Sensitivity Study of Design Parameters of a Railway Route under Vertical Deformations

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Abstract: Studying the interaction of components is basic for a railroad project, which is also very important for creating maintenance procedures based on predictions from a model that assumes adequate performance. To determine these interactions, which can have a great number of combinations, the use of a computational model is of vital importance, in this case, the program FERROVIA 1.0 (RAILROAD 1.0) was used. A critical study developed the program FERROVIA 1.0, and its variables were characterized based on values observed in the literature. After initial characterization was carried out, a comparative study was performed on the sensitivity between these variables and the indications of significant behavior for a railway deformed by the known load of a wheel. A statistical program was used to correlate the elements. The intention was to launch the program FERROVIA 1.0 and later use it for 2,187 combinations. The data used in these correlations corresponded to the normal values for the elements used in railroad engineering practice. Our main goal is to understand the behavior of the track vertical deflection according to the variation of the scaling of the various elements of the railway.

Key words: Railway, computational model, RAILROAD 1.0.

1. Introduction

The appropriate dimensions of railroads, such as the superstructure and infrastructure, are dependent on knowledge of elements that constitute the whole structure and the interaction between them. An important variable for understanding the interaction of these elements is the vertical deflection of the railroad.

The vertical deflection defines the need for maintenance procedures and thus the operating limits of the rail system. Therefore, it is a suitable parameter for railroad project designs.

By varying the dimensions of the track elements and comparing the final vertical deflection for each sleeper in accordance with boundary conditions, the optimal design with regard to structure and operation can be determined.

Preliminarily, the sensitivity of these elements in relation to vertical deflection of the track can also be determined as minor or major; this aids in

decision-making regarding the characteristics necessary for good system performance.

2. FERROVIA 1.0

The classic methodology for railroad design has low productivity for the development of a broader study of sensitivity. Thus, it is essential to use a computational method to ease the workload.

The program RAILROAD 1.0 is a software developed by Rodrigues [1] and was used in this study. This model seeks to make more realistic predictions of the tension than traditional computational models, according to Rodrigues [1], the latter's structural models are quite drastic simplifications, which makes them very dependent on empiricism with regard to validity. His program uses the finite element method for rails, sleepers, and fastenings and the method of finite layers for the infrastructure layers.

Rodrigues [1] described the mathematical process in his program, which uses the FERROVIA 1.0 inversion of the flexibility matrix to generate a stiffness matrix of the infrastructure. In order to

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calculate displacements and rotations of the nodal points, the stiffness matrix is added to the stiffness matrix of the superstructure, and the resulting system of linear equations is solved. The resilient deformation of the modules simulates the stress-strain behavior of soils and granular materials under the action of a moving load on the railroad. An iterative process is performed until convergence is achieved, the dependence of the resilient modulus on the generated stress state is considered to update the modules according to the tension. The effect of confinement generated by the self-weight of the layers is also considered.

The program FERROVIA 1.0 uses grade eleven sleepers (Fig. 1). The difference is that the grid along the track is a finite element mesh used for calculations [1]. To calculate the stresses acting in the layers of infrastructure, diagrams are used to illustrate the contact pressure between the sleeper and the top of the ballast; this pressure is related to the calculated displacements. This model is considered more appropriate than the stress estimated by Boussinesq because, for moving loads, the use of the latter does not produce satisfactory results when compared with the experimental values. The main features of the program are as follows [1]:

- (1) calculation of stresses and strains in the layers for the ballast, sub-ballast, sub-grade and rails;
- (2) low computational effort, even for a three-dimensional profile of direction;
- (3) increased model complexity compared with existing ones. Fig. 1: Grade eleven sleepers with the nodal points of the program based on FERROVIA 1.0 [1].

3. Sensitivity Study on the List of Variables for FERROVIA 1.0

3.1 Variables Used in This Study

The sensitivity study was conducted using the list of program variables for FERROVIA 1.0 presented in Table 1. Importantly, only the broad gauge value of

1,600 mm was considered for the parametric study in this work. The modulus of elasticity of the rail pattern depended on the material and did not differ for all analyses.

