Seismic Vulnerability of Building Construction Site

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Abstract: Building construction site plays an important role in the economic aspect of a region. So any disruption due to hazard event like earthquake can cause several direct and indirect damages. Direct damages can lead to the loss of equipments and qualified persons. Indirect damages can lead to inflation and loss of purchasing power. To deal with that situation, parameters that govern the vulnerability of building construction site have been identified. Using a MCDM (multiple criteria decision making) method, the priority order of these parameters has been set. Then a PVI (partial vulnerability index) and a GVI (global vulnerability index) are proposed. These indexes allow the classification of building construction sites according to their intrinsic vulnerability and seismic vulnerability respectively that is through the use of a given classification. Several Algerian building construction sites belonging to different seismic zones were considered in order to show the applicability of the method.

Key words: Seismic vulnerability, construction site, risk management, analytical hierarchy process, vulnerability index.

1. Introduction

One of the definition of vulnerability is the capacity of a system, a subsystem or system components to respond to a hazard or its impacts [1-3]. System vulnerability can be described as a regard to its exposure to a hazard, its capacity to resist hazard impacts, and its possibility to recover from the hazard [4]. It is necessary to identify a hazard event in order to give sense to the vulnerability. Vulnerability can be internally created and modified by organizational, social and economic factors, parameters which are not usually connected to hazard events in time and space. This means that vulnerability is not only dependent on the hazard event [5].

Even though certain factors can make damage on a system, making it vulnerable to hazard attack, these can not be dissociated from the system [2]. This is particularly true in the case of an earthquake event, affecting an industrial site (factory) where human and financial factors can not be dissociated [6-11]. The

purpose of vulnerability assessment is to make policies to improve adaptive capacities of a system to cope with hazard impacts [4, 2] especially in the case of building construction site [12, 13] subjected to seismic event.

The present study deals with seismic risk management of building construction sites and aims to improve the impact of factors having an influence on the realization process of buildings. For this purpose, a vulnerability index was developed taking into account factors identified from past earthquakes. These factors were quantified using the techniques of the AHP (analytical hierarchy process) [14, 15]. The obtained index allows the classification of buildings construction site according to its degree of vulnerability. This classification was done according a developed scale.

2. Project Risk Management

The main purpose of the project risk management is to identify, evaluate and control the risks that might delay the progress of the construction process of a project and as a result affect its success. The success of a given project can be difficult to evaluate due to the fact that the project itself is structured in stages.

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Many stakeholders have different criteria to evaluate project success stages. However, the project success criteria are generally measured by time overrun, cost overrun and technical performance [16-18]. In the case of a building construction site the success of the project is measured by the achievement of the construction in time and with the initial estimated cost.

To study the seismic vulnerability of building construction site, four steps were defined.

Step 1: Identification of the parameters that play an important role in the vulnerability of building construction site;

Step 2: Quantification of these parameters using a MCDM (multiple criteria decision making) method;

Step 3: Definition of a vulnerability index;

Step 4: Elaboration of a scale leading to classify the element according its vulnerability index.

3. Identified Parameters

The main factors which govern the vulnerability of a building construction sites must be identified first. Seismic feedback experiences have allowed identifying parameters as follows.

3.1 Human Component

The human component is the personal staff present on the project site. This factor will be denoted CH.

3.2 Equipments

The equipment requirement in a project implementation is based on the importance of the project. This factor will be denoted Eq.

3.3 Supplies

The supplies are all the goods and means needed by the project. This factor will be denoted Ap.

3.4 Organization

The organizations all the schedules made for the project realization. This factor will be denoted Og.

3.5 Site

A site is the location where the construction project will be implemented.

4. Quantification of the Parameters

For a given site or a given seismic zone, the above parameters should be quantified. Judgment and experience play an important role in this process. Multiple criteria decision making methods were often used for this purpose. In the present case, the AHP has been adopted [19, 20].

