Major and Minor Harmonic Keys: The Discrepancy in Chord Classification Under a Computational Tonality Analytical Method*

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Basing upon the tonality analytic method, the present paper aims at: (1) Drawing attention to the subordination of the minor key vs. the major key in the chord classification, using the same methodology; and (2) Showing the differences for the major key and the minor (harmonic) key in the classification of chords, as an aspect of importance for interpreting a piece's tonal structure diagram. The relations between chords appearing in the major and minor (harmonic) key will be shown by applying the comparisons of: (1) third-based chords; and (2) degrees in the C major and A minor keys, on which the same diatonic chords appear.

Keywords: tonality, modes, classification of diatonic chords, methodology

Introduction

In the Western music, the term “musical mode” may have dissimilar definitions. However, this notion is usually used as a concept concerning a kind of scale and melody (Powers, 2001). The invention of harmony in the Baroque period was significant source of polemics around music in the 17th and 18th centuries. Among those who investigated the foundations of harmony on a philosophical basis, including the legitimacy of the two modes, are the founders of new scientific methods (Kepler, 1619; Mersenne, 1637; Descartes, 1650). In the same period, both modes (i.e., major and minor) tend to be reduced to a single, i.e., major, scale—in that a minor is but a variety of the “perfect” major scale. Theoretical works on scale modes, justifying the existence of scales, show minor scales as subordinate to the major. In Helmholtz's (1863) approach, the minor scale is not part of the music's beauty; nor can it be classed under the natural or rational system. Also Rameau (1722) was of opinion that only the existence of the major mode is explainable in rational terms in the world of harmony. He considered the minor mode an unnatural variety of the major mode.

Our contemporary theoretical works on harmony, tonality, and methods of main key determination in a musical piece, maps of chord relations, etc., are indicative of certain problems with the minor key (Shepard, 1982; Krumhansl, 1990, 2002; Chew, 2000; Cambouropoulos, 2003; Honingh, 2007). The problem with the

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minor key issue also concerns the author's method of analysis of the tonal structure in pieces of music (Majchrzak, 2005, 2007, 2009).

Analytical Method

Analytical method (Majchrzak, 2005, 2009) consists in assignation of diatonic chords appearing in a piece of music to individual KR's (key ranges) being keys in their respective natural variety. We mark the keys with the consecutive integers: the sharp keys with positive numbers, the flat keys—with negative numbers. For instance, the number (3) marks the keys of A major and F sharp minor (natural); the number (-1)—the keys of F major and D minor (natural).

For any tone, we can determine the keys it appears in. For instance, the tone D appears in these keys: (-3, -2, -1, 0, 1, 2, 3)¹. The tone E appears in the following keys: (-1, 0, 1, 2, 3, 4, 5)². This is similarly for any and each chord. For example, the tones of the C major chord appear in the following keys: (-5, -4, -3, -2, -1, 0, 1), (-1, 0, 1, 2, 3, 4, 5), (-4, -3, -2, -1, 0, 1, 2).

The substratum for our chord classification is the arithmetic average of keys wherein the tones of a given diatonic chord appear:

$$\text{arithmetic average} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

See Examples 1-5:

Example (1) DF sharp:

$$\text{AA (arithmetic average)} = \frac{(-3 - 2 - 1 + 0 + 1 + 2 + 3) + (1 + 2 + 3 + 4 + 5 + 6 + 7)}{7 + 7} = 2$$

Example (2) BDFA:

$$\text{AA} = \frac{(0 + 1 + 2 + 3 + 4 + 5 + 6) + (-3 + 2 + 1 + 0 + 1 + 2 + 3) + (-6 - 5 - 4 - 3 - 2 - 1 + 0) + (-2 - 1 + 0 + 1 + 2 + 3 + 4)}{7 + 7 + 7 + 7} = 0.25$$

Example (3) A flat B flat DFG:

$$\text{AA} = \frac{(-9 - 8 - 7 - 6 - 5 - 4 - 3) + (-7 - 6 - 5 - 4 - 3 - 2 - 1) + (-3 - 2 - 1 + 0 + 1 + 2 + 3) + (-6 - 5 - 4 - 3 - 2 - 1 + 0) + (-4 - 3 - 2 - 1 + 0 + 1 + 2)}{7 + 7 + 7 + 7} = -2.8$$