3.2 Variable Values in This Study

To study the sensitivity based on the variables in Table 1 and the above observations, used the values presented in Table 2. These values were mostly obtained from the Brazilian technical literature: Brina [2] and Stopatto [3], to be specific. Some values were obtained by varying a default value by a percentage more or less than the classical value, for others, we used extreme values that were not established in the literature in order to study the limits for the change in strain and understand the amplitude of the sensitivity of certain variables. Table 2 summarizes preliminary results to separate the most sensitive variables from the less sensitive ones, this was done to further study the former. In Table 2, it is important to note that the value representing the soil behavior transmits requests to the modulus of linear elasticity.

Based on values studied by Barros [4], soils are represented by a modulus of elasticity of 70-3,500 kgf/cm², this may include platforms and some types of sub-ballast. A modulus of elasticity of 700-7,000 kgf/cm² represents flexible platforms and can be used for the ballast and other types of sub-ballast. For the ballast, Spada [5] performed triaxial tests to determine a modulus of elasticity of 1,200-2,200 kgf/cm².

The variables in bold in Table 2 correspond to the values used as a reference for preliminary analysis of elements in order to maintain a standard for all variations that will be performed, this will allow a comparison of results. When the values of the elements of the track are launched one at a time, these values in bold are fixed.

This study considered a standard track of two axes with a spacing of 1.50 times the wheelbase. The point of load application has the center of the large FERROVIA 1.0 program as its origin, as shown in Fig. 1.

Fig. 1 Grade eleven sleepers with the nodal points of the program based on FERROVIA 1.0 [1].

Table 1 Variables studied in the program RAILROAD 1.0.

Spacing of sleepers	k fastening	Inertia rail	Transverse section area of the rail
Rail base width	Modulus of elasticity of the sleepers	Inertia sleepers	Modulus of elasticity of the ballast
Modulus of elasticity of sub-ballast	Modulus of elasticity of the platform	Thickness of ballast	Thickness of sub-ballast
Thickness of the platform	Poisson's ratio of the ballast	Poisson's ratio of the sub-ballast	Poisson's ratio of the platform
Friction angle of the ballast	Friction angle of the sub-ballast	Friction angle of the platform	Applied load

Table 2 Values to be studied for initial qualification of sensitivity for each element with the program RAILROAD 1.0.

Element	Variations				
	1	2	3	4	5
Track gauge	Large	1,600 mm	-	-	-
K (kN/m)	50,000	70,000	90,000	-	-
Load (T)	10	20	30	40	-
Rail	TR57				
Inertia (cm ²)	2,184.4	2,457.45	2,730.5	3,003.55	-
Rail area (cm ²)	58.06	65.32	72.58	79.84	87.09
Width of rail base (cm)	13.5	13.97	14.5	-	-
Sleepers					
Length (m)	2.65	2.7	2.8	-	-
Height	0.01	16	17	18	25
Spacing of sleepers (cm)	50	55	60	65	70
Modulus of elasticity (kgf/cm ²)	94,000	105,000	154,000	-	-
Layers					
Platform					
Modulus of elasticity (kgf/cm ²)	100	200	400	700	-
Poisson's ratio	0.4	0.45	0.5	-	-
Cohesion	0.1	0.3	1	5	-
Friction angle	0.01	25	30	45	-
Thickness (cm)	10	15	20	-	-
Sub-ballast					
Modulus of elasticity (kgf/cm ²)	700	800	1,000	1,200	-
Poisson's ratio	0.25	0.3	0.35	-	-
Cohesion	0.1	0.3	1	5	-
Friction angle	0.01	25	30	45	-
Thickness (cm)	10	15	20	-	-
Ballast					
Modulus of elasticity (kgf/cm ²)	1,200	1,500	2,000	2,700	-
Poisson's ratio	0.25	0.3	0.35	-	-
Cohesion	0.1	0.3	1	5	-
Friction angle	0.01	25	30	45	-
Thickness (cm)	25	30	35	-	-

The other point depends on the spacing of sleepers and corresponds to the node of the finite element mesh close to the load line.

4. Sensitivity Study

An example of the process is presented in Table 3 with the variation in Poisson's ratio of the ballast. Table 4 summarizes the vertical deflection results for

each sleeper, and Fig. 2 shows the variation in deflection over the 11 dormant grids analyzed (20 T = 20,000 kgf).