4.1 AHP Principle

The AHP is a decision-support tool developed by Saaty [21, 22]. It aims to help the decision-maker facing a complex problem with multiple conflicting and subjective criteria (e.g., location or investment selection, projects ranking, and so forth).

The advantages of this approach is that it organizes tangible and intangible factors in a systematic way, and provides a structured yet relatively simple solution to the decision-making problems [21].

To perform the AHP several steps were defined [18], they are, construction of the hierarchy, setting pairs of comparison, prioritization and checking the logic consistency of the analysis [10, 21, 22]. A table was developed by Saaty (Table 1) in order to give a numerical value between two factors.

4.2 AHP Implementation

The hierarchy and the pair-wise comparisons being

Table 1 Gradation scale for quantitative comparison of alternatives [23].

Option	Numerical value (s)
Equal	1
Marginally strong	3
Strong	-
Very strong	7
Extremely strong	9
Intermediate value to reflect fuzzy inputs	2, 4, 6, 8
Reflecting dominance of second	Reciprocals
Alternative compared with first	-

performed [10, 21-23], a priority vector E_i must be determined, this one classify the priority in an increasing or a decreasing relative order:

$$E_i = \sum_{j=1}^n W_{ij} / n \quad (1)$$

with:

$$W_{ij} = a_{ij} / \sum_{i=1}^n a_{ij} \quad (2)$$

where, n is the matrix size and W_{ij} are the elements matrix, the a_{ij} are determined based on the seismic feedback experience using the relative scale measurement shown in Table 1.

Hierarchical synthesis is now used to weight the eigenvectors by the weights of the criteria and the sum is taken over all weighted eigenvector entries corresponding to those in the next lower level of the hierarchy.

Having made all the pair-wise comparisons, the consistency is determined by using the eigenvalue, λ_{max} , to calculate the consistency index, CI as follows:

$$CI = (\lambda_{max} - n) / (n - 1) \quad (3)$$

Judgment consistency can be checked by taking the CR (consistency ratio) of CI with the appropriate value in Table 2. The CR is acceptable, if its value does not exceed 0.10. If the value is greater, the judgment matrix is inconsistent.

5. Vulnerability Index Method

Within this work, two vulnerability indexes were defined. The first one was called “ PVI (partial vulnerability index)” and the second one was called “ GVI (global vulnerability index)”.

The PVI was defined as the sum of the vulnerability of each parameter that has an influence on the activity of the building construction site.

The GVI takes into account the seismic zone where the project is implemented.

So with:

E_i : priority order of the parameter calculated by the AHP;

P : potential risk related to the site or to the seismic zone;

p_i : parameter value;

n_i : referential parameter value.

The vulnerability of a parameter can be expressed by:

$$V_i = E_i * p_i / n_i \quad (4)$$

The PVI of a building construction site is then:

$$PVI = \sum V_i \quad (5)$$

The partial vulnerability index above does not take into account the seismic zone. This is done through the GVI .

It is defined as:

$$GVI = PVI/P \quad (6)$$

This one let the comparison between two building construction sites implemented in two different seismic zones.

According to the Algerian seismic code in use, five seismic zones are defined, from the less seismic zone to the most seismic one (0, 1, 2a, 2b, 3). These ones let the determination of the potential risk P .

6. Proposed Classification

A vulnerability classification of building construction site is proposed (Table 3):

The green interval means that the building construction site is not vulnerable to seismic action and therefore there is no risk of major disruptions. The

Table 2 Average random consistency (RI) [21, 22].

Size of matrix	Random consistency
1	0
2	0
3	0.58
4	0.9
5	1.12
6	1.24
7	1.32
8	1.41
9	1.45
10	1.49

Table 3 Classification of building construction site.

Class	Red	Orange	Green
VI*	0 ÷ 0.25	0.25 ÷ 0.75	0.75 ÷ 1

* Vulnerability index.

red interval means that the building construction site is vulnerable to seismic action and therefore it might suffer from very important disturbances. The orange interval is an intermediate situation.

