Analytical Solution for Evaluation of Voltage Surge Distribution along Transformer Windings through the Method of Residues

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Abstract: Computer programs have definitely become indispensable for designing power transformer. Among several applications, computer programs are mostly used for electric field calculation and thus electrical insulation concerns. In consequence, studies based on analytical approach to basic studies of correlated problems have become even more important because they form the very basis of knowledge that is necessary to every transformer designer in view of taking all the advantages of computational analyses. On the other hand, one of the most important basic studies consists in the evaluation of voltage surge distribution along transformer windings for which the method of separation of variables has been extensively used thanks to some simplifying assumptions. With this aim, authors have developed and previously published works that show the applicability of an alternative and useful analytical method that is the method of the residues, which requires no simplification to be assumed. In this work, another important step is taken towards proofing the total applicability of this promising method that is through a practical problem. A comparison to the numerical method TLM (transmission line method) is also performed and concordance with TLM and experimental data confirms the proposal of the method of residues can be also applicable to several others problems of electromagnetism.

Key words: Transformer winding, method of residues, electrical transient, voltage surge.

1. Introduction

Since every power transformer is inherently vulnerable to incidence of lightning voltage surges, several studies regarding this fact and its effects have been published ever since transformer has been put in use [1]. In consequence, several numerical tools and software have been developed for modeling, studying and understanding this issue with different degrees of complexity [2, 3]. On the other hand, no matter how complex the transformer design can be or how sophisticated is the computational tool, the analysis of the behavior of its windings under the incidence of a lightning voltage surge is often based upon concepts

extracted from a classical study. This classical study consists in the analytical solution in time-domain of the differential equation that governs the voltage surge distribution along a continuous and homogeneous winding submitted to a step voltage [1]. Although it has been quite usual to apply the MSV (method of separation of variables) for solving the ruling differential equation, the authors have successfully proposed and proofed the better applicability of the MR (Method of Residues) for the solution of that same differential equation [4]. As a further development but with few space for details [5] shortly presents a new achievement that is the applicability of the MR to a practical case that consists in an air-core winding submitted to a step voltage. Thus, this extended work shows details of this new achievement that is that MR shows itself as being a quite applicable and practical

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tool for obtaining analytical solution in studies that serve as the fundamental basis for more complex problems that require computational tools. For the sake of a better comparison results from numerical method TLM (transmission line method) are also presented.

2. Theoretical Basis

In accordance with Refs. [1, 6], when a travelling voltage surge reaches a continuous winding of a transformer, this winding becomes vulnerable to insulation failure, immediately. This is due to the sudden voltage distribution that becomes established along it and that invariably gives origin to high values of electric field along its first turns. Such a sudden voltage distribution happens due to the instantaneous charging of parasitic capacitances of the winding, as schematically shown in Fig. 1, where capacitances are between adjacent turns, C_s , and between any turn and the grounded parts (core, for example) of the transformer, C_g . Typical values for both these capacitances depend on the composition of the dielectric material as well as on the geometrical features of the design of the transformer. The influence of both capacitances is represented by the product of the winding length, ℓ , by a constant that depends on the values of the capacitances, the number of turns of the winding, N , as well as its same length. This constant is α and it is given by:

$$\alpha = \sqrt{\frac{NC_g}{C_s \ell}} \quad (1)$$

Then, if no failure happens, the voltage distribution along this winding will tend to oscillate thanks to exchanges of energies among charged capacitances and

Fig. 1 Schematic representation of the parasitic capacitances of a transformer winding [1].

all the distributed mutual and series inductances of the winding. Even though the presence of series resistance along winding could cause reduction of overvoltage, due to energy dissipation, such a resistance is supposed as having a relatively low value and it can thus be neglected. Therefore, voltage oscillations will probably give origin to overvoltage along several points of the winding, until steady state may be reached. As an example, Fig. 2 shows the behavior of the per unit voltage distribution along a continuous and homogeneous winding that has $\alpha\ell = 100$ for the instants $t = 0$; 1.0; 1.75; 3.0 and 6.6 μs after the incidence of the step voltage, with amplitude V_o [1]. For its turn, Fig. 3 shows the behavior of the instantaneous voltage to ground in the middle of the winding. From both figures it can be noticed that the whole winding is submitted to voltage oscillations,

Fig. 2 Behavior of the per unit voltage distribution along a $\alpha\ell = 100$ winding at five different instants.

Fig. 3 Behavior of the per unit voltage along time at the middle of a $\alpha\ell = 100$ winding.

which can easily give origin to several dielectric failures along it.

At last, if no failure happened, the voltage distribution along this winding becomes uniformly decreasing along it.