Example (4) GBDF:

$$\text{AA} = \frac{(-4 - 3 - 2 - 1 + 0 + 1 + 2) + (0 + 1 + 2 + 3 + 4 + 5 + 6) + (-3 - 2 - 1 + 0 + 1 + 2 + 3) + (-6 - 5 - 4 - 3 - 2 - 1 + 0)}{7 + 7 + 7 + 7} = -0.25$$

Example (5) GC sharp:

$$\text{AA} = \frac{(-4 - 3 - 2 - 1 + 0 + 1 + 2) + (2 + 3 + 4 + 5 + 6 + 7 + 8)}{7 + 7} = 2$$

In this method: Arithmetic average space—all numeric values derivable from the above arithmetic-average formula. The arithmetic average space is divided into KR's, each of which is a KR with a given number of clef signs. E.g., the KR of one-flat keys (F major and D minor) encompasses the arithmetic average space's open-ended range, spanning between -0.5 and -1.5. The KR of two-sharp keys (D major and B minor)

¹ E flat major and C minor, B flat major and G minor, F major and D minor, C major and A minor, G major and E minor, D major and B minor, and A major and F sharp minor.

² F major and D minor, C major and A minor, G major and E minor, D major and B minor, A major and F sharp minor, E major and C sharp minor, and B major and G sharp minor.

³ Note: $x_1, x_2, x_3, \dots, x_n$: keys wherein the tones of a given diatonic chord appear; n : number of all keys.

encompasses the arithmetic average spanning between 1.5 and 2.5. The KR of four-sharp keys (E major and C# minor) encompasses the arithmetic average spanning between 4.5 and 5.5.

(1) Chords and KR. For example: The chord GBDF (AA = -0.25) belongs to KR 0. The chord EG#BC# (AA = 4) belongs to KR 4.

(2) 2KRs chord—any chord whose arithmetic average belongs to two adjacent KRs. E.g., the arithmetic average of the CEGB chord is 0.5; the chord belongs to both KR 0 (C major and A minor) and KR 1 (G major and E minor). The arithmetic average of the CDEFGA chord is -0.5; the chord belongs to both KR -1 (F major and D minor) and KR 0 (C major and A minor).

(3) N-D—non-diatonic chords.

Analysis of pieces can be displayed in Figure 1.

Figure 1. Chopin, Mazurka B flat major, Op. 17, No. 1.

Note: Horizontal Axis: KRs. For example: -4 (KR of A flat major and F minor), -2 (KR of B flat major and D minor), and 0 (KRs of C major and A minor); Vertical Axis: percentage domination of given KRs.

Third-Based Chords Built on Individual Degrees of C Major/A Minor Keys

As discussed hereinabove, the analytical method consists in assignment of diatonic chords to individual ranges of a key, which is followed by a quantitative comparison of the KRs. Let us take a look at the differences in assignment to KRs of triad appearing on individual grades of C major and A minor keys.

(1) C major

Figure 2. Triads appearing on individual grades of C major key.

(2) A minor

Figure 3. Triads appearing on individual grades of A minor (harmonic) key.

We could see in Figures 2-3 that all the triads built upon individual C major key grades are assigned to the key's three ranges, including: KR 0 (C major and A minor), KR -1 (F major and D minor), and KR 1 (G major and E minor). Triads created on individual grades of the A minor harmonic key belong to the key's four ranges whilst one of them belongs to the N-D group. Now, let us have a closer look at third-based chords built up on individual C major and A minor (harmonic) key grades (see Table 1).

(1) Triads: 1st, 4th: Chords based on 1st degree of C major and A minor keys are classed in the KR where they function as the keynotes, i.e., KR 0 (C major, A minor). The situation where triads built upon the same degree in the keys C major and A minor are part of a single KR is to be met only once: This concerns chords built on the fourth degree. The F major and D minor chords are part of KR -1. The situation is different for triads built on the remaining degrees of those keys.