The PVI is used as a value of the VI in order to classify building constructions sites located in the same seismic zone. In this case, the seismic hazard is not taken into account.

In order to deal with the seismic aspect, the GVI is then used.

To calculate the GVI, the potential risk P , must be determined. This one is deduced from the acceleration coefficient taken from the Algerian seismic code (RPA99) [24] (Table 4).

In the RPA99, the use group is a classification of structures according to their importance. So four groups are distinguished [24]:

- Group 1A: for strategic structures and buildings of very high importance;
- Group 1B: for important structures and buildings higher than 48 m. Water towers are included in this group;
- Group 2: for useful structures and buildings lower than 48 m;
- Group 3: for structures of low importance and temporary buildings.

Based on the previous table, the values of P are

Table 4 Acceleration coefficient according the seismic zone [24].

Use group	Seismic zone			
	1	2a	2b	3
1A	0.15	0.25	0.30	0.40
1B	0.12	0.20	0.25	0.30
2	0.10	0.15	0.20	0.25
3	0.07	0.10	0.14	0.18

Table 5 P values.

Use group	Seismic zone				
	0	1	2a	2b	3
1A	1	1.15	1.25	1.30	1.40
1B	1	1.12	1.20	1.25	1.30
2	1	1.10	1.15	1.20	1.25
3	1	1.07	1.10	1.14	1.18

Table 6 Project' characteristics.

Project No.	Description	Location	Use group
1	1,100 building units	Blida	2
2	Secondary school with 2,000 training capacity	Blida	1B
3	Sixteen levels tower with a basement including three levels	Blida	1B
4	Office building with six floors	Tissemssilt	2
5	Secondary school with 1,200 training capacity	Tissemssilt	1B
6	Water tower with 300 m ³ capacity	Tissemssilt	1B

taken according to Table 5. In the case of Zone 0, the GVI and the PVI are the same, this is correct because there is no seismic risk in this area, so only intrinsic characteristics of the building construction site play a role in its vulnerability.

7. Applications

The proposed method has been applied on several projects in Algeria. Three of the projects are located in the district of Blida, an area with a high potential of seismicity, Zone 3 according to the Algerian Seismic Code [24], and the three others located in the district of Tissemssilt, with lower seismic potential classified, Zone 2a according to the Algerian Seismic Code [24]. The characteristics of each project are given in Table 6.

The parameter value (p_i) is a score out of ten obtained by the building company for each considered parameter, taking into account the means at its disposal to carry out the project (this is a result of the tender process).

7.1 Project 1

The estimated cost of this project (Fig. 1) is around 30 million US\$ and the completion period is 48 months. Detailed calculations are given in Appendix 1.

7.1.1 Priority Factors and Consistency Index

Using the AHP the comparison pairs is established and E_i (Table 7) is derived according to Eq. (2):

According to Eq. (3), $CI = 0.0163$ and $RI = 0.9$, so $CR = 0.018$. This value is less than 10%, so the

judgment matrix is consistent.

7.1.2 Parameters Evaluation

After an in situ evaluation, the parameters value was derived for the different factors and the results are given in Table 8.

7.1.3 Vulnerability Assessment

The PVI is computed and the results are given in Table 9.

$$GVI = PVI/P = 0.7542/1.25 = 0.6$$

The PVI shows a value which is located between the two classes, the green and the orange. This shows that the risk of disturbance is not very high. However, in case of a seismic event it must be deal with it carefully since the GVI belongs to the orange range.

Fig. 1 Project 1 in progress.

Table 7 Project 1 decision matrix.

	CH	Eq	Ap	Og	E_i
CH	1	5	3	4	0.543
Eq	1/5	1	1/3	1/2	0.085
Ap	1/3	3	1	2	0.233
Og	1/4	2	1/2	1	0.14

Table 8 Parameters value of the first case study.

	Human component CH/10	Equipment Eq/10	Supply Ap/10	Organization Og/10
p_i	8	8	6	8

Table 9 Vulnerability of the first case study.

