Unified Aerodynamic-Acoustic Formulation for Aero-Acoustic Structure Coupling

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Received: January 9, 2013 / Accepted: February 7, 2013 / Published: April 25, 2013.

Abstract: Conventional coupled BE/FE (Boundary-Element/Finite-Element) method and modeling of structural-acoustic interaction has shown its promise and potential in the design and analysis of various structural-acoustic interaction applications. Unified combined acoustic and aerodynamic loading on the structure is synthesized using two approaches. Firstly, by linear superposition of the acoustic pressure disturbance to the aeroelastic problem, the effect of acoustic pressure disturbance to the aeroelastic structure is considered to consist of structural motion independent incident acoustic pressure and structural motion dependent acoustic pressure, which is known as the scattering pressure, referred here as the acoustic aerodynamic analogy. Secondly, by synthesizing the acoustic and aerodynamic effects on elastic structure using an elegant, effective and unified approach, both acoustic and aerodynamic effect on solid structural boundaries can be formulated as a boundary value problem governed by second order differential equations which lead to solutions expressible as surface integral equations. The unified formulation of the acousto-aeroelastic problem is amenable for simultaneous solution, although certain prevailing situations allow the solution of the equations independently. For this purpose, the unsteady aerodynamic problem which was earlier utilizes well-established lifting surface method is reformulated using Boundary Element (BE) approach. These schemes are outlined and worked out with examples.

Key words: Active flutter control, acousto-aeroelasticity, aero-acoustic structure coupling, fluid structure interaction.

1. Introduction

Structural-acoustic interaction, in particular the vibration of structures due to sound waves, has been addressed in Refs. [1-2] as a significant issue found in many applications. Among the examples cited are the acoustic excitation and corresponding acoustic loads of the aircraft structure of main concerns on B-52 wing during takeoff as reported by Edson [3], which reaches acoustic sound pressure levels as high as 164 dB. Modern new and relatively lighter aircrafts may be subject to higher acoustic sound pressure level, such as predicted for the NASP [4]. Typical structural acoustic and high frequency vibration problems can severely and adversely affect spacecraft structures and their payloads has lucidly been elaborated by Eaton [5] and Smith et al. [6]. For many classes of structures exhibiting a plate-like vibration behavior, such as antennas and solar panels, their low-order mode response is likely to be of greatest importance. Assessment of combined acoustical and quasi-static loads may be significant. Modeling of structural-acoustic interaction has been attempted by many investigators, among others, Mei and Pates [7], by using coupled BE/FE Method, who also considered the control of acoustic pressure using piezoelectric actuators. With this consideration, it is possible to couple the boundary element method and the finite element method to solve structure-aero-acoustic interaction problem.

Earlier works by Djojodihardjo and his colleagues [1-2, 8-11] addressed progressive stages of the development of coupled BE/FE method and modeling of structural-acoustic interaction. The computational
approach based on the formulation of the boundary element computation of the acoustic load following the governing Helmholtz equation for acoustic disturbance propagation on the structure allows the incorporation of the direct or hydrodynamic part of the acoustic influence on the aeroelastic stability as a specific problem investigated. Various components of the method have been validated and the method produced results in qualitative agreement with well established analytical and experimental results of Holström [12], Marquez, Meddahi and Selgas [13], and Huang [14].

To address the problem associated with fluid structure interaction, in particular the vibration of structures due to sound waves, aerodynamics and their combined effects, a generic approach to the solution of the acousto-aerelastic-dynamic interaction has been followed. The development of the foundation for the computational scheme for the calculation of the influence of the acoustic disturbance to the aeroelastic stability of the structure [8-11, 15], starts from a rather simple and instructive to a more elaborate FE-BE fluid-structure interaction model. The logical sequence of the formulation of the problem to treat the aeroacoustic effects on aeroelastic structure is summarized in Fig. 1. It is of interest at this point to recall the concluding remarks made by Farassat [16] that linear unsteady aerodynamics should be viewed as part of the problem of aeroacoustics.

