Development of an Accurate Numerical Methodology for Predicting the First Three Modes of Vibration of an Electric Motor Fixed on a Rigid Base

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Abstract: This paper proposes a numerical methodology for the prediction of the first three modes of vibration of an electric motor fixed on a rigid base. A deep literature review supported the production of four ad hoc prototypes that aided the development of the proposed approach. Tests carried out with the prototypes led to the procurement of the modal parameters be used to calibrate the numerical models, as well as the FRF (frequency response function) curves be used to validate the numerical solution. The validated model allowed structural changes to be then promoted on the prototypes, in order to make them more robust to variations in manufacturing and assembling processes. The mentioned adjustments and structural changes were accomplished by means of a process of structural optimization using Genetic Algorithm. The solution was developed based on the commercial finite element code ANSYS. The practical results obtained in this study show that a numerical model for modal analysis of an electric motor fixed on a rigid base with errors less than 3% for the first three modes of vibration can be achieved, allowing positive structural changes to be performed in the machine design that result in the minimization of manufacturing reworks associated with the dynamic behavior of the studied motor.

Key words: Modal analyses, numerical methodology, finite element.

1. Introduction

According to Tustin [1], impact and vibration often accelerates the failure of industrial machinery and equipment. Consequently, minimize or control these effects may delay a premature failure. This statement can be observed in the industry through increasingly stringent specifications from normative organizations of rotating electrical machines.

The obtaining of vibration levels in electric motors that meet the criteria set in standards and/or special criteria, military applications, for example, it becomes more difficult to achieve as the motor's power increases. As the power increases the electric machine size

become proportionally larger, which inevitably affects the ratio between stiffness per mass and hence its natural frequencies and associated mode shapes. Moreover, the excitation source keeps the same, increasing the probability to match with some natural frequency. In such cases, it is essential the full understanding of the physical behavior associated with the problem and its susceptibility to intrinsic variations in the manufacturing process. There remains the question if these changes will cause undesirable and potentially harmful/destructive effects to the motor. Based on these arguments, it is essential for an electric motors manufacturer the ability to predict the dynamic behavior of their product, taking into account boundary conditions and project robustness, still in the conceptual phase.

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1.1 Critical Aspects of an Electric Motor with Respect to Vibration

The assembly of an electric motor, with the exception of the stator, is relatively simple, but two motors theoretically equal probably will not present the same values of natural frequencies for their respective modes of vibration. This occurs because the contact between parts, for example, increases the problem complexity by inserting a number of variables in the manufacturing process. The following analyzes the main peculiarities, from the standpoint of the manufacturer and of some authors that should be considered in the study of natural frequencies in an electric motor.

1.2 Stator

Due to its orthotropic characteristics (Gonçalves [2], Roivainen [3], Wang [4], Gieras et al. [5], Garvey et al. [6], Long et al. [7] and Delves [8]) and the way that the coils are mounted in their slots, the stator is undoubtedly a complicating factor in the study of electric motor natural frequencies. According to Kukula [9], the interface between the stator slots and coils winding is a strongly non-linear contact with the stiffness strongly dependent on an insulating film which is inserted between the slots and the coils. On the other hand, the interference between the stator and frame, besides being a nonlinear contact produces an assembly's tensioning. Gieras et al. [5] calculates this effect with a pre-static analysis, known as pre-stress, to then calculate the natural frequencies. This means that the frame stiffness can change and, consequently, their natural frequencies.

1.3 Rotor

With the objective to simplify the numerical modal analysis of a complete motor, Kukula [9] considered only the inertia effects of the rotor and its components, neglecting the effects of damping and stiffness. Kukula's consideration with relation to rotor, suggests that rotor influence on the physical system represented

by the motor stator and end shields is limited to addition of mass, without significant stiffness and damping effects. This argument is based by observation of rotor assembly. In the standard process, the bearings are installed at the rotor ends. These bearings are the only connection between the rotor, the end shields and the rest of assembly. Consequently, the rotor mass will be supported between the end shields.

1.4 End Shields and Frame

The effects of end shields or contacts between the frame and end shields, is an issue with scarce bibliography. A pioneering work to approach this issue was developed by Cai et al. [10] using reluctance motors. His goal was to verify the behavior of motor's natural frequencies values before and after the end shields installation and find a way to reproduce this effect in a numerical model. Despite the simplicity of the idea, the results presented by the author are interesting. Experimentally, there was an increase of 25% in natural frequencies associated with modes of second order. This behavior was expected, since the end shields are mounted with a certain interference and adds more stiffness compared to mass. The obvious conclusion indicated that the effects of contact between end shields and frame should be considered in the physical prototype.