Factors	E_i	p_i	n_i	V_i
CH	0.543	8	10	0.4344
Eq	0.085	8	10	0.068
Ap	0.233	6	10	0.1398
Og	0.14	8	10	0.112
PVI				0.7542

7.2 Project 2

The estimated cost of this project (Fig. 2) is around 4 million US\$ and the completion period is 26 months.

7.2.1 Priority Factors and Consistency Index

Using the AHP the comparison pairs is established and E_i (Table 10) is derived according to Eq. (2):

According to Eq. (3), $CR = 0.03278$. This value is less than 10%, so the judgment matrix is consistent.

7.2.2 Parameters Evaluation

After an in situ evaluation, the parameters value was derived for the different factors and the results are given in Table 11.

Fig. 2 The secondary school in progress.

Table 10 Project 2 decision matrix.

	CH	Eq	Ap	Og	E_i
CH	1	4	3	3	0.501
Eq	1/4	1	1/3	1/2	0.094
Ap	1/3	3	1	2	0.247
Og	1/3	2	1/2	1	0.158

Table 11 Parameters value of the second case study.

	Human component CH/10	Equipment Eq/10	Supply Ap/10	Organization Og/10
p_i	8	7	6	8

Table 12 Vulnerability of the second case study.

Factors	E_i	p_i	n_i	V_i
CH	0.502	8	10	0.4016
Eq	0.094	7	10	0.0658
Ap	0.247	6	10	0.1482
Og	0.158	8	10	0.1264
PVI				0.742

7.2.3 Vulnerability Assessment

The PVI is computed and the results are given in Table 12. $GVI = PVI/P = 0.742/1.30$ and $GVI = 0.57$.

This project has a medium vulnerability since its PVI and GVI belong to the orange range. The main parameters increasing its vulnerability are human component and supplies. So these two parameters should be taken into account in order to decrease the vulnerability of the project.

7.3 Project 3

The estimated cost of this project (Fig. 3) is around 5 million US\$ and the completion times are 48 months.

7.3.1 Priority Factors and Consistency Index

Using the AHP, the comparison pairs is established and E_i (Table 13) is derived according to Eq. (2):

According to Eq. (3), $CR = 0.03287$. This value is less than 10% so the judgment matrix is consistent.

7.3.2 Parameters Evaluation

After an in situ evaluation, the parameters value was derived for the different factors and the results are given in Table 14.

7.3.3 Vulnerability Assessment

The partial vulnerability index is computed and the results are given in Table 15. $GVI = PVI/1.30$, $GVI = 0.59$.

Since the PVI belongs to the green range, the project has a small vulnerability. But in case of a seismic event, it must be deal with it carefully since the GVI belongs to the orange range.

7.4 Project 4

The estimated cost of this project (Fig. 4) is around 15 million US\$ and the completion times are 48 months.

7.4.1 Priority Factors and Consistency Index

Using the AHP, the comparison pairs is established and E_i (Table 16) is derived according to Eq. (2):

According to Eq. (3), $CR = 0.068$. This value is less than 10% so the judgment matrix is consistent.

Fig. 3 Tower in progress.

Table 13 Project 3 decision matrix.

	CH	Eq	Ap	Og	E_i
CH	1	4	3	4	0.525
Eq	1/4	1	1/3	1/2	0.093
Ap	1/3	3	1	2	0.239
Og	1/4	2	1/2	1	0.143

Table 14 Parameters value of the third case study.

	Human component CH/10	Equipment Eq/10	Supply Ap/10	Organization Og/10
p_i	9	8	6	6

Table 15 Vulnerability of the third case study.

Factors	E_i	p_i	n_i	V_i
CH	0.502	9	10	0.4518
Eq	0.094	8	10	0.0752
Ap	0.247	6	10	0.1482
Og	0.158	6	10	0.0948
PVI				0.77

7.4.2 Parameters Evaluation

After an in situ evaluation, the parameters value was derived for the different factors and the results are given in Table 17.