There are still many problems in aeroacoustics as well as in unsteady aerodynamics to be solved by combined analytic and numerical methods. The efforts devoted here is only a small attempt in that direction. The paper is organized as follows: Section 2 deals with comprehensive derivation of the governing acoustic effects on aeroelastic problem; section 3 comprehensively elaborates the author’s unified linearized unsteady aerodynamics and acoustics differential equations approach from the present state of the art; followed by section 3, the boundary element formulation of the solution of the linearized unsteady aerodynamics equations is introduced in section 4; the unified treatment of aerodynamic-acoustic forcing in the acousto-aerelastic boundary element formulation is elaborated in section 5; another unified approach in grouping the acoustic disturbance with the unsteady aerodynamic loading is treated in section 6, elaborating the author’s acoustic-aerodynamic analogy approach; generic examples as proof of concept are dealt with in section 7, case study and numerical results; section 8 summarizes the supporting findings that have been gained thus far.

2. Governing Equation for Acousto-Aerelastic Problem

Referring to the development already carried out in earlier works [2, 8-11], the governing equations for the acousto-aerelasticity problem for an elastic wing submerged in a fluid and subject to acoustic pressure disturbance can be represented by the structural dynamic equation of motion with a general form of

\[
\left[ [K] - \omega^2 [M] + [A_{\text{aero}}] + [P_{\text{aero}}] \right] \{x\} = \{F_{\text{ext}}\} \quad (1)
\]

where, with appropriate considerations on their actual form:
• K—stiffness related terms;
• M—inertial mass related term;
• $F_{\text{Aero}}$, $A_{\text{Aero}}$—aerodynamic related term;
• $F_{\text{Acou}}$, $P_{\text{Acou}}$—acoustic related term;
• F—external forces other than those generated aerodynamically or acoustically.

The acoustic pressure is governed by Helmholtz equation, in discretized form can be written as

$$ \mathbf{H}\{P_{\text{Acou}}\} = i\rho_0\omega\mathbf{G}\{v\} + \{P_{\text{exc}}\} \tag{2} $$

where, with appropriate considerations on their actual form:

• $\mathbf{H}$—acoustic pressure influence coefficient;
• $\mathbf{G}$—acoustic velocity influence coefficient;
• $v$—acoustic velocity;
• $P_{\text{Acou}}$—acoustic pressure.

The acoustic domain is schematically depicted in Fig. 2.

The unified treatment which was the focus of the earlier works capitalizes on the followings:

1. The set of Eqs. (1) and (2) represents the unified state of affairs of acousto-aeroelastic problem being considered, which in principle can be solved simultaneously. However, prevailing situations may allow the solution of the two equations independently;
2. The acoustic term incorporated in Eq. (1), as can be seen in subsequent development, is considered to consist of three components to be elaborated below, and are also treated in unified fashion;
3. Analogous to the aerodynamic terms, the total acoustic pressure in the acoustic term incorporated in Eq. (1), will be separated into elastic structural motion dependent and independent parts, which is referred to here as acoustic-aerodynamic analogy. Such approach allows the incorporation of the structural motion dependent part to the aerodynamic term in Eq. (1).

In the work of Huang [9], by introducing acoustic pressure field using vibrating membrane of loud-speaker, his theoretical prediction of the direct influence of the acoustic pressure field to the aerodynamically induced structural vibration has been demonstrated to agree with his experiments. On the other hand, Lu and Huang [17] and Nagai et al. [18] have introduced the influence of the acoustic pressure field to the aerodynamic flowfield, recognizing the influence of of trailing edge receptivity not accounted for in the earlier works.

With such perspective in mind, the unsteady aerodynamic loading can be considered to consist of the following three components:

1. The unsteady aerodynamic load component induced by the elastic structural motion in the absence of acoustic excitation;
2. The unsteady aerodynamic load due to structural vibration which is induced by the ensuing acoustic pressure loading ($P_{\text{exc}}$);
3. The unsteady aerodynamic flow-field induced by acoustic pressure disturbance.