Table 1

Third-Based Chords Built on Individual Degrees of C Major and A Minor (Harmonic) Keys

Dyads				
	C major		A minor	
Degree	Chord	KR	Chord	KR
1st	CE	KR 0	AC	KR -1, KR 0
2nd	DF	KR -2, KR -1	BD	KR 1, KR 2
3th	EG	KR 0, KR 1	CE	KR 0
4th	FA	KR -1	DF	KR -2, KR -1
5th	GB	KR 1	EG#	KR 4
6th	AC	KR -1, KR 0	FA	KR -1
7th	BD	KR 1, KR 2	G#B	KR 4, KR 5
Triads				
	C major		A minor	
DEGREE	CHORD	KR	CHORD	KR
1st	CEG	KR 0	ACE	KR 0
2nd	DFA	KR -1	BDF	KR 0
3th	EGB	KR 1	CEG#	N-D
4th	FAC	KR -1	DFA	KR -1
5th	GBD	KR 1	EG#B	KR 4
6th	ACE	KR 0	FAC	KR -1
7th	BDF	KR 0	G#BD	KR 3
Four-note chords				
	C major		A minor	
Degree	Chord	KR	KR	KR
1st	CEGB	KR 0, KR 1	ACEG#	N-D
2nd	DFAC	KR -1	BDFA	KR 0
3th	EGBD	KR 1	CEG#B	N-D
4th	FACE	KR -1, KR 0	DFAC	KR -1
5th	GBDF	KR 0	EG#BD	KR 3
6th	ACEG	KR 0	FACE	KR -1, KR 0
7th	BDFA	KR 0	G#BDF	N-D

(Table 1 continued)

Five-note chords				
	C major		A minor	
Degree	Chord	KR	KR	KR
1st	CEGBD	KR 0	ACEG#B	N-D
2nd	DFAEC	KR 0	BDFAC	KR 0
3th	EGBDF	KR 0	CEG#BD	N-D
4th	FACEG	KR -1	DFAEC	KR 0
5th	GBDFA	KR 0	EG#BDF	N-D
6th	ACEGB	KR 1	FACEG#	N-D
7th	BDFAC	KR 0	G#BDFA	N-D
Six-note chords				
	C major		A minor	
Degree	Chord	KR	KR	KR
1st	CEGBDF	KR 0	ACEG#BD	N-D
2nd	DFAECG	KR -1, KR 0	BDFACEG#	N-D
3th	EGBDFA	KR 0	CEG#BD	N-D
4th	FACEGB	KR 0	DFAECG#	N-D
5th	GBDFAC	KR 0	EG#BDFA	N-D
6th	ACEGBD	KR 0, KR 1	FACEG#B	N-D
7th	BDFACE	KR 0	G#BDFAC	N-D

2nd: The chord built up on the 2nd degree of the C major, i.e., the subdominant of the 2nd degree belongs, as shown in Table 1, to KR -1, the range to which the C major subdominant chord belongs as well. In the A minor, the chord built on the 2nd degree is part of the same KR as the chord built on 1st degree (i.e., the minor keynote), that is, KR 0. Having said that, why should the triad built on the 2nd degree of A minor key belong to PT 0? The BDF chord may be considered as a dominant seventh without the root in C major key. It then appears in the KR within which the chord appears into which it is resolved (according to the classic theory of harmony, the BDF chord may be resolved to the C major chord).

3th: The chord built up on the 3rd degree of the C major, composed of EGB tones, belongs to KR 1, and so, to the KR where the G major chord appears. The chord on the 3rd degree in the A minor is an augmented chord, which means that it is not assigned to a KR. Instead, it is classed under a separate group of N-D chords.

5th: The triad built on the 5th degree (the dominant) in the C major, i.e., the G major chord, is classified in KR 1. The range is situated right of the keynote's range (KR 0 in the C major). In A minor key, the minor keynote appears in KR 0. In turn, the chord built on the 5th degree of the A minor, i.e., the major dominant, is classified as KR 4. Then, how should the dominant's situation be explained, in a range fixed as many as four ranges away from the range where the minor keynote in the A minor is classed? The E major chord may act as a keynote for the E major key. Hence, it is contained within KR 4, similarly as the C major chord in KR 0 or the A flat major chord KR -4.