In Fig. 2, Series 1-3 correspond to the series in Table 4.

It is noted that more variables could have been regarded as sensitive, but that would go beyond the traditional values of their characteristics, or power

Table 3 Cover for calculation of the deformations with the variation in Poisson's ratio of the ballast.

Element	Variations				
	1	2	3	4	5
Track gauge	Large	1,600 mm	-	-	-
K (kN/m)	50,000	70,000	90,000	-	-
Load (T)	10	20	30	40	-
Rail	TR57				
Inertia (cm ²)	2,184.4	2,457.45	2,730.5	3,003.55	-
Rail area (cm ²)	58.06	65.32	72.58	79.84	87.09
Width of rail base (cm)	13.5	13.97	14.5	-	-
Sleepers					
Length (m)	2.65	2.7	2.8	-	-
Height	0.01	16	17	18	25
Spacing of sleepers (cm)	50	55	60	65	70
Modulus of elasticity (kgf/cm ²)	94,000	105,000	154,000	-	-
Layers					
Platform					
Modulus of elasticity (kgf/cm ²)	100	200	400	700	-
Poisson's ratio	0.4	0.45	0.5	-	-
Cohesion	0.1	0.3	1	5	-
Friction angle	0.01	25	30	45	-
Thickness (cm)	10	15	20	-	-
Sub-ballast					
Modulus of elasticity (kgf/cm ²)	700	800	1,000	1,200	-
Poisson's ratio	0.25	0.3	0.35	-	-
Cohesion	0.1	0.3	1	5	-
Friction angle	0.01	25	30	45	-
Thickness (cm)	10	15	20	-	-
Ballast					
Modulus of elasticity (kgf/cm ²)	1,200	1,500	2,000	2,700	-
Poisson's ratio	0.25	0.3	0.35	-	-
Cohesion	0.1	0.3	1	5	-
Friction angle	0.01	25	30	45	-
Thickness (cm)	25	30	35	-	-

Table 4 Strain values for the eleven sleepers of the program grid FERROVIA 1.0 according to the variation in Poisson's ratio of the ballast.

	Series 1	Series 2	Series 3
Poisson platform	0.49, 0.35, 0.35	0.45, 0.35, 0.35	0.4, 0.35, 0.35
Nodal points	0.49	0.45	0.4
146	0.01015092	0.01036511	0.01046264
148	0.01065648	0.01083492	0.01087964
150	0.002893097	0.002993601	0.002959552
152	(-)0.0249193	(-)0.02495101	(-)0.02509844
154	(-)0.07871997	(-)0.07891775	(-)0.079191911
156	(-)0.1288259	(-)0.1291784	(-)0.1295641
158	(-)0.1215488	(-)0.1220021	(-)0.1224605
160	(-)0.124463	(-)0.1239397	(-)0.1253931
162	(-)0.1359708	(-)0.1373735	(-)0.136773
164	(-)0.08292893	(-)0.08321244	(-)0.08352727

Fig. 2 Graph of deformation of the grid for eleven 1.0 FERROVIA ties that were programmed according to the variation in Poisson's ratio of the ballast.

would incur in unrealistic situations to create false sensitivity. A typical case is the Poisson coefficient for the infrastructure layers.

Using the values available in the literature, we found that the ballast values could have exceeded the 5% range but would be based on unrealistic Poisson values for this type of layer. The same methodology was used to calculate the strains in relation to the variation in Poisson's ratio of the ballast for the analysis of the other elements. Figs. 3-5 illustrate the results for the modulus of elasticity of the tie, spacing of railway sleepers and applied loads.

For the sensitivity study (Fig. 3), the moduli of elasticity of the following sources were used for the

sleeper ties: Peroba Rosa (94,000 kgf/cm²), Parana Pine (105,000 kgf/cm²), and Ipe Tobacco or Yellow (154,000 kgf/cm²) [6].

The sensitivity to the spacing of the sleepers following deformation was obtained as shown in Fig. 4. For the sleepers, increased spacing led to an increase in strain. This result was observed for all spacing variations from 50 cm to 70 cm in 5 cm increments between the two extreme values.

To determine the load sensitivity (Fig. 5), loads of 10-40 tons (10,000-40,000 kgf) representing light passenger trains on one end to overweight freight trains on the other were tested.