The combined acoustic and aerodynamic loading on the structure is then synthesized by linear superposition principle of small oscillation for the acoustic pressure disturbance to the aeroelastic problem, the latter due to aerodynamic-structure interaction. To facilitate the solution of the acousto-aeroelastic stability equation solved by V-g method, a novel approach is undertaken. Analogous to the treatment of dynamic aeroelastic stability problem of structure, in which the aerodynamic effects can be distinguished into motion independent and motion induced aerodynamic forces, the effect of acoustic pressure disturbance to the aeroelastic structure (acousto-aeroelastic problem) is considered to
consist of structural motion independent incident acoustic pressure and structural motion dependent acoustic pressure, which is the scattering pressure. This is referred to in this work as the acoustic aerodynamic analogy. The governing equation for the acousto-aeroelastic problem is then formulated incorporating the total acoustic pressure (incident plus scattering pressure), and the acoustic aerodynamic analogy. A generic approach to solve the governing equation as stability or dynamic response equation is formulated allowing a unified treatment of the problem.

3. Linearized Unsteady Aerodynamics and Acoustics Differential Equations

The present work dwells further into the unified formulation of the unsteady aerodynamics and the acoustic terms.

The unsteady velocity potential \( \Phi \) satisfies the gas-dynamic equation [11]:

\[
\nabla^2 \Phi - \frac{1}{a^2} \left[ \frac{\partial^2 \Phi}{\partial t^2} + \frac{\partial}{\partial t} \left( \nabla \Phi \right)^2 \right] = 0
\]

(3)

For flows having free-stream velocity \( U_\infty \) in the direction of the positive x-axis, and assuming \( \phi \) to be the perturbation velocity potential which satisfies

\[
\Phi = \phi + U_\infty x
\]

(4)

\[
u = \frac{\partial \phi}{\partial x} \quad \nu \ll U_\infty
\]

(5)

Then one obtains (Ashley and Landahl [19], Morino and Kuo [20])

\[
\nabla^2 \phi - \left( \frac{1}{a_\infty} \right)^2 \left[ \frac{\partial}{\partial t} + U_\infty \frac{\partial}{\partial x} \right]^2 \phi = F \approx 0
\]

(6)

where \( F \) is the contribution of non-linear terms, which for practical purposes consistent with perturbation linearization assumption, can be neglected. Furthermore, it will be convenient to consider harmonic motion as an element of any unsteady motion which can be built up using the principle of superposition. Then one may assume

\[
\phi(x,t) = \bar{\phi}(x,t) e^{i\omega t}
\]

(7)

Substitution into Eq. (6) gives

\[
\frac{\partial^2 \bar{\phi}}{\partial t^2} - \frac{1}{a^2} \left[ i\omega + U_\infty \frac{\partial}{\partial x} \right]^2 \bar{\phi} = 0
\]

(8)

Defining \( M \equiv \frac{U_\infty}{a} \), Eq. (8) can be recast into

\[
(1-M^2) \frac{\partial^2 \bar{\phi}}{\partial t^2} + \frac{\partial^2 \bar{\phi}}{\partial x^2} - 2i\omega M \frac{\partial \bar{\phi}}{\partial x} - \omega^2 \bar{\phi} = 0
\]

(9)

Define a new variable \( \Phi^* \) following Prandtl-Glauert Transformation as

\[
\bar{\phi} = \Phi^* e^{i\omega t}
\]

(10)

and substitute into Eq. (9), then one obtains

\[
\left\{ (1-M^2) \left[ \frac{\partial^2 \Phi^*}{\partial x^2} + \frac{\partial^2 \Phi^*}{\partial z^2} \right] + \Phi^* \left[ \frac{M^2}{(1-M^2)} \frac{\omega^2}{U_\infty^2} \right] \right\} \times \exp \left[ i \frac{M^2}{(1-M^2)} \frac{\omega}{U_\infty} x \right] = 0
\]

(11)

