it in one curve. The approximate function of indicial response of mean heat flux on inner surface of basement is shown as follows:

$$\varphi(t) = 1.18 - 0.75e^{-0.00022t} + 0.27e^{-0.00034t} - 0.70e^{-0.0043t} \quad (t: \text{hour}) \quad (9)$$

6. The Linearized Prediction by Standard Approximate Indicial Response

The overall heat transfer load was calculated using Eq. (9) and outdoor air temperature in Tokyo. In consideration of calculating the usual heat load calculation using the meteorological data obtained discretely, it was calculated by the response factor method.

Fig. 7 shows the comparison between the calculation result by the response factor method and the exact calculation result (nonlinear solution) by nonlinear simultaneous heat and moisture transfer

Fig. 6 The indicial responses of mean heat flux on inner wall of basement by the outdoor air temperature variation.

equation. As a reference, the calculation result of simple heat diffusion is also shown in Fig. 7. However, this calculation result by the simple heat diffusive equation was calculated considering the following as conditions so that the solution nearest to the exact solution might be obtained [4]:

(1) The physical properties of concrete and soil are assumed to be in saturated water conditions;

(2) The meteorological condition considers only the outdoor air temperature.

If the result was in summer, when the non-linearity of physical properties of soil is stronger due to large amounts of rainfall and evaporation on the ground surface is excluded. Then the sensible heat flux on inner wall of basement by the response factor method is roughly predictable as shown in Fig. 7, through the year. In addition, it is clarified that the accuracy by a response factor method was higher than the calculation results by the simple heat diffusion.

When heat load calculation of this basement for one year was performed, the computation time took tens of hours when the nonlinear simultaneous heat and moisture transfer equations were used. On the other hand, it was shortened in a few seconds by this approximation response factor method.

7. The Linear Approximation Indicial Responses about Each Part of Single-Walled Concrete

In the previous section, from a viewpoint of shortening the computation time, the simple load calculation method by the approximation indicial response of the liner heat flux to outdoor air temperature change, and its validity were showed for the experimental basement, which is laid underground completely.

In this section, in order to apply this method to the thermal load calculation of a more realistic basement, the approximation indicial response in each position of the wall and floor were calculated for the basement as shown in Fig. 8.

Fig. 7 Comparison of entire heat flux on inner wall of basement under standard meteorological conditions in Tokyo.

Fig. 8 Sectional plan of the housing-sized basement.

7.1 Calculation Conditions

In order to minimize the tedious and complexity of nonlinear numerical analyses, the numerical analysis of a housing-sized basement is treated as a two-dimensional coordinate system and performed by the finite difference method, with assumed homogeneity of the ground soil/concrete. The expanded AMeDAS weather data of Tokyo was used for this numerical analysis and other calculation conditions are the same for this computation, as described in the previous section.

7.2 Heat Flux Comparison by the Difference of Depth of Basement

The difference of heat flux by the difference of depth of housing-sized basement was investigated in this section.

The depth of floor position of the basement of three cases is 2 m, 3 m and 4 m were calculated. As shown

in Figs. 9 and 10, the positions of the computed heat flux were in five points (A-E) at the floor and three points (A-C) at the wall.

8. The Linearized Prediction of the Heat Load in Basement

8.1 Indicial Response of Heat Flux by Each Part of Wall and Floor

The original function $\varphi(t)$ of the approximation indicial response to the outdoor air temperature change at the depth of 3.0 m calculated by using the heat flux on each part of wall and floor investigated in the previous section are as follows:

Wall-A

$$\varphi(t) = 2.12 - 1.04e^{-0.00015t} + 0.94e^{-0.00017t} - 1.58e^{-0.0129t} \quad (10)$$

Wall-B

$$\varphi(t) = 1.26 - 0.64e^{-0.00024t} + 0.4e^{-0.000244t} - 1.10e^{-0.0072t} \quad (11)$$

Wall-C(3.0m)

$$\varphi(t) = 0.90 - 0.42e^{-0.00020t} + 0.026e^{-0.00049t} - 0.58e^{-0.0031t} \quad (12)$$

Wall-E(3.0m)

$$\varphi(t) = 1.05 - 1.22e^{-0.00023t} + 0.44e^{-0.00037t} - 0.34e^{-0.0022t} \quad (13)$$

Floor-A(3.0 m)

$$\varphi(t) = 0.74 - 1.52e^{-0.00025t} + 0.79e^{-0.00040t} - 0.024e^{-0.0030t} \quad (14)$$

Floor-B(3.0 m)

$$\varphi(t) = 0.86 - 1.11e^{-0.00023t} + 0.28e^{-0.00067t} - 0.032e^{-0.0061t} \quad (15)$$

Floor-C(3.0 m)

$$\varphi(t) = 1.09 - 1.30e^{-0.00020t} + 0.25e^{-0.00028t} - 0.03e^{-0.0043t} \quad (16)$$

8.2 Linearized Prediction by Indicial Response

The overall heat transfer loads of all the area on the wall and the floor of basement were treated as a two-dimensional section in the three kinds of floor depths 2.0 m, 3.0 m and 4.0 m, which were calculated by a convolution calculation of the response factor and outdoor air temperature. Figs. 11-13 show the results of overall heat transfer load in the basement section in the cases of floor depth positions which are G.L-2.0 m, 3.0 m and 4.0 m, respectively. In these figures, “exact solutions” are based on the nonlinear simultaneous heat and moisture transfer equations. As these figures show that the linearized approximation solution agreed

Fig. 9 Schematic diagram of wall and floor division.