6th: In the C major, the sixth-grade keynote (ACE) appears within the same KR as the keynote (CEG), i.e., KR 0. In the A minor, the triad built on the sixth degree is situated in KR -1, that is, a range located left of the range wherein the keynote chord appears.

7th: In both C major and A major key, a diminished chord appears upon degree 7th. We have already come across the BDF chord on the grounds of A minor key (as its 2nd-degree chord). As for the G sharp BD chord appearing on the 7th degree of the A minor, the following question arises: How can we interpret the position of a chord built up on the 7th degree of A minor key in the range of A major and F sharp minor keys (KR 3)? The chord composed of the notes G sharp BD may be deemed to be the dominant seventh without the root for A major key, i.e., KR 3. Thus, the second of the diminished triads built upon the A minor degrees better corresponds with the major key (A major—KR 3) than with a minor one (A minor—KR 0).

(2) Four-note chords: 1st: The arithmetic average of the keys where the notes appear of the four-part chord built on 1st degree of the C major (i.e., CEGB) equals 0.5. Thus, the chord belongs to both KR 0 and KR 1⁴. As for the minor key, one has to do with a chord whose notes are not reducible to a single key of the natural variety (BDFG sharp)⁵, and hence, we will not take it into consideration for the purpose of assignment to individual KRs.

2nd: The four-note chord built on the 2nd degree of the C major belongs to KR -1. This chord can be considered as the minor keynote with a small seventh added in the D minor, or, the keynote with a great sixth added in the F major. The four-part chord BDFA appearing on degree 2nd of the A minor is classed under KR 0. Within this same range, C major chord appears, to which BDFA chord, being the C major key's dominant ninth without the root, gets most frequently resolved.

3rd: The structure of four-note chord EGBD is identical to that of DFAC chord, whereas the CEG sharp B chord is put in our breakdown in a separate column (N-D), and not assigned to an individual KR.

4th: In the C major, the four-note chord built on degree 4th has a structure identical as the four-note chord built on the key's 1st degree (the chord belongs to both KR -1 and KR 0). In the A minor, the four-note chord built on the 4th degree is of an identical structure as the one built on degree 1st of the C major (the chord belongs to KR -1).

5th: The GBDF chord is part of KR 0, i.e., to that major-KR in which it operates as the dominant seventh. In the case of the A minor dominant 7th, we come across a troublesome case. This chord does not, namely, belong to the KR wherein the A minor keynote appears (i.e., KR 0), but is part of KR 3 instead. A similar situation was the case when it came to discussing the G sharp BD chord. The GBDF chord belongs to the range where the major tonic appears (CEG chord appearing in KR 0), whereas it is not part of the range where the major tonic appears to which this chord can be resolved (CE flat G appears in KR -3). A similar thing happens with EG sharp BD chord, which appears in the range where the major tonic is classed to which it is resolved, that is, in KR 3 (The A major chord to which EG#BD chord gets resolved appears in KR 3).

6th, 7th: Chords build like: ACEG, FACE, and BDFA have already been discussed. The diminished four-part chord is a N-D chord.

(3) Five-note chords: Most five-note chords built on A minor key degrees are part of the N-D group. In the C major, most of the chords belong to the key's main range, and two of them belong to KR -1 and KR 1 respectively.

(4) Six-note chords: All the six-note chords created on individual degrees of the A minor, in its harmonic variety, contain an augmented four-part chord. This means that these are not assigned to KRs (N-D). As for the

⁴ 2 KRs.

⁵ N-D.

(5) Dyads: To end with, let us take a look on third-sized dyads. As it can be seen in Table 1, the differences are remarkable also for the dyads.

The previous section discussed third-based chords built upon individual C major and A minor keys' degrees. Some of the minor-key chords were unclassified with respect to the KRs, as their tones could not be reduced to a single natural key. Now, let us turn attention to diatonic chords with a diverse interval structure. The subsequent table lines specify chords belonging to KR 0 (C major, A minor) and the C major/A minor degrees whereupon the chords are created.