The strain was observed to vary linearly in relation

Fig. 3 Graph of deformation for the variation in the modulus of elasticity of sleepers (kgf/cm²). Strains are shown in order to facilitate visualization of centimeter deformation.

Fig. 4 Graph of deformation for the eleven sleepers of the program grid RAILROAD 1.0 obtained by varying the spacing between the sleepers.

Fig. 5 Graph of deformation for the eleven sleepers of the program grid RAILROAD 1.0 obtained by varying the axle load (1 T = 1,000 kgf).

to the load, as shown in Fig. 6.

The deformations are in centimeters to facilitate visualization of the variation in deformation (1 cm = 0.01 m).

5. Evaluation of the Sensitivity of Each Variable

The process for selecting the variables was established via the statistical test. This adjustment was used to divide variables as being more or less sensitive. The statistical test was an adaptation of the level of significance test.

According to Spiegel [7], the probability should be

chosen before the extraction of any sample so that the results do not influence the choice. A significance level of 0.05 was chosen, which means that the odds

Fig. 6 Scatter plot for deformations with varying axle loads.

are 100:5 that the hypothesis would be rejected. In general, it means that there is 95% confidence that the decision is correct.

Whereas, the reference variables were examined one by one, the variables evaluated in this analysis were divided into three groups. The less sensitive variables varied by less than 5%, those with low sensitivity varied by 5%-10%, and very sensitive variables varied by more than 10%, as indicated in Table 5. Thus, Table 6 lists the variables with ratios according to the results obtained in the program FERROVIA 1.0, whereas the values in Table 2.

The values in Table 6 were obtained using the following procedure:

- For a given infrastructure, each element is varied one at a time. An example of a variation component is presented in Table 3, where Poisson's ratio of the ballast is varied and all other values are maintained;
- There was a primary reason for the change in the

element;

- The second reason was found by deformation of the first element, and the second value was calculated by the program RAILROAD 1.0;
- The resulting division of the variable ratio by the deformation ratio for each rail element is indicated in Tables 5 and 6 together with the non-sensitive, slightly sensitive, and very sensitive variables.

For instance, when the inertia value was 2,184.40 cm⁴, this was comparable with the inertia of the track at 2,457.45 cm⁴. From this analysis, the first reason was derived. The ratio of rail deformation at the same inertia was compared. A variation of 12.5% in the inertia of the rail resulted in an almost 10% variation in the deformation, so the variable was considered to be very sensitive. Closer ratios of the resulting unit value or those below unity meant greater sensitivity.

Units in centimeters are normal for technical use of railways.

Table 5 Construction of the sensitivity control table (relative values).

Variable	Value base (Vb)	Comparative value (Vc)	Deflection in sleepers d1			
			Vc/Vb	DVb	DVc	DVc/DVb
Inertia of rail (cm ⁴)	2184.40	2475.45	1.125	0.008323010	0.009134151	1.097
Spacinf of sleepers (cm)	50	55	1.100	0.018348860	0.013658798	0.744
E of sleepers (kgf/cm ²)	94,000.00	154,000.00	1.638	0.009896578	0.008721457	0.881
Fixing of K (kgf/cm ²)	50,000.00	90,000.00	1.800	0.012222367	0.008715002	0.713
Poisson's ratio of ballast	0.25	0.35	1.4	0.009843040	0.010365110	1.053
Poisson's ratio of sub-ballast	0.25	0.35	1.4	0.010469300	0.010365110	0.990
Poisson's ratio of platform	0.40	0.49	1.225	0.010462640	0.010150920	0.970
E of ballast (kgf/cm ²)	1,200.00	1,500.00	1.250	0.005852514	0.007200256	1.230
E of sub-ballast (kgf/cm ²)	700.00	1,200.00	1.714	0.009896578	0.010251200	1.102
E of platform (kgf/cm ²)	400.00	700.00	1.750	0.009896578	0.003272004	0.331
Cohesion of ballast(kgf/cm ²)	0.01	0.3	30.000	0.010365110	0.010365110	1.000
Cohesion of Sub-ballast (kgf/cm ²)	0.01	0.3	30.000	0.010365110	0.010365110	1.000
Cohesion of platform (kgf/cm ²)	0.01	0.3	30.000	0.010365110	0.010365110	1.000
Friction angle of ballast (°)	25	45	1.800	0.010365110	0.010365110	1.000
Friction angle of sub-ballast (°)	25	45	1.800	0.010365110	0.010365110	1.000
Friction angle of platform (°)	25	45	1.800	0.010365110	0.010365110	1.000
Width of rail base (cm)	13.50	14.50	1.074	0.009896578	0.009896578	1.000
Rail area (cm ²)	58.06	87.09	1.500	0.009896578	0.009896578	1.000
Load (T)	10.00	40.00	4.00	0.00494761	0.009896578	2.000
Height of sleeper (cm)	16.00	18.00	1.125	0.010365110	0.009468560	0.914