7.4.3 Vulnerability Assessment

The partial vulnerability index is computed and the results are given in Table 18. $GVI = PVI/P = 0.77/1.15$, so $GVI = 0.67$.

Since the PVI belongs to the green range, the project has a small vulnerability. But this vulnerability increase in the case of seismic event. This is highlighted by the GVI which belongs to the orange range.

Fig. 4 Office building in progress.

Table 16 Project 4 decision matrix.

	CH	Eq	Ap	Og	E_i
CH	1	3	4	5	0.520
Eq	1/3	1	3	4	0.268
Ap	1/4	1/3	1	3	0.141
Og	1/5	1/4	1/3	1	0.071

Table 17 Parameters value of the forth case study.

	Human component CH/10	Equipment Eq/10	Supply Ap/10	Organization Og/10
p_i	8	8	7	6

Table 18 Vulnerability of the forth case study.

Factors	E_i	p_i	n_i	V_i
CH	0.520	8	10	0.416
Eq	0.268	8	10	0.2144
Ap	0.141	7	10	0.0987
Og	0.071	6	10	0.0426
PVI	0.7717			

7.5 Project 5

The estimated cost of this project (Fig. 5) is around 6 million US\$ and the completion times are 48 months.

7.5.1 Priority Factors and Consistency Index

Using the AHP the comparison pairs is established and E_i (Table 19) is derived according to Eq. (2).

According to Eq. (3), $CR = 0.063$. This value is less than 10%, so the judgment matrix is consistent.

7.5.2 Parameters Evaluation

After an in situ evaluation, the parameters value was derived for the different factors and the results are given in Table 20.

Fig. 5 Project 5 in progress.

Table 19 Project 5 decision matrix.

	CH	Eq	Ap	Og	E_i
CH	1	5	4	3	0.543
Eq	1/5	1	1/2	1/3	0.086
Ap	1/4	2	1	2	0.189
Og	1/3	2	1/2	1	0.182

Table 20 Parameters value of the fifth case study.

	Human component CH/10	Equipment Eq/10	Supply Ap/10	Organization Og/10
p_i	7	8	8	7

Table 21 Vulnerability of the fifth case study.

Factors	E_i	p_i	n_i	V_i
CH	0.543	7	10	0.3801
Eq	0.086	8	10	0.0688
Ap	0.189	8	10	0.1512
Og	0.182	7	10	0.1274
PVI	0.7275			

7.5.3 Vulnerability Assessment

The partial vulnerability index is computed and the results are given in Table 21. $GVI = PVI/P = 0.7275/1.20$, so $GVI = 0.61$.

The PVI and the GVI belongs to the orange range, indicating that the project has a medium vulnerability.

7.6 Project 6

The estimated cost of this project (Fig. 6) is around 8 million US\$ and the completion times are 48 months.

7.6.1 Priority Factors and Consistency Index

Using the AHP, the comparison pairs is established and E_i (Table 22) is derived according to Eq. (2).

According to Eq. (3), $CR = 0.060$. This value is less than 10% so the judgment matrix is consistent.

7.6.2 Parameters Evaluation

After an in situ evaluation, the parameters value was derived for the different factors and the results are given in Table 23.

7.6.3 Vulnerability Assessment

The partial vulnerability index is computed and the results are given in Table 24. $GVI = PVI/P = 0.8119/1.20$, so $GVI = 0.68$.

The PVI belongs to the green range, the project has a small intrinsic vulnerability, but seismic risk increase it vulnerability, so according to the GVI , this building construction site is classify orange.

Fig. 6 Water tower in progress.

Table 22 Project 6 decision matrix.

	CH	Eq	Ap	Og	E_i
CH	1	6	4	3	0.556
Eq	1/6	1	1/2	1/3	0.079
Ap	1/3	3	1	2	0.189
Og	1/3	2	1/2	1	0.175

Table 23 Parameters value of the sixth case study.