C major key degrees:

			1	2	3	4	5	6	7	
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Tones:

G	A	B	C	D	E	F	G	A	B	C
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A minor key degrees:

	1	2	3	4	5	6	7			
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Figure 4. Selected triad.

Table 2

Diads													
Chord											C major key degrees		A minor key degrees
G	A	B	C	D	E	F	G	A	B	C	1, 3 (CE)	3, 5 (CE)	
G	A	B	C	D	E	F	G	A	B	C	3, 5 (EG)	5, 7 (EG)	
G	A	B	C	D	E	F	G	A	B	C	1, 6 (CA)	1, 3 (AC)	
G	A	B	C	D	E	F	G	A	B	C	5, 6 (GA)	1, 7 (AG)	
G	A	B	C	D	E	F	G	A	B	C	1, 7 (CB)	2, 3 (BC)	
G	A	B	C	D	E	F	G	A	B	C	3, 4 (EF)	5, 6 (EF)	
G	A	B	C	D	E	F	G	A	B	C	4, 7 (FB)	2, 6 (BF)	
G	A	B	C	D	E	F	G	A	B	C	2, 6 (DA)	1, 5 (AD)	
G	A	B	C	D	E	F	G	A	B	C	2, 5 (DG)	4, 7 (DG)	
Triads													
Chord											C major key degrees		A minor key degrees
G	A	B	C	D	E	F	G	A	B	C	1, 3, 5 (CEG)	3, 5, 7 (CEG)	
G	A	B	C	D	E	F	G	A	B	C	1, 3, 6 (CEA)	1, 3, 5 (ACE)	
G	A	B	C	D	E	F	G	A	B	C	2, 5, 6 (DGA)	1, 4, 7 (AGD)	
G	A	B	C	D	E	F	G	A	B	C	2, 4, 7 (DFB)	2, 4, 6 (BDF)	