All of these important and consolidated results are obtained from the solution of the differential equation that rules the surge voltage distribution along the winding, in time, $V(x, t)$, that is:

$$\frac{\partial^2 V(x, t)}{\partial x^2} = R_s \left(C_g \frac{\partial V(x, t)}{\partial t} + \frac{C_s \ell}{N} \frac{\partial^3 V(x, t)}{\partial t \partial x^2} \right) + L_s \left(C_g \frac{\partial^2 V(x, t)}{\partial t^2} + \frac{C_s \ell}{N} \frac{\partial^4 V(x, t)}{\partial t^2 \partial x^2} \right) \quad (2)$$

in which R_s is the longitudinal resistance and L_s is the effective series inductance, both in per unit of the length of the winding, whereas C_g , C_s , ℓ and N have already been described above. The continuous and homogeneous winding lies along the x -axis, it is grounded at its end, $x = \ell$, and the step voltage is applied at $x = 0$. For the sake of making easier the solution of Eq. (2) through the MSV method and since it represents a conservative approach the resistance of the winding is usually neglected [1]. On the other hand, although this assumption may be very practical, the authors have proposed in Ref. [5], the application of the MR as an alternative method for solving Eq. (2) by taking such a resistance into account. This method is based on Cauchy's Residue Theorem [7], which implies in the use of the Laplace transform to Eq. (2). The qualitative expression that is the basis of this method is:

$$V(x, t) = L^{-1}[V(x, s)] = \sum \text{residues of } V(x, s)e^{st} \quad (3)$$

in which L^{-1} is the operator of the inverse of the Laplace transform.

After application of direct Laplace transformer to Eq. (2), its solution in the transformed space is:

$$V(x, s) = \frac{V_o}{s} \frac{\sinh[\gamma(\ell - x)]}{\sinh \gamma \ell} \quad (4)$$

for which γ is the constant of propagation and it is given by:

$$\gamma^2 = \frac{R_s C_g s + L_s C_g s^2}{1 + \frac{R_s C_s \ell s + L_s C_s \ell s^2}{N}} \quad (5)$$

Now, since the function $\sinh \theta$ can be represented by an infinite series of products [7] as follows:

$$\sinh \theta = \theta \left(1 + \frac{\theta^2}{\pi^2} \right) \left(1 + \frac{\theta^2}{4\pi^2} \right) \left(1 + \frac{\theta^2}{9\pi^2} \right) \dots \quad (6)$$

Therefore, Eq. (4) can be also presented as:

$$V(x, s) = \frac{V_o}{s} \frac{\gamma(\ell - x) \left(1 + \frac{\gamma^2(\ell - x)^2}{\pi^2} \right) \left(1 + \frac{\gamma^2(\ell - x)^2}{4\pi^2} \right) \left(1 + \frac{\gamma^2(\ell - x)^2}{9\pi^2} \right) \dots}{\gamma \ell \left(1 + \frac{\gamma^2 \ell^2}{\pi^2} \right) \left(1 + \frac{\gamma^2 \ell^2}{4\pi^2} \right) \left(1 + \frac{\gamma^2 \ell^2}{9\pi^2} \right) \dots} \quad (7)$$

Thus, $V(x, s)$ is a function of the complex frequency, s , that presents one simple pole at $s = 0$ and several other poles at values of s that satisfy the following condition:

$$\gamma^2 = -\left(\frac{n\pi}{\ell} \right)^2; \quad \text{For } n = 1, 2, 3, \dots \quad (8)$$

Since poles are known, by taking forward the process of evaluation of the inverse of Laplace transformation, as presented in details in Ref. [4], solution of Eq. (2) on time domain is:

$$V(x, t) = V_o \left(\frac{\ell - x}{\ell} \right) + \frac{2V_o \beta_1}{\pi \beta_2} \sum_{n=1}^{\infty} \left\{ \frac{\sin\left(\frac{n\pi x}{\ell}\right)}{n} \left(\frac{A_{n_1} + \frac{\beta_2}{s_1 \beta_1}}{A_{n_1} + \frac{s_1}{\beta_1}} \right) s_1 A_{n_1} e^{s_1 t} + \left(\frac{A_{n_2} + \frac{\beta_2}{s_2 \beta_1}}{A_{n_2} + \frac{s_2}{\beta_1}} \right) s_2 A_{n_2} e^{s_2 t} \right\} \quad (9)$$

in which:

$$\beta_0 = \frac{NC_g}{\ell C_s}; \quad \beta_1 = \frac{R_s}{L_s}; \quad \beta_2 = \frac{N}{L_s C_s \ell}; \quad A_{n_1} \quad (10)$$

$$\beta_1 = \beta_1 + s_1; \quad A_{n_2} = 1 + \frac{s_2}{\beta_2}$$

And s_1 and s_2 are obtained from:

$$s_{1,2} = \frac{-\beta_1 \pm \sqrt{\left(\frac{\beta_1}{2} \right)^2 - \frac{\beta_2 \left(\frac{n\pi}{\ell} \right)^2}{\beta_0 + \left(\frac{n\pi}{\ell} \right)^2}}}{\beta_0 + \left(\frac{n\pi}{\ell} \right)^2} \quad (11)$$

A succeeded comparison between MR and MSV is also presented in Ref. [3], as well as a study of the influence of the series resistance, R_s , which can be usually neglected.

