Peculiarities of Calculating Bridges with Seismic Isolation Including Spherical Bearings and Hydraulic Dampers in Russia

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Abstract: The peculiarities of calculating isolated structures with spherical bearings are analyzed in this paper. Some of peculiarities are caused by the lack of data at the moment when engineering solutions had to be made. Other peculiarities are connected with physical peculiarities of the device behaviour. To provide the analysis of structure behaviour under the condition of the lack of input information, two types of design models of seismic protection devices were considered. They are the dampers linearization and the modelling of real dampers by dry friction ones. The dampers linearization makes it possible to use the existing software for calculating linear strongly-damped systems. To calculate structures with dry friction dampers, a new software was worked out. In this case, the structure is described as a piecewise-linear system of a relay-type. The investigations of the structure oscillations take into account both horizontal and vertical components of earthquake input. Under this condition, horizontal oscillation equations of structures are the Mattie-Hill ones. The input and structure parameters which caused the structure instability are estimated. To exclude the structure instability, high damping devices should be used. These methods were used for seismic resistant analysis of bridges with spherical bearings and hydraulic dampers applied in Sochi.

Key words: Seismic isolation, spherical bearings, hydraulic dampers, calculations.

1. Introduction

Spherical bearings and hydraulic dampers began to be used in Russia for seismic strengthening structures especially for bridges in the last decade. But the existing Russian guidelines do not provide for calculating such structures. In accordance with the Russia Building Code, special technical guidelines are to be worked out for each structure. A problem which arises during this work is connected with lack of data about earthquake input in the process of designing as these data can be available only at the end of the design process.

This paper presents and summarizes the authors' experience in this field in respect to motor road bridges with seismic protection devices of Maurer

Söhnes and FIP Industriale firms. In calculating these protection devices, we had to consider four problems:

(1) The first problem is connected with using the response spectra method for a preliminary analysis of structures with seismic protection. Such analysis is necessary for the first design stage when seismological data are not available and the designer of the structure has to choose the type of seismic protection and estimate its parameters. Most part of isolating bearings can be considered as linear ones and their stiffness is usually stated by their manufacturers. But hydraulic dampers have a nonlinear force-displacement diagram. Generally, it can be described by the following formula [1, 2]:

$$Q = -b \cdot |V|^v \cdot \text{sign}(V) \quad (1)$$

where, Q is the damping force, b and v are the damper parameters, V is the system velocity.

The damping force depends on two parameters b

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and ν . If $\nu = 0$, Eq. (1) describes dry friction damping, if $\nu = 1$, it describes viscous damping, and if $\nu = 2$, it describes turbulent dissipation. For hydraulic dampers of Maurer Söhnes and FIP Industriale firms, $0.1 < \nu < 0.2$. To use the response spectra method, one has to linearize Eq. (1);

(2) The second problem is similar to the first one. To make a preliminary analysis of the behaviour of the structure with hydraulic dampers, it may be convenient to substitute the dry friction force for the real damping force. After such substitution, the well-known methods and software can be used;

(3) The third problem is to set earthquake input without sufficient information about earthquake conditions on the building site;

(4) The forth problem concerns the influence of vertical excitation on the horizontal structure oscillations. This problem can be important for spherical bearings, because their horizontal stiffness depends on their vertical reactions which depend on the vertical excitation.

The solution of the above-mentioned different problems was necessary to be able to solve the main problem, i.e., to substantiate the efficiency of bridge seismic isolation and to set its parameters. The abovementioned problems with the methods of their solving and the results obtained are considered below.

2. Linearizing Motion Equations for Structures with Spherical Bearings and Hydraulic Dampers

The equation of oscillations of an isolated structure under consideration with one degree of freedom can be presented as follows:

$$q'' + \mu N \text{sign}(y') + b|q'|^\nu \text{sign}(q') + k^2 q = 0 \quad (2)$$

where, q is the structure displacement, μ is the friction ratio of spherical bearings, N is the bearing reaction, b and y is the maximum oscillation displacement, k is the isolated structure frequency.

$$k^2 = \frac{Hg}{(H-r)^2} \quad (3)$$

where, H is the distance between centres of circle of the spherical surface of the bearing plate and that of the movable spherical element, r is the radius of the movable spherical element and g is the acceleration of gravity.

The kinematics of the bearing under consideration is described by Uzdin, Fedorova and Durseneva [1] and Uzdin, Elizarov and Belash [2].