If one defines

\[
K \equiv \frac{M^2}{(1-M^2)} \frac{\omega^2}{U_\infty^2}
\]

(12)

then the linearized unsteady aerodynamic Eq. (11) can be recast as

\[
\nabla^2 \Phi^* + K^2 \Phi^* = 0 \quad \text{or} \quad \left\{ \nabla^2 + K^2 \right\} \Phi^* = 0
\]

(13)

which has the wave-equation form similar to the Helmholtz equation for the acoustic pressure disturbance [1-2, 8 -11]:

\[
\nabla^2 + k^2 \rho = 0
\]

(14)

4. Boundary Element Formulation of the Solution of the Linearized Unsteady Aerodynamics Equations

To solve for the differential Eq. (13), boundary integral formulation can be obtained by the application of the Green’s theorem, similar to the approach taken for the solution of the acoustic pressure Eq. (14).

Applying the Green’s Theorem to Eq. (13) [9, 12, 21], then one obtains

\[
\int_{\partial S} \nabla \Phi^* \cdot \nabla \Phi^* \ ds = \int_{\partial \bar{V}} \Phi^* \frac{\partial \Phi^*}{\partial n} \ ds
\]

(15)
in which $\Phi^*_1$ is the transformed potential of interest and $\Phi^*_2$ the corresponding free-space Green’s function, i.e.,

$$\{\nabla^2 + K^2\} \Phi^*_1 = 0$$  \hspace{0.5cm} (16)

and

$$\{\nabla^2 + K^2\} \Phi^*_2 = \delta(R - R_i)$$  \hspace{0.5cm} (17)

Then it follows that [6, 15-16]

$$\Phi^*_1 = \frac{1}{4\pi} \int \Phi^*_1 \left( \frac{\partial}{\partial n} \left( \rho \frac{e^{-ik|R-R_i|}}{|R-R_i|} \right) \right) dS$$  \hspace{0.5cm} (18)

Here the free-space Green’s function with the general form

$$G(|R-R_i|) = \frac{e^{-ik|R-R_i|}}{4\pi|R-R_i|}$$  \hspace{0.5cm} (19)

is used, where refers to the $R$ coordinate of the field point of interest and $R_i$ the coordinate of the source element [8-12], and $\mathbf{U}$ (or $n$, as appropriate) is the unit outward normal vector of a surface element on $S$.

The boundary condition of no-flow through the surface of the body (and wing) defines the normal velocity or downwash $w$ at the surface, and is given by

$$\frac{\partial \phi}{\partial n} = \frac{\partial \phi}{\partial z} = w$$  \hspace{0.5cm} (25)

Substituting Eqs. (24) and (25) into Eq. (21) (with appropriate algebraic manipulation) gives

$$w = -\frac{1}{8\pi} \int \frac{\Delta \overline{\mathbf{P}}(x_i, y_i)}{\rho U_{\infty}^2} K(x-x_i, y-y_i) dS$$  \hspace{0.5cm} (26)

where

$$K(x, y) = \lim_{z \to 0} \left[ -\frac{i\omega}{U_{\infty}} x \right] \exp \left[ \frac{1}{1-M^2} \frac{1}{U_{\infty}} \frac{\partial}{\partial z} \left( \frac{e^{-ikx}}{r} \right) d\xi \right]$$  \hspace{0.5cm} (27)

which is the well known standard Kernel function representation in lifting surface or panel method, and leads to further utilization of Doublet Lattice Method (DLM) or Doublet Point Method (DPM) [24] in the unsteady aerodynamic problem as solution of Eq. (13) with appropriate boundary conditions. The detail of the formulation is elaborated in Refs. [25-26].