2. Methodology

The experiments presented in this work were performed in order to provide data to assist in evaluation, adjustment and validation of numerical methodology. The used method was experimental modal analysis, where the modal parameters were extracted from four prototypes and from parts that compose them, and extensometry to measure the deformation's field during the assembly process. The object of study was an electric motor of size IEC 225 S/M, two poles (60 Hz/3,600 rpm), 440 V, 60 hp, 320 kg without connection box.

Initially, each component was tested on “free” condition. This condition excludes influences inherent to manufacturing process, such as interference fitting, torque of fixing screws or from the test base. Thus, the initial numerical model setting was simplified and focused on geometric variations between the physical parts and CAD (computer aided design) models and the properties of manufacturing materials.

A second stage began with a gradual assembly, where, for each new component inserted into the prototype, was conducted an information collection about the variables involved in process and a new modal parameters extraction was performed on both boundary conditions (free and fixed on an IEC 60034-14 [11] rigid base).

Among the variables evaluated are the screw’s mounting torque for end shields and frames feet, the mechanical interference magnitude between parts and the deformation caused by the machining process. The concern with such details is justified by reducing doubt credibility about the numerical model, by safely optimize the computational simulation cost and by indicate which project characteristics are sensitive to the manufacturing process.

3. Main Experimental Results

3.1 Mechanical Interference Data Collection from Manufacturing Process

The dimensional data of each component were collected after the machining process; the regions of greatest interest were contacts between end shields and frames, the frame’s internal diameter and stator’s outer diameter. These regions can change the problem boundary conditions. Tables 1 and 2 summarize the amounts of interference that each part (theoretically) contributed on the final prototypes.

Note that the interference between the stator and frame of prototype n° 2 are higher than others, that was intentionally performed to verify the impact of a higher interference on a possible change of the prototype’s natural frequencies.

Table 1 Mechanical interference between end shield and frame.

Prototype	Component	Medium interference (mm)
1	Front end shield n° 2	0.02
	Frame n° 3	—
	Back end shield n° 1	-0.07
2	Front end shield n° 4	0.03
	Frame n° 4	—
	Back end shield n° 2	0.04
3	Front end shield n° 5	0.06
	Frame n° 1	—
	Back end shield n° 3	0.08
4	Front end shield n° 6	0.07
	Frame n° 2	—
	Back end shield n° 4	0.07

Table 2 Mechanical interference between stator and frame.

Prototype	Component	Medium interference (mm)
1	Stator n° 1	0.24
	Frame n° 3	
2	Stator n° 2	0.44
	Frame n° 4	
3	Stator n° 3	0.14
	Frame n° 1	
4	Stator n° 4	0.15
	Frame n° 2	

3.2 Extensometry

The SGs (strain gages) were installed on frame to measure the radial deformation, resulting from stator assembly, and axial, derived from end shields assembly. Fig. 1 describe the location and identification of each SG installed on frame and end shields.

The Table 3 extensometry results indicates that with insertion of stator, the SG 6CM showed a deformation magnitude of approximately three times higher on frame n° 4 when compared to the others.

This behavior was expected, since the frame n° 4 had an internal diameter smaller than others frames (more interference).

Taking the average results of SG 1T and 2T installed on the end shields, the back end shield’s magnitude deformation was about four up to five times higher

Fig. 1 Location and identification of installed SG.

Table 3 Results of SG 6CM after end shields installation on frame.

Component	Deformation (SG 6CM)
Frame 01 + Stator 04	0.17%
Frame 02 + Stator 03	0.18%
Frame 03 + Stator 01	0.22%
Frame 04 + Stator 02	0.62%

Table 4 Results of SG 1T and 2T after end shields installation on frame.

Component		Medium deformation		Medium from SG 1T and 2T
		SG 1T	SG 2T	
Frame n° 3	Front end shields	0.002%	0.005%	0.004%
	Back end shields	0.004%	0.037%	0.021%
Frame n° 4	Front end shields	0.003%	0.017%	0.010%
	Back end shields	0.007%	0.071%	0.039%

relative to front end shields, as shown in Table 4. It can be associate this difference to absence of fins on back end shield, making it more susceptible to deformation on the regions of contact with the frame.

3.3 Experimental Modal Analysis—Main Results

Fig. 2 summarizes the percentage variation of natural frequencies considering as reference the medium value of natural frequency of first four vibration modes found for samples of various components and assemblies on free condition and Fig. 3 on fixed condition (rigid base).