Table 6 Table sensitivity control variables (relative values).

Element	First sleeper's grid			Identification		
	Ratio variable	Ratio deformation	Final ratio	No sensible (< 5%)	Little sensible (5-10%)	Sensible (> 10%)
Inertia rail	1.125	1.097	1.026			Yes
Spacing sleepers	1.1	1.34	0.821			Yes
E sleepers	1.64	1.13	1.451		Yes	
K fastening	1.8	1.40	1.286			Yes
Poisson's ballast	1.4	1.05	1.333		Yes	
Poisson's sub-ballast	1.4	1.01	1.386	Yes		
Poisson's platform	1.22	1.03	1.184		Yes	
E ballast	1.25	1.23	1.016			Yes
E sub-ballast	1.71	1.10	1.555		Yes	
E platform	1.75	3.02	0.579			Yes
Cohesion ballast	30	1.00	30	Yes		
Cohesion sub-ballast	30	1.00	30	Yes		
Cohesion platform	1.8	1.00	30	Yes		
Friction angle ballast	1.8	1.00	1.80	Yes		
Friction angle sub-ballast	1.075	1.00	1.80	Yes		
Friction angle platform	1.5	1.00	1.80	Yes		
Width rail base	4.00	1.00	1.075	Yes		
Rails area	1.56	1.00	1.50	Yes		
Load	4.00	4.00	1.00			Yes
Height's ties	1.56	1.38	1.13			Yes

6. Conclusions and Suggestions for Future Research

The results showed that the following variables are more sensitive: inertia of the rail, spacing of sleepers, fixing of K, modulus of elasticity of the ballast, modulus of elasticity of the platform, load height and sleeper. Slightly sensitive variables include modulus of elasticity of the sleepers, Poisson's ratios of the ballast and platform, and modulus of elasticity of the sub-ballast. Non-sensitive variables were Poisson's ratio for the sub-ballast; cohesion of the ballast, sub-ballast and platform; friction angles of the ballast, sub-ballast and platform; width of the rail base and rail area.

It is noted that more variables could have been regarded as sensitive, but this would have gone beyond their traditional characteristic values, or power would incur unrealistic situations to create false sensitivity. A typical case is Poisson's ratio of the infrastructure layers. Using the values available in the

literature, we found that the value for the ballast could have exceeded the 5% range but would be based on unrealistic values for this type of layer.

Importantly, the values were similar to those obtained through the computational model of Tarabji and Thompson [8], this demonstrates that the program RAILROAD 1.0 produces results consistent with other computational models for railroad design.

The results reinforced the importance of variables such as the inertia of the rail, which is represented by the profile and spacing for the ties and determines the maximum number of sleepers per kilometer of road. The height of the sleeper, which can also be represented by its inertia, is also important in studies on track-like structures. Such components have a direct effect on the conduct of load demands for the various infrastructure layers. Also of importance is the influence of the degree of rigidity of the rail mountings on the process.

The moduli of elasticity for the ballast and platform, which represent the material characteristics of the

infrastructure, proved to greatly influence the vertical deflection of the track, this was already expected. However, the inverse relationship that occurs in the ballast is important. A higher stiffness generates a greater deformation, this can be partly explained by the loss of elasticity this layer causes to the track structure. The platform had a proportional relationship, i.e., a lower stiffness generated greater deformation, which is also normal.

The load showed deviation from proportionality of the deformations. A two-fold increase in the amount of charge led to deformation with the same proportion.

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