5. Unified Treatment of Aerodynamic-Acoustic Forcing in the Acousto-Aeroelastic Boundary Element Formulation

The treatment of the aerodynamic problem elaborated above motivates further elaboration of the unified approach of the aerodynamic and acoustic part of the forcing function on the elastic structure, which leads to another form of the governing Eq. (1) as

$$\left[ \mathbf{K}^* \right] \mathbf{q} = \left\{ \mathbf{F} \right\}_{\text{inc}}$$  \hspace{0.5cm} (28)

where

$$\Delta \mathbf{P} = \Delta \mathbf{P} e^{i\alpha}$$  \hspace{0.5cm} (23)

Using the method of variation of parameter [23], the solution of the Bernoulli Eq. (23) can be obtained as

$$\bar{\phi} = \int_{-\infty}^{x} \frac{\Delta \mathbf{P}}{\rho U_{\infty} e^{i\omega (x-x)}} d\lambda$$  \hspace{0.5cm} (24)

The boundary condition of no-flow through the surface of the body (and wing) defines the normal velocity or downwash $w$ at the surface, and is given by

$$\frac{\partial \phi}{\partial n} = \frac{\partial \phi}{\partial z} = w$$  \hspace{0.5cm} (25)

Substituting Eqs. (24) and (25) into Eq. (21) (with appropriate algebraic manipulation) gives

$$w = -\frac{1}{8\pi} \int \frac{\Delta \overline{\mathbf{P}}(x_i, y_i)}{\rho U_{\infty}^2} K(x-x_i, y-y_i) dS$$  \hspace{0.5cm} (26)

where

$$K(x, y) = \lim_{z \to 0} \left[ -\frac{i\omega}{U_{\infty}} x \right] \exp \left[ \frac{1}{1-M^2} \frac{1}{U_{\infty}} \frac{\partial}{\partial z} \left( \frac{e^{-ikx}}{r} \right) d\xi \right]$$  \hspace{0.5cm} (27)

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and
\[ \rho_o \omega^2 \left[ G_{ii} \right] \left[ \mathbf{T} \right] \left[ \Xi \right] \{ q \} + \left[ \mathbf{H}_{ii} \right] \{ p_s \} + \left[ \mathbf{H}_{ii} \right] \{ p_s \} = i \rho_o \omega \left[ G_{ii} \right] \left\{ v_s \right\} + \left\{ p_{m_i} \right\} \] (29)
\[ \rho_o \omega^2 \left[ G_{21} \right] \left[ \mathbf{T} \right] \left[ \Xi \right] \{ q \} + \left[ \mathbf{H}_{21} \right] \{ p_s \} + \left[ \mathbf{H}_{22} \right] \{ p_s \} = i \rho_o \omega \left[ G_{22} \right] \left\{ v_s \right\} + \left\{ p_{m_i} \right\} \] (30)

where
\[ \left[ K^* \right] = \Xi^T \left[ K \right] \Xi \] (31)
\[ \left[ M^* \right] = \Xi^T \left[ M \right] \Xi \] (32)
\[ \left[ F_{w_i}(ik) \right] = \Xi^T \left[ L \right] \left[ P_{w_i}(ik) \right] \Xi \] (33)
\[ \left[ A(i\kappa) \right] = \Xi^T \left[ G_{w_i} \right] \left[ A(i\kappa) \right] \left[ G_{w_i} \right] \Xi \] (34)

where \( \Xi \) is the Eigen-modes obtained from modal analysis of the purely elastic structure governed by
\[ \left[ K - \omega^2 \mathbf{M} \right] \{ x \} = \{ 0 \} \] (35)
and where \( \mathbf{L} \) is a coupling matrix of size \( \mathbf{M} \times \mathbf{N} \), where \( \mathbf{M} \) is the number of FE degrees of freedom and \( \mathbf{N} \) is the number of BE nodes on the coupled boundary \( a \), depicted in Fig. 4. For each coinciding boundary and finite element, the acoustic pressure elemental coupling term is given by Djojodihardjo and Safari [2, 8], Holström [12] and Marquez et al. [13].
\[ \mathbf{L}_w = \int_{S_a} \mathbf{N}_w^T \mathbf{n} \mathbf{N}_w dS \] (36)
where \( \mathbf{N}_w \) and \( \mathbf{N}_\theta \) are the shape function of the coinciding Finite Element and Boundary Element, respectively, and \( \mathbf{n} \) is the normal vector to the corresponding coinciding surface. Here \( \{ x \} \) is the structural deformation and transformed to the generalized coordinates \( \{ q \} \) using the following relationship:
\[ x = \Xi q \] (37)
where \( \Xi \) is the modal matrix as solution to the Eigen-value problem corresponding to the free vibration of the structure.