	Human component CH/10	Equipment Eq/10	Supply Ap/10	Organization Og/10
p_i	9	7	8	6

Table 24 Vulnerability of the sixth case study.

Factors	E_i	p_i	n_i	V_i
CH	0.556	9	10	0.5004
Eq	0.079	7	10	0.0553
Ap	0.189	8	10	0.1512
Og	0.175	6	10	0.105
PVI				0.8119

As can be seen from the results, the building construction sites of Zone 2a are less vulnerable than those of Zone 3. The most vulnerable project is the project 2, the least one is Project 6.

8. Conclusions

What can be drawn from this study is that seismic vulnerability studies carried on buildings construction sites can highlight weak points and help project managers to reduce/or protect the site from any seismic resulting hazard. Parameters such as human component, organization, supplies, equipment are main components defining the seismic vulnerability of a given construction site. Indeed, defining the priority order of such component is of great importance to approach its vulnerability. In this study, this order has been obtained using the AHP method. More ever, vulnerability classification was made on the base of the developed parameters PVI and GVI .

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Appendix A: Project 1 Detailed Calculation.

Let put: $C_{ij} = \sum_{i=1}^n a_{ij}$; here $n = 4$, so:

$$C_{11} = 1 + 1/5 + 1/3 + 1/4 = 107/60$$

$$C_{21} = 5 + 1 + 3 + 2 = 11$$

$$C_{31} = 3 + 1/3 + 1 + 1/2 = 29/6$$

$$C_{41} = 4 + 1/2 + 2 + 1 = 15/2$$

According Eq. (2):

$$w_{ij} = \begin{bmatrix} \frac{1}{107/60} & \frac{5}{11} & \frac{3}{29/6} & \frac{4}{15/2} \\ \frac{1/5}{107/60} & \frac{1}{11} & \frac{1/3}{29/6} & \frac{1/2}{15/2} \\ \frac{1/3}{107/60} & \frac{3}{11} & \frac{1}{29/6} & \frac{2}{15/2} \\ \frac{1/4}{107/60} & \frac{2}{11} & \frac{1/2}{29/6} & \frac{1}{15/2} \end{bmatrix}$$

Then applying Eq. (1) leads to:

$$E_i = \frac{1}{4} \begin{bmatrix} \frac{1}{107/60} + \frac{5}{11} + \frac{3}{29/6} + \frac{4}{15/2} \\ \frac{1/5}{1/3} + \frac{1}{11} + \frac{1/3}{29/6} + \frac{1/2}{15/2} \\ \frac{107/60}{1/3} + \frac{11}{3} + \frac{29/6}{1} + \frac{15/2}{2} \\ \frac{107/60}{1/4} + \frac{11}{2} + \frac{29/6}{1/2} + \frac{15/2}{1} \end{bmatrix}$$

Then:

$$CR = \frac{CI}{RI}$$

CI is calculated according Eq. (3) with:

$$\lambda_{\max} = \frac{1}{n} \sum_{i=1}^n \sum_{j=1}^n a_{ij} \frac{E_i}{E_j}$$

and RI is determined from Table 2, so $RI = 0.9$, then:

$$\lambda_{\max} = \frac{1}{4} \begin{bmatrix} \frac{1}{0.543} (1 \times 0.543 + 5 \times 0.085 + 3 \times 0.233 + 4 \times 0.14) + \\ \frac{1}{0.085} (1/5 \times 0.543 + 1 \times 0.085 + 1/3 \times 0.233 + 1/2 \times 0.14) + \\ \frac{1}{0.233} (1/3 \times 0.543 + 3 \times 0.085 + 1 \times 0.233 + 2 \times 0.14) + \\ \frac{1}{0.14} (1/4 \times 0.543 + 2 \times 0.085 + 1/2 \times 0.233 + 1 \times 0.14) \end{bmatrix} \lambda_{\max} = 4.049$$

$$CI = \frac{4.049 - 4}{3} = 0.0163; \quad CR = \frac{0.0163}{0.9} = 0.018$$