Moment-Rotation Characteristics for Flexible Beam-to-Column Steel Joint Exposed to Fire

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Abstract: Analytical models used to describe behaviour of steel frame loadbearing structures in fully developed fire usually do not allow for reduced joint stiffness due to increased member temperature. Joints previously designed as nominally rigid tend to become flexible in fire situation, with degree of flexibility increasing during fire development. Reliable analysis of this phenomenon and its influence on the redistribution of internal forces result in the need for developing appropriate characteristics, describing relationship between bending moment applied to the joint and joint rotation. Characteristics of such type, specified for fire conditions, depend on steel temperature. In the current work, the authors propose a practical approach to develop such characteristics, based on the knowledge of analogous characteristic prepared for persistent design situation. The developed technique does not require to generalize the classical component method to the case of fire, which may be difficult in practical situations. The proposed computational algorithm has been tested on an example of a typical beam-to-column joint.

Key words: Steel joint, fire, temperature, flexibility, stiffness, rotation.

1. Introduction

The development of a computational algorithm allowing for specification of the bending moment-rotation relationship describing compliance of the considered steel beam-to-column joint subject to accidental fully-developed fire scenario is the purpose of this paper. Determination of the reference characteristic calibrated for the considered joint in persistent design situation, without any fire interaction, is the starting point in such analysis. Such reference characteristic is usually determined by a standard analytical approach known as component method [1]. It is based on a detailed analysis of potential failure modes assigned to individual, conceptually isolated joint components, and addition of effective stiffnesses for such components. In the current work, the authors suggest the application of simpler formal models, using a simple function, to describe joint behaviour while subject to loads. The joint displayed in Fig. 1 is analysed in the example presented below. The popular Richard-Abbott model [2] is chosen for the analysis, for which relationship between bending moment in the joint and its rotation is expressed as follows:

\[
\phi = \frac{M}{S_{j,ini}} \left[ 1 - \left( \frac{M}{M_{j,R}} \right)^{\frac{1}{c}} \right]
\]

(1)

where, \( c = 1.216 \text{ when } \log(\phi) \leq -2.81 \) and \( c = 1.73 \log(\phi) + 6.077 \text{ when } \log(\phi) > -2.81 \).

Symbol \( M_{j,R} \) stands for loadbearing capacity of the considered joint while \( M \) denotes bending moment applied to the joint. In the next stage, initial joint stiffness \( S_{j,ini} \) associated with room temperature and rotation of the joint exhibiting such stiffness are estimated. Those factors are determined according to the canons of classical component method by the formulae:

\[
S_{1,20} = S_{1,ini} \quad \text{and} \quad \phi_{1,20}^y = \frac{M_{1,20}^y}{S_{1,20}}
\]

(2)

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where, $E_{20}$ stands for steel longitudinal elasticity modulus in normal conditions (temperature of 20 °C), and $z$ for the level arm of the effective bending moment applied to the joint. Bending moment $M_{1,20}$ represents the limit plastic moment determined at the $S_{1,20}$ stiffness. Components $k_i$ stand for compliance of $i$-th joint component, in particular: column web subject to shear; column web, beam flange and web subject to compression; column flange and beam endplate subject to bending; column and beam webs as well as bolts subject to tension in the head on beam-to-column connection. Compliance of individual components may be interpreted as characteristics of replacement springs. The spacing of such springs is determined by the value of $z$.

The stiffness is modified by changing the angle of piecewise linear curve at the point in which straight thin solid line determined by such stiffness intersects the reference curve on the moment-rotation diagram. After such change, the next intersection point is obtained, thus forcing successive stiffness correction. In this manner, the joint stiffness is modified in consecutive steps, using rotation increment control, as to derive the piecewise linear curve approximating the initial moment-rotation curve determined on the basis of a priori assumed formal model. As a consequence, this piecewise linear curve is treated as the reference line and transformed in the following steps to satisfy the conditions of fully developed fire scenario.

2. Moment-Rotation Curve Transformation to the Accidental Design Situation of Fully Developed Fire Scenario

In the developed fire conditions, steel yield point as well as the longitudinal elasticity modulus for this material is reduced as the result of fire exposure. Reduction degree is expressed by $\psi_{y\theta}$ and $\psi_{E\theta}$ coefficients, respectively, as given in the EN 1993-1-2 [3] code. It is assumed that the slope angle change with respect to the horizontal axis on the bending moment-rotation diagram, expressing joint compliance, occurs every time after yield point is reached in a spring modelling behaviour of consecutive joint component. Ground rules of such recursive approach were presented in Ref. [4, 5]. As the result of consecutive yielding, when first and $s$-th joint components yield, respectively, the following happens (Fig. 2):

$$S_{1,\theta} = k_{E\theta}E_{20}z^2 = k_{E\theta}S_{1,20}$$

and

$$S_{s,\theta} = k_{E\theta}S_{s,20}$$

Besides, if the force in an equivalent spring is equal to $F_{i,\theta}$ and after reduction of the whole complex
set of springs to a couple of forces (compressive and tensile), this value changes to \( F_{r,\Theta} \), the following occurs:

\[
M_{\Theta} = F_{r,\Theta}z = k_y\,\Theta M_{20}
\]

and

\[
F_{i,\Theta} = k_y\,\Theta F_{i,20}^y
\]

and also:

\[
M_{s,20}^y = k_y\,\Theta M_{s,20}^y
\]