These figures present the sensitivity to assembly variations of various components or group of components under conditions that were evaluated. The tightening torque of end shields screws was 79 Nm and for frame feet screws was 98 Nm. These results allowed the main test items to be stratified, leading to a better understanding of the deviation causes.

3.4 Experimental Modal Analysis—Machined Frames with Stators

Fig. 4 presents the results of natural frequencies after inserting the stators into the frames. The fact of assembly's interference of frame n° 4 with stator n° 2 to be higher, was not reflected in a significant increase of natural frequencies values when compared with other assemblies with different interferences.

3.5 Experimental Modal Analysis—Machined Frames Assembled with Stators and End Shields

An important observation to be made before evaluate the results after installing both end shields, is the cancellation of the higher interference effect for frame n° 4 with the stator n° 2.

Without end shields, the set of components with higher magnitude of interference between the stator and frame discreetly assumed higher values of natural frequencies for some modes, but this difference disappeared with installation of end shields (Fig. 5).

Observing the data in Table 1 and comparing them to the results of Figs. 4b and 5, it can be seen that the higher interference between the frames n° 1 and n° 2 and the respective end shields presented natural frequencies values slightly higher or near the other two mounting assemblies.

Fig. 2 Percentage variation of the experimental results of the first four natural frequencies of each component or set of components of the motor in free condition.

Fig. 3 Percentage variation of the experimental results of the first four natural frequencies of each component or set of components of the motor in fixed condition.

Therefore, the influence of interference's magnitude between frame and stators on the natural frequencies of the set, depend on the interference's magnitude between end shields and frame.

3.6 Experimental Modal Analysis—Others Components Assembly and Full Prototypes

In free condition, results deviations for

pre-machined frames were higher when compared to fixed condition. When evaluating these data separately, it is realized that the greatest contributions for that difference were the results obtained with frame n° 3. If frame n° 3 results were excluded, the percentage variation of experimental results falls from 8.99% to 3.14%. Evaluating the mass and dimensional data, one concludes that frame n° 3 had some mass difference.

(a) Free condition

(b) Fixed condition

Fig. 4 Results of the natural frequencies of the frames with stators.

Fig. 5 Results of the natural frequencies of the frames with stators and the both end shields in the fixed condition.

The variations relative to assemblies of frames with end shields on both conditions (free and fixed) are also associated to mass difference of frame n° 3.

The variations relative to end shields, stators and full-assembled prototypes can be considered satisfactory, because they are smaller than the configurations previously evaluated. Additionally, what are evaluated are the influence of fixing screw tightening torque of end shields and frames feet, as well as the process of stress relief after frame pre-machining. The results indicated that both the torque and the heat treatment have no major influence to change natural frequencies of vibration modes of parts and assembly sets. Full results can be found in Ref. [2].

4. Numerical Simulation

To simulate the proposed problem the commercial program ANSYS WB (workbench) 2.0-Version: 14.0.0 was used. First, the CAD model had its geometry simplified eliminating unnecessary details. Then, the CAD model was imported to simulation environment and all parts except the rotor were modeled with solid elements which present square displacement behavior. The sensitivity of modal analysis results to changes in mesh size was evaluated using the element as parameter. The error due to sensitivity, associated with the natural frequencies calculation and the processing time, was used as condition for setting the default size of elements to be adopted for all simulations (Fig. 6).

The deviations were calculated using the first four natural frequencies of frame on free condition. The results obtained with the finer mesh model (element size of 10 mm) were used as reference to calculated deviation and presented at Fig. 6. Using the Fig. 6 was defined the element size of 25 mm, because the deviations related to the four first modes presented low values and an adequate processing time. With the element size defined, it was evaluated the mesh characteristics using different generation methods. From the results of this last step, it was decided to use the automatically generated mesh method with the geometry divided into several main bodies or a multi-bodies geometry (Fig. 7).

Based on the concepts presented in Section 1.1

Fig. 6 Graph comparing the processing time of the diversion number of the carcass in a free condition.

(a) Motor frame model

(b) Stator model

Fig. 7 Mesh appearance using automatic multi-body meshing method.

relating to the rotor, it is believed, from dynamic point of view, that the installation process of the rotor at the motor has an adding mass effect to mechanical system composed by frame, stator and end shields. The mass addition is proportional to rotor mass. For this reason, it is believed that the stiffening effect due to the rotor assembly is smaller and negligible. This means from numerical point of view, that the rotor and all its components should be added to model as Kukula [9], considering only the effects of inertia forces acting on end shields. Thus, the rotor was regarded as load, proportional to its weight, distributed on the two bearings house.