6
7

Triads												
Chord											C major key degrees	A minor key degrees
G	A	B	C	D	E	F	G	A	B	C	4, 6, 7 (FAB)	1, 2, 6 (ABF)
G	A	B	C	D	E	F	G	A	B	C	4, 5, 7 (FGB)	2, 6, 7 (BFG)
G	A	Bb	C	D	E	F	G	A	Bb	C	3, 6, 7b (EABb)	1, 2b, 5 (ABbE)
G	A	B	C	D	E	F#	G	A	B	C	1, 4#, 5 (CF#G)	3, 6#, 7 (CF#G)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 6 (CDA)	1, 3, 4 (ACD)
G	A	B	C	D	E	F	G	A	B	C	2, 3, 5 (DEG)	4, 5, 7 (DEG)
G	A	B	C	D	E	F	G	A	B	C	1, 5, 7 (CGB)	2, 3, 7 (BCG)
G	A	B	C	D	E	F	G	A	B	C	3, 4, 6 (EFA)	1, 5, 6 (AEF)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 3 (CDE)	3, 4, 5 (CDE)
G	A	B	C	D	E	F	G	A	B	C	2, 3, 4 (DEF)	4, 5, 6 (DEF)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 7 (CDB)	2, 3, 4 (BCD)
Four-note chords												
Chord											C major key degrees	A minor key degrees
G	A	B	C	D	E	F	G	A	B	C	2, 4, 5, 7 (DFGB)	2, 4, 6, 7 (BDFG)
G	A	B	C	D	E	F	G	A	B	C	2, 4, 6, 7 (DFAB)	1, 2, 4, 6 (ABDF)
G	A	B	C	D	E	F	G	A	B	C	2, 3, 5, 6 (DEGA)	1, 4, 5, 7 (ADEG)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 5, 6 (CDGA)	1, 3, 4, 7 (ACDG)
G	A	B	C	D	E	F	G	A	B	C	1, 3, 5, 6 (CEGA)	1, 3, 5, 7 (ACEG)
G	A	B	C	D	E	F	G	A	B	C	1, 3, 5, 7 (CEGB)	2, 3, 5, 7 (BCEG)
G	A	B	C	D	E	F	G	A	B	C	1, 3, 4, 6 (CEFA)	1, 3, 5, 6 (ACEF)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 3, 6 (CDEA)	1, 3, 4, 5 (ACDE)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 3, 5 (CDEG)	3, 4, 5, 7 (CDEG)
G	A	B	C	D	E	F#	G	A	B	C	1, 2, 4#, 5 (CDF#G)	3, 4, 6#, 7 (CDF#G)
G	A	B	C	D	E	F	G	A	B	C	2, 3, 6, 7b (DEABb)	1, 2b, 4, 5 (ABbDE)
G	A	B	C	D	E	F	G	A	B	C	1, 4, 6, 7 (CFAB)	1, 2, 3, 6 (ABCF)
G	A	B	C	D	E	F	G	A	B	C	3, 4, 5, 7 (EFGB)	2, 5, 6, 7 (BEFG)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 5, 7 (CDGB)	2, 3, 4, 7 (BCDG)
G	A	B	C	D	E	F	G	A	B	C	4, 5, 6, 7 (FGAB)	1, 2, 6, 7 (ABFG)
G	A	B	C	D	E	F	G	A	B	C	1, 3, 4, 7 (CEFB)	2, 3, 5, 6 (BCEF)
G	A	B	C	D	E	F	G	A	B	C	2, 3, 4, 7 (DEFB)	2, 4, 5, 6 (BDEF)
G	A	Bb	C	D	E	F	G	A	Bb	C	3, 5, 6, 7b (EGABb)	1, 2b, 5, 7 (ABbEG)
G	A	B	C	D	E	F#	G	A	B	C	1, 4#, 5, 6 (CF#GA)	1, 3, 6#, 7 (ACF#G)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 4, 7 (CDFB)	2, 3, 4, 6 (BCDF)
G	A	B	C	D	E	F	G	A	B	C	1, 5, 6, 7 (CGAB)	1, 2, 3, 7 (ABCG)
G	A	B	C	D	E	F	G	A	B	C	3, 4, 5, 6 (EFGA)	1, 5, 6, 7 (AEFG)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 6, 7 (CDAB)	1, 2, 3, 4 (ABCD)
G	A	B	C	D	E	F	G	A	B	C	2, 3, 4, 5 (DEFG)	4, 5, 6, 7 (DEFG)
Five-note chords												
Chord											C major key degrees	A minor key degrees
G	A	B	C	D	E	F	G	A	B	C	1, 2, 3, 5, 6 (CDEGA)	1, 3, 4, 5, 7 (ACDEG)
G	A	B	C	D	E	F	G	A	B	C	2, 4, 5, 6, 7 (DFGAB)	1, 2, 4, 6, 7 (ABDFG)
G	A	Bb	C	D	E	F	G	A	Bb	C	2, 3, 5, 6, 7b (DEGABb)	1, 2b, 4, 5, 7 (ABbDEG)
G	A	B	C	D	E	F#	G	A	B	C	1, 2, 4#, 5, 6 (CDF#GA)	1, 3, 4, 6#, 7 (ACDF#G)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 3, 4, 6 (CDEFA)	1, 3, 4, 5, 6 (ACDEF)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 3, 5, 7 (CDEGB)	2, 3, 4, 5, 7 (BCDEG)

(Table 2 continued)