Thus:

\[
\phi_{1,\Theta}^y = \frac{M_{1,\Theta}}{S_{1,\Theta}} = \frac{k_y\,\Theta}{k_E\,\Theta} \phi_{1,\Theta}^y
\]

and

\[
\phi_{s,\Theta}^y = \frac{k_y\,\Theta}{k_E\,\Theta} \phi_{s,20}^y
\]

and following:

\[
\bar{\phi}_{s+1,\Theta}^y = \phi_{s+1,\Theta}^y - \phi_{s,\Theta}^y
\]

As the consequence:

\[
\bar{S}_{s+1,\Theta} = \frac{M_{s+1,\Theta} - M_{s,\Theta}}{\phi_{s+1,\Theta}^y} \Rightarrow \bar{\phi}_{s+1,\Theta}^y = \frac{M_{s+1,\Theta}^y - M_{s,\Theta}^y}{\bar{S}_{s+1,\Theta}^y}
\]

As may be inferred from the above, joint stiffness change in fire conditions is proportional to the reduction in longitudinal elasticity modulus for steel. On the other side, the transferred bending moment is diminished in a degree proportional to the reduction in the material yield point. As a result of these interdependences, rotation in a joint depends on the
relationship between those two reduction components enumerated above.

3. Force-Displacement Relationship in Fire

Determination of parameters describing compliance of a single joint component in fire conditions must take into account elastic-plastic character of its working conditions. Because of this, a bilinear force-displacement relationship is assumed for the joint component (Fig. 3). In this way, one can distinguish between \( K_{i,\Theta}^{el} = k_E \cdot k_{i,20}^{el} \) stiffness, adequate in elastic working regime, while \( F^* < F_{i,\Theta}^y \), and \( K_{i,\Theta}^{pl} = k_E \cdot k_{i,20}^{pl} \) stiffness, used to describe spring response in an inelastic area, when \( F^{**} \geq F_{i,\Theta}^y \).

In the first case, the following occurs:

\[
\Delta_{i,\Theta}(F^*) = \frac{F^*}{K_{i,\Theta}^{el} k_{E,\Theta} k_{i,20}^{el}} = \frac{1}{k_{E,\Theta} k_{i,20}^{el}} \Delta_{i,20}(F^*) \tag{12}
\]

subject to:

\[
A_{i,\Theta}(F^{**}) = \frac{F_{i,\Theta}^y}{K_{i,\Theta}^{el} k_{E,\Theta} k_{i,20}^{el}} = \frac{1}{k_{E,\Theta} k_{i,20}^{el}} \Delta_{i,20}(F^{**}) = \frac{1}{k_{E,\Theta} k_{i,20}^{el}} \Delta_{i,20}(F_{i,\Theta}^y) \tag{13}
\]

while in the second case:

\[
A_{i,\Theta}(F^{**}) = \Delta_{i,\Theta}^{**} = \Delta_{i,\Theta}^y + \frac{1}{k_{E,\Theta} k_{i,20}^{pl}} \left( F^{**} - F_{i,\Theta}^y \right) \tag{14}
\]

One may easily note that displacement increase under applied load in fire conditions is inversely proportional to the reduction degree in the longitudinal modulus of elasticity. It has to be noted that the boundary defining elastic working regimen of the considered joint component changes as well, since it depends on the ratio of both reducing factors defined above. The superscript \( f \) in Eq. (14) is to be associated with element failure.

4. Obtained Results and Conclusions

Bending moment-rotation characteristics for the analyzed joint are presented in Fig. 4. The shape of those characteristics depends on assumed element temperature. The initial reference curve, which is based
Fig. 4  Moment-rotation characteristics determined for fire scenario, assuming Richard-Abbott model based relationship curve derivation: (a) reference curve; (b) joint compliance characteristics.

on the assumed Richard-Abbott model, and in this paper approximated by a piecewise line, is presented in Fig. 4a.

As it can be observed, application of the approach proposed by the authors, resulting in simplified bending moment-rotation characteristics assumed as reference curves based on the assumption of appropriate formal model as the basis for analysis, results in efficient derivation of the same relationships for an accidental design scenario of fully developed fire. Credibility of relationships determined in the manner described above obviously depends on the credibility of characteristics selected to describe the joint behaviour in persistent design scenario. One may easily observe that compliance of the joint increases slowly, as long as the temperature of joint components does not exceed 400 °C. In such a temperature, longitudinal modulus of elasticity for steel decreases, but yield point reduction is not observed. In higher temperature, when bearing capacity of steel is affected to a greater degree, increase in joint compliance is more pronounced. Comparison of curves obtained for a developed fire scenario with an initial reference curve allows for an assessment of the importance of the joint stiffness change, usually omitted in the analysis, on the final fire resistance of the considered frame. It has to be clearly noted, though, that the proposed calculation methodology constitutes a simplified approach. At the current stage of research, one may attempt to undertake a generalization effort for an appropriate extension of classical component method, in which individual potential failure modes will be analyzed with due attention paid to the specifics of fire conditions. Detailed discussion of these issues constitutes a task for separate study, definitely more extensive and complex.

References