3. A Practical Case of Study and Its Numerical and Analytical Solutions

For a comparison between MR and experimental results, an apparatus was assembled for the application of step voltage to a practical air core winding. This winding presents nine taps with specific functions for some of them. Tap 1 is for the application of the step

voltage, tap 9 is to be grounded and all the others are for the measurement of instantaneous voltage. Schematic diagram of the apparatus is shown in Fig. 4, whereas in Table 1 main features of the winding are presented. Still in regard to Fig. 4, the inner cylinder is made of aluminum, it is solidly grounded and its function is to enable capacitances to ground. It somehow represents the grounded frame of the magnetic circuit of a transformer, although it cannot represent its magnetic characteristics, but only electric

Distances and dimensions are also presented, as well the step voltage source that is a workbench square-wave generator with $50\ \Omega$ output impedance.

Dimensions of this winding are typical for a 45 kVA transformer.

At last, since the numerical method TLM has presented itself as a very applicable method, it was taken for modelling and simulation of this problem for better comparison to MR and experimental results. Thus, the modelling of the practical winding consisted in dividing it into several segments with related capacitances and inductances. Fig. 5 shows the aspect of the circuit for TLM modelling and simulation of the winding presented in Fig. 4.

Although with TLM modelling it could be possible to represent partially those unavoidable non-homogeneities of any winding by setting slightly different values of capacitances and inductances for different segments, it was not done. The reason is that this could not be done with the analytical method, since its assumption is that winding is absolutely homogeneous. In this way, MR results are expected to be closer to TLM results than to practical results.

4. Comparison between MR Method and Experimental Data

The practical air-core winding of Fig. 4 has the following parameters, experimentally evaluated: $C_g = 4$ nF/m; $\ell = 0.4$ m; $C_s \ell / N = 6.37$ pF·m/turn and $L_s = 0.4$ mH/m. In accordance with practical concerns R_s was set to $0\ \Omega/\text{m}$. All these values were applied to Eqs. (9)-(11) on an Excel[®] spreadsheet to perform the

Fig. 4 Schematic diagram of the experimental apparatus.

Table 1 Main constructive characteristics of the air-core winding.

Bare conductor diameter (mm)	1.0
Length of the enamel layer (mm)	0.1
Number of turns (copper)	248
Inner diameter	200
External insulation	Presspan paper
Winding length (m)	0.3
Inner insulation	Kraft paper

Fig. 5 Equivalent circuit for practical winding.

MR simulation with value of n set to 21. In regard to TLM simulation with a program developed by the authors of Ref. [5] the number of segments was set in 10, in accordance with Fig. 5.

As simulation results, Fig. 6 presents the behaviour of instantaneous voltage to ground at the first quarter of the winding that corresponds to tap 3, in Fig. 4. The TLM simulation is also presented for the sake of a reinforcement of the comparison, as above mentioned. Thus, in accordance with Fig. 6 it can be noticed that results between TLM and MR are very close, whereas experimental results present a small but natural difference in comparison to both. This obvious

Fig. 6 Voltage to ground along time for the first quarter of the winding.

difference is a time shift and it is principally due to the inherently non-homogeneity of the real winding and to the fact that a real voltage source is not an ideal step voltage. In consequence, stray capacitances and internal resistances of voltage source contribute for a time delay in the experimental response. However, besides such a time shift results from all three simulations have a significant coherence.

Still in regard to graphs of Fig. 6 it is important to consider that differences between TLM and MR results may lie in the choice of the number of segments in the TLM modelling as well as the value of n in the series presented in Eq. (9). However, additional simulation of the MR method with n set as 100 presented no significant influence. Thus, although this simulation can be done in an electronic spreadsheet due to its simplicity, the use of more terms in Eq. (9) is not necessary to be greater than 21 if an approximate evaluation is required. On the other hand, more than 100 terms will be necessary for the exact results between TLM and MR methods.

4. Conclusions

Presented results from a practical simulation of a transformer winding show how effective the proposed method of the residues can be for analytically solving classical problems of Electromagnetism related to transformers. Although some minor real concerns

could not be taken into account, unless under a tremendous increase in complexity, the MR presents results with significant coherence. In this way, obtained results encourage the use of this method for analyses of several other problems with time dependence, in Electromagnetism. After all, this method is simple in such a degree that its calculations can be done in a simple electronic spreadsheet, with no significant computational effort, as well as it allows the evaluation of the influence of the resistance of the conductor. Regarding power transformers, all the obtained results encourage future studies for evaluating the substitution of the air core winding by real and practical one, with ferromagnetic core. These presented results also encourage application of the method in other areas of engineering in view of consolidating applicability of this important and method.

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