4.1 Models Adjustment

Each numerical model was adjusted using the free condition comparing the numerical data with their respective average experimental data. To perform the comparison, ANSYS provides some optimization algorithms in the DX (design explorer) tool. The DX is a statistical tool for parametric analysis that integrates the WB virtual environment. The first step was to define the parameters to be used. For the frame and end shields was used the cast iron data, to stator was used the orthotropic data presented on Roivainen [3] work (Fig. 8).

Properties of Outline Row 4: Coroa Estator				
	A	B	C	D E
1	Property	Value	Unit	
2	Density	7700	kg m ⁻³	
3	Orthotropic Elasticity			
4	Young's Modulus X direction	2,12E+11	Pa	
5	Young's Modulus Y direction	2,12E+11	Pa	
6	Young's Modulus Z direction	1,8E+09	Pa	
7	Poisson's Ratio XY	0,3		
8	Poisson's Ratio YZ	0,3		
9	Poisson's Ratio XZ	0,3		
10	Shear Modulus XY	8,2E+10	Pa	
11	Shear Modulus YZ	6,3E+10	Pa	
12	Shear Modulus XZ	6,3E+10	Pa	

(a) Ansys data input table

(b) Stator model

Fig. 8 Stator orthotropic data initially used into DX.

The next step consists on correlate all these parameters to identify the models sensibility for each parameter change. With this procedure was it possible to choose the most adequate parameter and exclude the ones with less importance, minimizing the variables number and computational costs. The tool used by DX is called parameter correlation and the correlation chosen was spearman type.

From the correlation result, it was determined that the most cast iron significant parameter for the natural frequencies calculation is the elastic modulus. With respect to stator, the results indicate that the coil material, as proposed by Roivainen [3] exerts low influence on the natural frequency values of the

respective modes, and may be used without any modification from original values.

Already the data from orthotropic material model representing the stator stack indicated, with relation to the natural frequencies of the respective modes, that: (1) the Poisson coefficients have low influence; (2) the elastic modulus associated with axial compression must be low (5E9 N/m²), but a variation of $\pm 10\%$ from this value also exerts low influence; (3) the elastic modulus, and the shear modulus associated with the circumferential deformations, are the main parameters that exert influence.

The most significant parameters were subdivided by a DX statistical tool named DOE (design of experiments) to develop an efficiently series of simulations that represent possible solutions of numerical model's optimization and adjustment. This development was done by CCD (central composite design) method.

The points identified at this process were then calculated and interpolated to create a response surface. This response surface represents the combinations of solution produced by the DOE and solved by ANSYS. The interpolation method used was kriging. The response surface provides approximate values for output parameters at any point on space created by the CCD without performing a complete solution [12].

The final models adjustment was conceived from the response surface with a DX tool named GDO (goal drive optimization), using genetic algorithm named MOGA (multi-objective genetic algorithm). The basic idea behind this algorithm is similar to Darwin's evolution theory, where natural selection is produced from a group of individuals (in this case from response surface) oriented to achieve a certain goal.

The parameter's selection is made by three basic operations: selection, crossover and mutation, and generates the population increase of suitable candidates until the stop criterion is reached [13]. The stopping criterion used was the lowest value of the objective function with up to twenty iterations of an initial

number of a hundred samples. In this case, the objective function used was the quadratic penalty. That calculation used the experimental data as reference.

Fig. 9 shows an example of how the natural frequencies values, for the first three modes, have been introduced into MOGA tool and how were presented the candidate parameter sets for the solution. The full model adjustment result, on fixed condition, can be conferred in Table 5.

4.2 Contact and Boundary Conditions

The fixed condition was defined according to Kukula [9], zero displacement for all cartesian directions on frame feet's lower surface. The stress related to interference between end shields and frame and stator and frame, was added into the model as a pre-tension. From this definition, the contacts between end shields and frame were defined as in Ref. [10], where the contact areas near end shield's fixing screws presented identical movements ("bonded"), while the remaining contact areas could vibrate independently

(Fig. 10). In WB, this definition is equivalent to use contacts that allow relative motion between the surfaces in the normal separation direction, transferring tangential and compressive stresses in its entirety.

For comparison was performed a numerical modal analysis using the contacts configuration as defined by Cai et al. [10] and with all the contacts "bonded". With contacts "bonded" the natural frequency for the first mode, on fixed condition, of the frame with the front end shield installed, showed a value 14% higher than medium experimental value. Using the contacts similar to those used by Cai et al. [10], the numerical deviation with respect to medium experimental value was below 4%.