The schematic of BE-FE fluid-structure interaction domain represented as a quarter space with defined boundary conditions on the boundaries is exhibited in Fig. 3 (with the computational philosophy for the solution of acousto-aeroelastic problem elaborated in Refs. [2, 8-11]).

By appropriate mathematical derivation, it was shown by Djojodihardjo [9, 11] that
\[ \{ p_s \} = -\rho_o \omega^2 \left[ P_{w_i}(ik) \right] \{ x \} \] (38)

With such formulation, a similar procedure as elaborated in Refs. [9-11] is then followed.

6. Acoustic-Aerodynamic Analogy Formulation

Analogous to the treatment of dynamic aeroelastic stability problem of structure, in which the aerodynamic effects can be distinguished into motion independent (self-excited) and motion induced aerodynamic forces, the effect of acoustic pressure disturbance to the aeroelastic structure (acousto-aeroelastic problem) can be viewed to consist of structural motion independent incident acoustic pressure (excitation acoustic pressure) and structural motion dependent acoustic pressure, which is known as the scattering pressure. However the scattering acoustic pressure is also dependent on the incident acoustic pressure.

Solution of Eq. (28) will be facilitated by the use of modal approach, i.e., transforming \( \{ x \} \) into the generalized coordinate \( \{ q \} \) following the relationship \( x = \Phi q \), where \( \Phi \) is the modal matrix. In the examples worked out, a selected lower order natural modes will be employed for \( \Phi \). Pre-multiplying Eq. (28) by \( \Phi^T \) and converting dynamic pressure \( q_w \) into reduced frequency \( (\kappa) \) as elaborated in Refs. [2, 8-11], Eq. (28) can then be written as

![Fig. 3 A schematic of BE-FE fluid-structure interaction domain represented as a quarter space with defined boundary conditions on the boundaries.](image)
\[
\Phi'^T \left[ \Phi' - i\alpha^2 \left( M + \rho \frac{L}{2} \left( \frac{1}{k} \right)^2 \left[ A(ik) \right] + \rho_c \left[ F_{sc} \left( k_w \right) \right] \right) \right] \nabla x \Phi'^T \Phi' = \Phi'^T \Phi' + \Phi'^T F
\]

(39)

Since all of the acoustic terms are functions of wave number \( k_w \), Eq. (39) will be solved by utilizing iterative procedure. Incorporation of the scattering acoustic component along with the aerodynamic component in the second term of Eq. (39) can be regarded as one manifestation of what is referred to here as the acoustic-aerodynamic analogy followed in this approach.

7. Case Study and Numerical Results

Generic case of a fixed flat plate with a pulsating monopole acoustic source located at certain point on its upper surface has been considered for benchmarking in earlier works [2, 8-11]. The present work will elaborate the application of the in-house Doublet Point Method code written in MATLAB\textsuperscript{\textregistered}, which has been rederived using boundary integral formulation in sections 3 and 4. Further formulation of the acoustic-aerodynamic analogy as implied by Eq. (39) will account for the change in the flow boundary conditions due to the presence of the acoustic pulsating monopole source as was carried out in preceding work [6]. The use of modified BAH wing\textsuperscript{2} [2-4] as carried out previously will be utilized for benchmarking for the three dimensional case. Results from this approach, along with the previous results for the direct acoustic effect on the acoustic influence on aeroelastic stability [4-6], will be discussed. At this point, a remark is in order. The work concerned here focuses on proof of concept, and in many examples, the agreements are qualitative or better. No particular efforts are devoted to uncertainties and high accuracies.