The absence of design earthquakes at the beginning of designing process makes it necessary to use the response spectra method in the preliminary structure analysis. To estimate oscillations parameters of the system using the response spectra method, the linearization of Eq. (2), i.e., the replacement of Eq. (2) by the equivalent linear one, is necessary. The equivalent linear equation has the following form:

$$q'' + 2n_e q' + k_e^2 q = 0 \quad (4)$$

where, k_e and n_e are frequency and damping parameters of the equivalent linear system.

To characterize damping parameters, the authors used the damping ratio in portion of the critical one $\zeta = n_e/k_e$.

The equivalent damping ratio for the first damping member in Eq. (2) was estimated using the harmonic linearization method. The value of the ratio is given in the Maurer technical information [3] and can be presented as below:

$$\zeta = \frac{2}{\pi} \frac{\mu}{\mu + \frac{y_{\max}}{r}} \quad (5)$$

where, $y_{\max}$ is the maximum oscillation displacement and r is the radius of the bearing spherical surface.

To estimate the equivalent damping ratio for the second damping member in Eq. (2), the authors also used harmonic linearization method. In this case, the following dependence can be written as below:

$$2n_e = \frac{S}{A} \quad (6)$$

where, $A = q_{\max}$ is the amplitude of oscillations.

S is calculated by Eq. (7):

$$S = \frac{1}{\pi} \int_0^{2\pi} \left[Ak^2 \cos \theta + (A\omega)^\nu b(-\sin \theta)^\nu \right] \sin \theta d\theta \quad (7)$$

where, ω is the oscillation frequency and θ is the integration variable.

This integral can be transformed into the following:

$$S = \frac{1}{\pi} \int_0^{2\pi} \left[Ak^2 \cos \theta + (A\omega)^\mu b(|-\sin \theta|)^\mu \right] \sin \theta d\theta =$$

$$\frac{1}{\pi} \int_0^{2\pi} \left[Ak^2 \cos \theta \right] \sin \theta d\theta + \frac{b}{\pi} \int_0^{2\pi} \left[(A\omega)^\mu (\sin \theta)^\mu \right] \sin \theta d\theta$$

$$= \frac{4b(A\omega)^\mu}{\pi} \cdot 2^\mu \cdot B\left(\frac{\mu+2}{2}, \frac{\mu+2}{2}\right) \quad (8)$$

where, $B(x, y)$ is the β -function with arguments x and y .

Using Eq. (8), we can write Eq. (9) as follows:

$$2n_e = \frac{4b(A\omega)^\mu}{\pi A\omega} \cdot 2^\mu \cdot B\left(\frac{\mu+2}{2}, \frac{\mu+2}{2}\right) =$$

$$\frac{1}{\pi} 2^{\mu+2} A^{\mu-1} \omega^{\mu-1} b B\left(\frac{\mu+2}{2}, \frac{\mu+2}{2}\right) \quad (9)$$

In its turn, β -function is expressed by using the Γ -function [4]:

$$B\left(\frac{\mu+2}{2}, \frac{\mu+2}{2}\right) = \frac{\Gamma\left(1+\frac{\mu}{2}\right) \cdot \Gamma\left(1+\frac{\mu}{2}\right)}{\Gamma(2+\mu)} = \frac{\left(\Gamma\left(1+\frac{\mu}{2}\right)\right)^2}{\Gamma(2+\mu)}$$

The dependence of β -function B on the parameter μ is given in Fig. 1.

In accordance with the results shown above, a seismoisolating element including the pendulum

isolator and the hydraulic damper can be replaced in calculations by a spring with the rigidity defined by Eq. (3) and two dampers. The first damper has the damping parameter defined by Eq. (5) and it characterises energy losses caused by friction in the pendulum isolator. The second damper has the damping parameter defined by Eq. (9) and it characterises energy losses in the hydraulic damper.

The proposed systems linearization allows us to carry out engineering calculations of constructions using the response spectra method, taking into account damping parameters. The technique to be used for this purpose was proposed in Refs. [2, 5].

3. Modelling Hydraulic Dampers by the Sum of Viscose and DFD

To approximately analyze the behaviour of seismic isolated systems, the software developed in the former USSR (Soviet Union) [6, 7] for calculating systems with DFD (dry friction dampers) was used. The considered systems are piecewise-linear ones. At any moment, the nonlinear connection of the system, like the relay, can be in one of the two states. A dry friction damper can be opened (be sliding) or closed (be jammed). Shutting down connections can be intact or destroyed and the moving restrictor can be opened

Fig. 1 Dependence of β -function (the value of B) on the non-dimensional parameter μ

(the gap is open) or closed (the gap is closed). If the system includes n such connections, the system state is described by a system status vector of length n , consisting of 0 and 1. If 1 is on the k -th place of this vector, the k -th connection is closed, otherwise it is opened. For example, the system with three dry friction dampers and the system status vector $V = [1 \ 0 \ 1]$ have two closed dampers (the first and the third) and one opened damper. If the system status vector is regarded as the binary representation of a number, we can give an unequivocal number for each system state. In the example considered above, this number $s = 5$. The greatest number of the system state corresponds to the vector with completely closed connections and is equal to $s_{\max} = 2^n - 1$. The least number $s_{\min} = 0$ (all connections are opened). Thus, the system can be in one of the 2^n conditions.