Table 2 (continued)												
Five-note chords												
Chord										C major key degrees	A minor key degrees	
G	A	B	C	D	E	F	G	A	B	C	1, 3, 4, 6, 7 (CEFAB)	1, 2, 3, 5, 6 (ABCEF)
G	A	B	C	D	E	F	G	A	B	C	1, 3, 4, 5, 7 (CEFGB)	2, 3, 5, 6, 7 (BCEFG)
G	A	B	C	D	E	F	G	A	B	C	2, 3, 4, 5, 7 (DEFGB)	2, 4, 5, 6, 7 (BDEFG)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 4, 6, 7 (CDFAB)	1, 2, 3, 4, 6 (ABCDF)
G	A	B	C	D	E	F	G	A	B	C	1, 4, 5, 6, 7 (CFGAB)	1, 2, 3, 6, 7 (ABCFG)
G	A	B	C	D	E	F	G	A	B	C	3, 4, 5, 6, 7 (EFGAB)	1, 2, 5, 6, 7 (ABEFG)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 3, 4, 7 (CDEFB)	2, 3, 4, 5, 6 (BCDEF)
G	A	B	C	D	E	F	G	A	B	C	2, 3, 4, 5, 6 (DEFGA)	1, 4, 5, 6, 7 (ADEFG)
G	A	B	C	D	E	F	G	A	B	C	1, 3, 4, 5, 7 (CEFGB)	2, 3, 5, 6, 7 (BCEFG)
Six-note chords												
Chord										C major key degrees	A minor key degrees	
G	A	B	C	D	E	F	G	A	B	C	1, 2, 4, 5, 6, 7 (CDFGAB)	1, 2, 3, 4, 6, 7 (ABCDFG)
G	A	B	C	D	E	F	G	A	B	C	2, 3, 4, 5, 6, 7 (DEFGAB)	1, 2, 4, 5, 6, 7 (ABDEFG)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 3, 5, 6, 7 (CDEGAB)	1, 2, 3, 4, 5, 7 (ABCDEG)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 3, 4, 5, 6 (CDEFGA)	1, 3, 4, 5, 6, 7 (ACDEFG)
G	A	B	C	D	E	F	G	A	B	C	1, 3, 4, 5, 6, 7 (CEFGAB)	1, 2, 3, 5, 6, 7 (ABCEFG)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 3, 4, 6, 7 (CDEFAB)	1, 2, 3, 4, 5, 6 (ABCDEF)
G	A	B	C	D	E	F	G	A	B	C	1, 2, 3, 4, 5, 7 (CDEFGB)	2, 3, 4, 5, 6, 7 (BCDEFG)

Conclusions

The examples discussed enable us to draw attention to the differences in the assignment of chords to the KRs conditional upon the key's mode. Basing upon the examples quoted in Table 1 (C major and A minor keys, in our case), tentative conclusions may be drawn with respect to a superiority of the major key over the minor harmonic.

In major key, dominant forms are frequently contained within the KR in which the major tonic appears to which they are resolvable. In the minor, dominant forms (also dominant chord!) are distant from the range wherein the minor chord (minor keynote) appears to which they are resolved.

In minor key (harmonic variety), chords appear that are not assignable to KRs (which also refers to dominant forms, e.g., dominant ninth with a small ninth, or, diminished four-part chord).

In C major key (Table 1) all triads are classified within three KRs—KR 0, KR -1, and KR 1. In the case of A minor harmonic key the triads are classified under numerous KRs—KR 0, KR -1, KR 1, KR 3, and KR 4. This seems to be not very natural, that selected triads related to the A minor key are classified within KRs distanced from KR 0.

All chords in C major belong to the diatonic material. In A minor harmonic key only all dyads belong to the diatonic material. Among triads, four-note chords and five-note chords we are able to find many N-D chords. In the case of third-based six-note chords all the chords are not classified under specific KRs (N-D material—N-D).

Material presented in Table 2 shows also differences in the interpretation of relationship between chords and their classification. For example:

Let us have a look at Figure 3. Triad CDE belongs to the KR 0 (C major and A minor). In the C major, it is built on degrees 1st, 2nd, and 3rd. These degrees appear more important than those in the case of A minor key (3rd, 6th, and 5th). The first three notes of the C major comprise the tonic's prime and third, whereas in the A minor, these are the keynote's third and fifth with an added fourth.

Let us have a look at four-note chord DFAB. It can be considered as a tonic chord with added major 6th in D minor key (In D minor key appear tone Bb, not B. However, other tones from this chord belong to the most recognized chord in one-flat key). Hence, how to explain the classification of chord DFAB within KR 0, not in the KR -1? We know that counterpart of this chord in major key (CEGA) is classified in the KR 0. Hence, if dominant 9th without the root is more important harmonic function then tonic with added interval?

Presented examples show the problem with a interpretation of chord's classification in the case of minor harmonic key. We can find examples, where more natural seems to be a classification of chords in minor harmonic key. However, such situations become visible in the case of chords appearing less frequent in given musical pieces. Hence, such examples are not very important when analysing a piece of music.

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