4.3 Numerical Models Validation

The final numerical models validation was made by comparing the experimental and numerical FRF (frequency response function) curves, of each part, on free condition and the main sets on fixed condition. The numerical FRF curves were extracted at the same

	P14 - DS_Raio_Interno .Part	P15 - DS_Reforco_Axial .Part	P16 - DS_Raio_Externo .Part	P17 - Total Deformation 7 Reported Frequency (Hz)	P18 - Total Deformation 8 Reported Frequency (Hz)	P19 - Total Deformation 9 Reported Frequency (Hz)
Optimization Domain						
Lower Bound	45	37	5			
Upper Bound	54	40	15			
Optimization Objectives						
Objective	No Objective	No Objective	No Objective	Seek Target	Seek Target	Seek Target
Target Value				385,58	513,78	895,5
Importance				Default	Higher	Higher
Constraint Handling						
Candidate Points						
Candidate A	53,429	37,256	5,9561	★★★ 391,78	★★ 504,84	★★ 941,84
Candidate B	53,915	38,452	5,764	★★★ 386,03	★★ 500,03	★★★ 928,55
Candidate C	50,783	37,766	7,1724	★★★ 387,32	★★ 501,58	★★ 942,7

Fig. 9 Result example for MOGA's adjust process.

Table 5 Full numerical model adjustment result.

	Medium experimental data (Hz)	Numerical deviation
Mode nº1	276.17	2.06%
Mode nº2	317.01	-3.38%
Mode nº3	409.66	-1.87%

(a) Contacts regions between end shields and frame

(b) Contacts regions for end shields fixing screws

Fig. 10 Contacts Localization details between end shields and frame.

Fig. 11 Numerical and experimental FRF curves for fully assembled prototypes fixed on a rigid base—damping factor = 1.87%.

points of their experimental peers and the method used was the modal superposition. The excitation force magnitude, as well as the damping values, were the same used and extracted from the experimental tests. Fig. 11 presents the FRF curve comparison for the fully assembled prototype fixed on a rigid base.

Although, from Fig. 11, the peak representing the third mode exhibits a deviation difference compared to the experimental ones, the curves show good correlation. Such behavior is attributed to stator's

simplification. Disregarding all the complexity of coils, insulating films and stack, seems to have directly influenced the accuracy of numerical model, particularly with respect to the third mode, but comparing to results from Tab. 4, it can be concluded that the final numerical model, using the parameters that produce the best results, is adequate for the purposes proposed in this work. Fig. 12 shows the first three modes of vibration, on the fixed condition, for the motor studied in this work.

Fig. 12 First three experimental and numerical vibration modes of studied electric motor fixed on a rigid base.

5. Conclusions

With the exception of stator, the adjustment results and model's validation showed low sensitivity ratio using data and factory tolerances, indicating that variations in the production process should not result in significant differences in frequency domain. With respect to the stator, the results showed that the method used to fit the model is appropriate and that the experimental determination of orthotropic characteristics is of fundamental importance for numerical model accuracy. The elasticity modulus found, associated with axial compression, differs from that found by the authors studied (Roivainen [3] and Delves [8]). It is believed that this difference is due to peculiarities of stator manufacture, as the lamination stack's pressing force and the impregnation method.

From these results, it is evident that the numerical solution is a powerful tool for first three vibration

modes prediction of a structure as complex as an electric motor, but is very dependent on the stator orthotropic characteristics and of the contacts that represent the fit between end shields and frame. The following steps must be followed for a proper motor numerical modeling according to the results found:

- (1) Determine the material properties used by stator with experimental validation, considering it an orthotropic system;
- (2) Using the known values of cast iron properties for frames and end shields;
- (3) Simplifying correctly the geometry models that compose the motor;
- (4) Using elements of appropriate size to form the mesh;
- (5) Assemble each component of numerical model using bonded contacts on the end shields fixing screws regions, and between end shields and frame and

between stator and frame use contact that transmits the tangential and compressive stress in its entirety, allowing relative movement between the surfaces in the normal separation direction;

(6) Restrict the movement of lower surface of frame model feet to eliminate any movement in three Cartesian axis directions and thus reproduce the condition “fixed on a rigid base”;

(7) The rotor effect should be considered on the pre-stress model in Item 5 before performing the numerical modal analysis;

(8) Perform the numerical modal analysis.

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