The equation of the system movement in the state “ s ” looks like the following:

$$M_s q'' + B_s q' + C_s q = V_p(t) + P_1 + P_2 \quad (10)$$

where, M_s , B_s , C_s are the inertia, damping and rigidity matrixes in the state “ s ”, $V_p(t)$ is the generalised force corresponding to external loading, P_1 is the generalised force corresponding to loads caused by friction forces in the opened DFD, P_2 is the generalised forces corresponding to loads caused by residual displacement in closed DFD.

To solve Eq. (10), an approximate or exact eigen-mode expansion of the Eq. (10) is made with the subsequent using the Duhamel integral for presenting the solution.

The use of the mentioned program demands that the nonlinear damper model be presented as two simple dampers. The first is a viscous damper with the damping ratio ζ and the second is a dry friction damper with the friction factor f . This problem is considered as follows:

$$b|q'|^\nu \text{sign}(q') \approx bq' + f\text{sign}(q') \quad (11)$$

To provide for approximate Eq. (3), minimization of the square of the difference of the right and left

parts of parity (3) at harmonious oscillations $q = q_0 \sin \omega t$ during the period $T = 2\pi/\omega$ is required, which means to minimize the function Φ .

$$\Phi = \int_0^T (b|q'|^\nu \text{sign}(q') - \beta q'' - f\text{sign}(q'))^2 dt \rightarrow \min \quad (12)$$

To fulfil conditions of Eq. (12), Eq. (13) should be written as follows:

$$\begin{aligned} \frac{\partial \Phi}{\partial \beta} &= -2 \int_0^T |q'| (B|q'|^\nu - \beta |q'| - f) dt = 0 \\ \frac{\partial \Phi}{\partial f} &= -2 \int_0^T (B|q'|^\nu - \beta |q'| - f) dt = 0 \end{aligned} \quad (13)$$

The following combined equations can be obtained using Eq. (13):

$$\begin{cases} \beta I_2 + f I_1 = b I_\nu \\ \beta I_1 + f T = b I_\nu \end{cases} \quad (14)$$

In this system, the following symbols are used:

$$I_1 = \int_0^T q' dt = \frac{2q'}{\omega}, \quad I_2 = \int_0^T q'^2 dt = \frac{\pi q'}{\omega}$$

$$I_\nu = \int_0^T q'^\nu dt = \frac{2}{\omega} 2^\nu B\left(\frac{\nu+1}{2}; \frac{\nu+1}{2}\right)$$

$$I_{\nu+1} = \int_0^T q'^{\nu+1} dt = \frac{4}{\omega} 2^\nu B\left(\frac{\nu}{2}+1; \frac{\nu}{2}+1\right)$$

The solution of Eq. (14) is:

$$\begin{cases} f = \frac{b I_\nu I_2 - b I_1 I_{\nu+1}}{T I_2 - I_1^2} \\ \beta = \frac{b T I_{\nu+1} - b I_1 I_\nu}{T I_2 - I_1^2} \end{cases} \quad (15)$$

By setting f and β parameters in accordance with Eq. (15), we can model real dampers by viscose and dry friction dampers.

4. Earthquake Input Setting

A very important problem was to generate design accelerograms using somewhat limited information about peculiarities of earthquake input on the building site, as only the atlas of seismic shakability for the

territory of USSR [8], including positions of seismic focuses and possible earthquake magnitudes, was available.

To build artificial design accelerograms, we presented seismic input as the sum of incoherent excitations of the original ground points, caused by the casual heterogeneity of the soil environment and the running frozen wave. The above-mentioned way was described by Dmitrovskaya and Uzdin [9] and Uzdin et al. [10]. The proposed input presentation makes it possible to write down Eq. (16) as follows:

$$u_s = u_s^{(inc)} + u_s^{(fw)} \quad (16)$$

where, u_s is the displacement of the s -th surface point; $u_s^{(fw)}$ is the displacement component of the s -th surface point caused by the running frozen wave; $u_s^{(inc)}$ is the displacement component of the s -th surface point caused by some accidentally additional process.