For this purpose, the in-house MATLAB program was developed following the Doublet Point (DPM) Scheme of Ueda and Dowell [27] for incorporation into the combined acousto-aeroelastic solution of flutter stability problem of concern. The validation of the in-house DPM program by the benchmarking application of the in-house DPM code the equivalent BAH wing considered in Refs. [2, 8-11] is elaborated in Refs. [25-26], and presented here. The Equivalent BAH wing discretization for unsteady aerodynamic load calculation is exhibited in Fig. 4. For the purpose of this study, the equivalent BAH wing planform was discretized into 40 aerodynamic boxes with 10 elements span-wise and 4 elements chord-wise, as depicted in Fig. 4. Fig. 4 is utilized to depict the computational grid utilized to solve the unsteady aerodynamic load problem.

To validate the method, calculations were carried out for several values of \( k \) (\( k = 0, 0.3, 0.6, 0.8 \)). Fig. 5a shows \( C_p \) distribution at station 1 for those \( k \) values. These results were then compared to typical \( C_p \) distribution from Zwaan [28] which is shown in Fig. 5b. Fig. 5a exhibits \( C_p \) distribution results obtained using the present method, for various values of \( k \); real part is shown on the left hand side while the imaginary part on the right hand side.

Fig. 5b exhibits the \( C_p \) distribution for various values of \( k \) reproduced and retouched from Zwaan [28] (real part-left, imaginary part-right). These figures indicate qualitative agreement, which serves to give confidence to the method at the present stage of development, which is also exemplified by a coarse numerical grid.

The calculation of unsteady aerodynamics on equivalent BAH wing was carried out for reduced frequency of \( k = 0.01 \), by taking heaving motion amplitude \( h = 1 \) and pitching motion amplitude \( \alpha = 1.7 \) rad. The results are presented in Fig. 6 as the distribution of pressure coefficient \( (C_p) \) over the wing surface and Fig. 7 as the chordwise distribution of \( C_p \) at station 1, 5 and 10. Fig. 6a shows the real part of \( C_p \) distribution, while Fig. 6b, the imaginary part of \( C_p \) distribution. Fig. 7a shows the real part while Fig. 7b the imaginary part of the pressure distribution on modified BAH wing \((k = 0.8)\).
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Fig. 4 Equivalent BAH wing discretization for unsteady aerodynamic load calculation.

Fig. 5 (a) Present work, Cp distribution with k variation (real part-left, imaginary part-right); (b) Cp distribution with k variation (real part, left, imaginary part, right) reproduced from Zwaan [28] (validation based on k).

0.01, M = 0.0, 1st bending). The computational result using the new in-house DPM program was also compared to earlier results (Djojodihardjo and Safari [2, 8]).

The in house DPM program was also applied to typical CN-235-like wing, and the results are shown in Fig. 8 with planform exhibited in Fig. 9, which exhibits the typical representation of CN-235-like Wing being studied.

The CN-235-like wing planform is utilized for benchmarking purposes, exhibiting sections 1, 5 and 10 of interest. Fig. 10 compares the Cp distribution results of the in-house DPM code applied to typical representation of CN-235 wing with that previously

Fig. 6 (a) Real part of Cp distribution; (b) Imaginary part of Cp distribution.

Fig. 7 Real (a) and imaginary (b) parts of pressure distribution on modified BAH wing (k = 0.01, M = 0.0, 1st bending) (associated with earlier work Ref. [3]).
Fig. 8  Cp distribution on sections 1, 5 and 10: (a) real part; (b) imaginary part.

Fig. 9  Schematic of wing planform typically representing CN-235-like wing utilized for benchmarking.

obtained by Soeherman [29], indicating only the real part. The results show good agreement. The results as exhibited by Figs. 6-10 show that the in-house code, developed through the reformulation of DPM method through rigorous boundary integral formulation allow a plausible unified approach for both unsteady aerodynamics and acoustic problem.

The present elaboration on the unsteady aerodynamics gives further rigorous foundation to the unified approach attempted in the present series of efforts.