Fig. 10 Annual variation of heat flux at floor-A.

Fig. 11 Comparison of heat load of basement at G.L-2 m.

Fig. 12 Comparison of heat load of basement at G.L-3 m.

well with the exact solutions throughout the year, regardless of depth difference of the floor positions.

Fig. 13 Comparison of heat load of basement at G.L.-4 m.

Fig. 14 Comparison of heat load of basement at G.L.-3.5 m.

The difference of the annual heat load and the maximum heat load between the linearized approximation solution and the exact solution were about 4.3%-9.6% and 2.1%-7.2%. It was clarified that the linearized approximation solution has a good predictable accuracy to the exact solution. Therefore, the heat load prediction of the basement using standard indicial response of all parts of wall and floor is applicable to a practical use calculation for a design.

9. Application Method under the Optional Depth Variation

In this section, the overall heat transfer load of the basement of an optional floor depth position was calculated using the indicial responses in each position of the wall and floor shown in the previous section.

A comparison of the approximate solution and the exact solution, of the overall heat transfer load, in the basement section in case, the floor depth position is 3.5 m is shown in Fig. 14. The indicial responses of

the floor position of 3 m and 4 m shown in the previous section were used for this approximate calculation.

The difference of the annual heat load and the maximum heat load between linearized approximation solution and exact solution were about 5% and 2.6%, respectively. Also, the value agreed well throughout a year.

10. Conclusions

To develop a simple prediction method of the heat load of basement with enough accuracy, the authors conducted the numerical analysis concerned with the linearized prediction of heat load in basement, under the optional time fluctuation of meteorological conditions. Also, it estimated linearized prediction by indicial response. The main conclusions in this paper are as follows:

- (1) Because the time-variance of indicial response of heat flux on inner wall of basement under the outdoor air temperature change was small, the indicial response computed by the nonlinear simultaneous heat and moisture transfer equations can be approximately expressed with a single function like a linear system;
- (2) The linear solution by a convolution calculation of the outdoor air temperature and the response factor method using by the approximation indicial response of sensible heat flux to the outdoor air temperature change agreed well with the nonlinear solution throughout one year, more so than the solution by simple heat diffusion;
- (3) The authors arranged the approximation indicial responses of each part of the basement of the dwelling house scale which consists of concrete single wall. It is clarified that the heat load calculation of the basement of the optional floor depth can be predicted by the approximation calculation using them with enough accuracy;
- (4) Judging from the above-points, if the approximation indicial responses are prepared beforehand, the heat load calculation of the basement

can be calculated very easily. This method is very effective as the design tool.

Consequently, the next phases of this study will focus on the arrangement of the indicial response according to the part of some actual wall structure including thickness difference of the thermal insulating materials.

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References

- [1] M. Matsumoto, D. Ogura, An analysis of heat and moisture behavior of underground space and its surrounding ground under natural condition, *Journal of Architecture, Planning and Environment Engineering* 474 (1995) 27-36.
- [2] D. Ogura, Study on the heat and moisture behaviour of the underground structure, Doctor Thesis, Kobe University, Japan, 2000, pp. 18-34.
- [3] K.S. Park, H. Nagai, T. Iwata, Study on the heat load characteristics of underground structures Part 1. Field experiment on an underground structure under an internal heat generation condition, *Journal of Asian Architecture and Building Engineering* 5 (2) (2006) 521-528.
- [4] K.S. Park, H. Nagai, Study on the heat load characteristics of underground structures Part 2. Computational analysis of the heat/moisture behavior and heat load of underground structures, *Journal of Asian Architecture and Building Engineering* 6 (1) (2007) 189-196.
- [5] M. Matsumoto, Simultaneous heat and moisture transfer, and moisture accumulation in buildings, Doctor Thesis, Kyoto University, Japan, 1978.
- [6] M. Matsumoto, *Physics of Architectural Environment*, Chapter 5, Heat Environment & Heat Transfer, Ohmsha, Japan, 1978, pp. 139-185.
- [7] S. Matsumoto, K. Soga, H. Akasaka, Development of data processing programs for the expanded AMeDAS weather data DVD, in: *Proceedings CD-ROM of the 2005 World Sustainable Building Conference (SB05Tokyo)*, Tokyo, 2005, pp. 97-104.
- [8] G. Hedenblad, Moisture permeability of mature concrete, cement and cement paste, Ph.D. Thesis, Division of Building, Materials Lund Institute of Technology, Sweden, 1993.
- [9] W. Jury, Simultaneous transport of heat and mass through a medium sand, Ph.D. Thesis, University of Michigan, USA, 1973.