The running wave can be presented in the form of a speed impulse, given in Ref. [11]. Accelerations, displacements and velocities of this process are presented as follows:

$$a(t) = \begin{cases} u_{\max}/t_0^2 \\ -u_{\max}/t_0^2 \\ 0 \end{cases}, \quad v(t) = \begin{cases} u_{\max}t/t_0^2 \\ \frac{u_{\max}}{t_0} \left(2 - \frac{t}{t_0} \right) \\ 0 \end{cases}, \quad (17)$$

$$u(t) = \begin{cases} \frac{u_{\max}}{2} \left(\frac{t}{t_0} \right)^2 \\ \frac{u_{\max}}{2} \left[-\left(\frac{t}{t_0} \right)^2 + 4 \left(\frac{t}{t_0} \right) - 2 \right] \\ u_{\max} \end{cases}$$

where, $u_{\max}$ is the residual displacement; t_0 is the duration of a half of the stepped process; t is the universal time.

The high-frequency component of the proposed input can be presented by various ways. The authors use the following presentation of this component:

$$y'(t)_{\text{noise}} = \sum_{i=1}^N e^{-\alpha_i t} \cdot (e^{-\beta_i t} - 1) \sin(\omega_i t) \quad (18)$$

where, α_i are parameters of the process attenuation; ω_i are characteristic frequencies of the input.

The number of members of the Eq. (18) depends on the complexity of the system under consideration. For seismic isolated systems, it is sufficient to keep two to three members in Eq. (18).

Frequencies ω_i are the predominant frequencies of predicted earthquakes. In the absence of these data, one can use the dangerous frequency for the structure as ω_i . For linear systems, it is fundamental structure frequencies. To estimate the dangerous frequency for nonlinear systems, the amplitude-frequency response of the system should be built.

The proposed input model includes $2N$ undetermined parameters α_i , β_i and three additional parameters for the running wave. For $N = 3$, we have nine undetermined parameters.

These parameters are defined by the minimization of misalignment M :

$$M = p_1 (I_A - I_A^0) + p_1 (\chi - \chi_0) \quad (19)$$

where, I_A is Arias intensity of the artificial accelerogram and I_A^0 is the value of Arias intensity, obtained on the basis of natural data:

$$\chi = \frac{y_0^{(\max)} y_0''^{(\max)}}{(y_0'^{(\max)})^2}$$

where, $y_0^{(\max)}$, $y_0'^{(\max)}$, $y_0''^{(\max)}$ are peak values of displacements, velocities and accelerations;

χ_0 is the value of this parameter obtained on the basis of natural data and given in Refs. [2, 5, 7];

p_1 and p_2 are weight factors, usually $p_1 = p_2 = 0.5$.

Thus, it is possible to generate dangerous (resonant) accelerograms with the given power characteristic for the structure under consideration.

5. Influence of Vertical Excitation on the Behavior of Seismic Isolation with Kinematic Bearings

An important peculiarity of spherical bearings is the dependence of bearing stiffness on the vertical bearing reaction. In this case, the vertical component of seismic impact can influence the horizontal structure oscillations. This fact was mentioned in some investigations [2, 12, 13], but there are no detailed

investigations in this field. The general qualitative analysis of this problem is considered below. On this basis, parameters of seismoisolating bearings are established.

At small displacements, the behaviors of structures on the considered kinematic bearings are described by the equation of the pendulum oscillations:

$$\varphi'' + \frac{g}{d} \varphi = \frac{\Phi(t)}{d} \quad (20)$$

where, φ is the angle of the bearing rotation, g is gravity acceleration, Φ is horizontal acceleration of the base, d is a certain bearing characteristic, having length dimension. For spherical bearings in accordance with Refs. [2, 10], d can be presented as follows:

$$d = \frac{(H-r)^2}{H} \quad (21)$$

If vertical oscillations are combined with horizontal ones, the oscillation equation should be written as follows:

$$\varphi'' + \frac{g + z''(t)}{d} \varphi = \frac{\Phi(t)}{d} \quad (22)$$

where, $z''(t)$ is the vertical excitation of the base.

To carry out the qualitative analysis of vertical component influence, the authors consider the harmonious two-frequency excitation of the system, i.e., the horizontal excitation with frequency ω (Eq. (23)) and the vertical one with frequency α (Eq. (24)):

$$\Phi(t) = A_h g \sin \omega t \quad (23)$$

$$z''(t) = A_v g \cos \alpha t \quad (24)$$

In Eqs. (23) and (24), A_v and A_h are accordingly vertical and horizontal accelerations of the base in portion of acceleration of gravity.