The computation and integration scheme for solving the acousto-aeroelastic problem follows the procedure depicted in Fig. 1.

The solution of Eq. (39) as a stability equation in a “unified treatment” already incorporates the total acoustic pressure, which has been “tuned” to behave like the aerodynamic terms in the modal approach.

Fig. 11 exhibits the application of the method using the reformulation of the aerodynamic problem for the flutter stability problem of modified BAH Wing, exhibiting (a) the damping and (b) the frequency diagram for BAH wing calculated using V-g method written in MATLAB® for the acousto-aeroelastic problem.

The V-g diagrams are drawn for pure aeroelastic as well as for acousto-aeroelastic cases. Fig. 11 illustrates the influence of the acoustic disturbance on the aeroelastic behavior of the structure considered.

8. Conclusions

The objective of the present work is to establish a unified approach for the acoustic and aerodynamic component of the acousto-aeroelastic problem using boundary integral approach, which will facilitate the unified treatment of the acousto-aeroelastic governing
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Eq. (35) carried out by reformulating the unsteady aerodynamic load calculation.

To this end, the governing unsteady gas dynamics equation for the perturbation velocity potential recast into a form similar to the Helmholtz wave equation for the acoustic pressure disturbance and the assumption of harmonic motion as an element of the unsteady motion allow the mean disturbance velocity potential be written in a boundary integral Eq. (20). Such formulation allows unified and combined treatment of the unsteady aerodynamic and acoustic terms in the combined Finite Element-Boundary Element expression of the acousto-aeroelastic governing Eq. (35) as well as facilitating the aerodynamic terms to be further transformed into the well established lifting surface type (kernel-function) expression relating the downwash on the surface of the structure to the pressure distribution there, as given by Eq. (26).

An in-house Doublet Point Method following the classical one has been developed and has been validated.

Accordingly, the computational scheme for the calculation of the influence of the acoustic disturbance to the aeroelastic stability of a structure follows the scheme that has been developed using a unified treatment and acoustic-aerodynamic analogy. By considering the effect of acoustic pressure disturbance to the aeroelastic structure (acousto-aeroelasto-dynamic problem) to consist of structural motion independent incident acoustic pressure and structural motion dependent acoustic pressure, the scattering acoustic pressure can be grouped together in the aerodynamic term of the aeroelastic equation. By tuning the incident acoustic pressure, it can also be incorporated along with the scattering acoustic term, forming the acousto-aeroelastic stability equation. For this purpose the problem domain has been defined to consist of those subjected to acoustic pressure only and that subject to acoustic structural coupling, which is treated as acousto-aeroelastic equation. Using BE and FE as
appropriate, an integrated formulation is then obtained as given by governing Eq. (39), which relates all the combined forces acting on the structure to the displacement vector of the structure. The solution of Eq. (35) and after using modal approach in structural dynamics, Eq. (39) can be obtained by solving it as a stability equation in a “unified treatment”. The disturbance acoustic pressure already incorporates the total pressure (incident plus scattering pressure), which has been “tuned” to behave like the aerodynamic terms in the modal Eq. (39). Such approach allows the application of the solution of the acousto-aeroelastic stability equation in the frequency domain using V-g method.

Acknowledgments

The work reported here has been carried out within research efforts initiated in 2004. Recent results reported was carried out under Universiti Sains Malaysia Fundamental Research Grant Scheme (FRGS) “Development of BEM-FEM Fluid-Structure Coupling Fluid-Structure Coupling for Basic Applications” granted in October 2007, the Universiti Putra Malaysia Research University Grant Scheme (RUGS) initiated in April 2010 and the current Universiti Putra Malaysia/Ministry of Higher Education Exploratory Research Grant Scheme (ERGS) Project Code: ERGS 5527088. The author would like to thank Universitas Al-Azhar Indonesia for the time granted to the author to carry out the work at Universiti Sains Malaysia and Universiti Putra Malaysia. Part of the work was carried out by Mr. Mahesa Akbar in his jointly supervised thesis work at Institut Teknologi Bandung.

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