If we introduce nondimensional time $\tau = kt$, after some transformations, Eq. (22) takes the following form:

$$\varphi'' + (1 + A_v \cos \chi \tau) \varphi = \sin \Omega \tau \quad (25)$$

where, the prime designates dimensionless time derivative and $\chi = \frac{\alpha}{k}$, $\Omega = \frac{\omega}{k}$.

The equation obtained relates to the equations of Mate presented as:

$$\varphi'' + (L + 2q \cos \chi \tau) \varphi = 0 \quad (26)$$

In our case, $L = 1$ and $2q = A_v$.

Taking into account the viscose resistance with damping $\gamma = 2\zeta$, the oscillation equation can be rewritten as follows:

$$\varphi'' + \gamma \varphi' + (1 + A_v \cos \chi \tau) \varphi = 0 \quad (27)$$

It is known [9] that the character of decisions of Eq. (27) depends on the parity of dimensionless factors L and q which define the stability of the movement. The area of change of these factors can be divided into subareas corresponding to steady (limited) and unsteady (infinitely increasing) solutions of Eq. (27), i.e., to steady and unsteady movements. Periodic solutions of Eq. (27) correspond to the borders between zones of stability and instability.

The borders of the first area of instability for $\gamma = 0$ and $\gamma = 0.2$, and the points corresponding to values of $A_v = 0.4$ and $A_v = 0.2$ are also shown in Fig. 2.

Thus, it becomes obvious that, during earthquakes at certain dynamic parameters of a construction, the phenomenon of a parametrical resonance may take place, i.e., a situation arises when the solution of Eq. (27) gets to the zone of dynamic instability and eventually the oscillation amplitude increases indefinitely.

The amplitude of oscillations in the mode of the main parametrical resonance was studied by Indeykin et al. [14]. Its change is described by the following law:

$$\varphi(t) = a(t) \cdot \sin \frac{\alpha t}{2} + b(t) \cdot \cos \frac{\alpha t}{2} \quad (28)$$

where, $a(t) = a_0 e^{ht}$, $b(t) = b_0 e^{ht}$.

The characteristic indicator h is calculated using the formula:

$$h = \frac{\mu k}{2} - n \quad (29)$$

where, μ is excitation factor, $\mu = \frac{A_v}{2}$, $n = \frac{\gamma k}{2} = \frac{\delta k}{2\pi} = \frac{\delta}{T} = \zeta k$, δ is the logarithmic decrement, T is the oscillation period.

The character of oscillation amplitude mainly depends on the value of indicator h . If $h < 0$, the

Fig. 2 Borders of the first area of instability: the lower curve is for $\gamma = 0$ and the upper curve is for $\gamma = 0.2$. Points (circles) show the values of parameters in the equation under consideration of Eq. (27).

Fig. 3 Dependence of the time θ of oscillation amplitude doubling on damping parameter γ

amplitude decreases (oscillations fade); if $h = 0$, the amplitude is constant; if $h > 0$, the amplitude increases, i.e., there appears the phenomenon of parametrical resonance.

The value γ at which the phenomenon of parametrical resonance does not appear can be written as follows:

$$\frac{\gamma k}{2} > \frac{\mu k}{2} \quad (30)$$

whence:

$$\gamma > \mu = \frac{A_{\text{sep}}}{2} \quad (31)$$

If $\gamma < A_{\text{sep}}/2$, parametrical resonance takes place. In this case, one can define the time when the oscillation amplitude increase by “ m ” times using the following condition:

$$m \cdot e^{ht} = e^{h(t+\theta)}$$

whence:

$$\theta = \frac{\ln m}{h} = \ln m \frac{1}{h} \quad (32)$$

Eq. (31) defines the damping ratio, necessary for excluding parametrical resonance for the system with the kinematic seismoisolation. Therefore, for an earthquake with accelerations of more than 0.4g, the minimal damping should not be less than 10% of the critical one ($\zeta = 0.1$; $\gamma = 0.2$). At smaller damping, the doubling of amplitudes can occur quickly enough. It is illustrated in Fig. 3. Hence, for damping with $\gamma = 0.15$, the doubling of the amplitude will occur every 2.5 s which is catastrophic for structures.

6. Conclusions

The results obtained made it possible to conclude:

- First, the presented models of seismic isolating devices make it possible to simplify their designing and setting their parameters;
- Second, the presented input model provides the safety of seismic isolation. Using this model, it is necessary to take into account the vertical input component while setting devices damping parameters;

- In general, the obtained results provide to set parameters of isolating devices for motor road bridges constructed in the city of Sochi.

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