Stability of Motion "In the Large" of Some Class of Phase Systems

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Abstract: This article focuses on the study of stability of motion of the phase systems described by differential equations whose right-hand sides are periodic in the angular coordinate. The article deals with the mathematical model which has been investigated for stability "in the large" using the second Lyapunov method. Based on the theoretical results obtained in the work, the computational experiments on concrete examples of electric power systems, which showed the sufficient efficacy of the proposed method for the studied phase system, were conducted.

Keywords: Stability of motion "in the large", Lyapunov function, phase system, electric power system, mathematical model.

1. Introduction

The mathematical model of the modern electric power complex, which consists of turbogenerators and complex multiply power units, is a system of nonlinear ordinary differential equations. The optimization problem of the functioning of these complexes, as well as the creation of algorithms for motion stability for such systems still attract the attention of many researchers and are relevant.

The industrial development of the modern society leads to the constant increase in electricity consumption. To meet these growing needs, complex electrical power systems are created. In mathematical modeling of such complexes it is required to solve a number of theoretical and practical issues. Ensuring the sustainability of movement is the most important problem at the phase of design and operation of the studied systems.

It should be noted that the study of global asymptotic stability of motion of phase systems is discussed in [1].

2. Statement of the Problem

Let's consider the system:

$$\frac{d\delta_i}{dt} = S_i, \quad \frac{dS_i}{dt} = w_i - K_i S_i - f_i(\delta_i),$$

$$w_i = C_i^* x_i, \quad (1)$$

$$\frac{dx_i}{dt} = A_i x_i + q_i S_i + b_i u_i + R_i(S_i, x_i) \quad (2)$$

where δ_i - the angular coordinate; S_i - angular velocity; x_i - n_i - the state vector of the regulator; w_i - the control action of the regulator; $K_i > 0$ - damping factor; c_i, q_i, b_i - constant n_i - dimensional vectors; A_i - constant $(n_i \times n_i)$ matrix; u_i - control of the feedback type. The symbol (*) denotes transposition. Second order differential equations (1) describe the processes in the object of control, and the vector differential equation (2) determines the state of the regulator of the i -th isolated subsystem, in which the phase trajectories tend to the particular stable equilibrium position.

The resistance of movement of phase systems of modern models was studied in [2].

The investigation of stability "in the large" of the system (1), (2) ($K_i = D_i, R_i = 0$) will be

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conducted in the band $G_{0i} = \{(\delta_i, S_i, x_i) | \delta_{-1i} < \delta_i < \delta_{0i}, S_i \in R_i^1, x_i \in R_i^{n_i}\}$ using the second Lyapunov method [3].

It should be noted that the work of Wyman M.J. [4] deals with the Lyapunov function, which has the form:

$$v = \sum_{i=1}^n \frac{H_i y_i^2}{2} - \sum_{i=1}^n \int_{x_{i0}}^{x_i} (P_{Ti} - P_i \sin x) dx + \sum_{j=2}^n \sum_{i=1}^{j-1} \int_{x_{ij0}}^{x_{ij}} P_{ij} \sin x dx$$

We examined the Lyapunov function of the "kinetic energy plus potential energy" type in the form of:

$$\tilde{v}_{0i}(\delta_i, S_i, x_i) = \frac{1}{2} x_i^* H_i x_i + \frac{\varepsilon_{1i}}{2} S_i^2 + v_{0i}(\delta_i, S_i), \quad (3)$$

which is defined in the space $R_i^{(n_i+2)}(\delta_i, S_i, x_i)$ in the band G_{0i} . Where the function:

$$v_{0i}(\delta_i, S_i) = \frac{1}{2} (S_i + \alpha_i D_i \delta_i)^2 + \frac{1}{2} \alpha_i D_i^2 (1 - \alpha_i) \cdot \delta_i^2 + F_i(\delta_i) + 2D_i \sqrt{\alpha_i(1 - \alpha_i)} \tilde{F}_i(\delta_i) = \frac{1}{2} (S_i + \alpha_i D_i \delta_i)^2 +$$

$$\int_0^{\delta_i} N_i(\delta_i) d\delta_i,$$

We are placing for consideration the symmetric $(n_i \times n_i)$ -matrix H_i , n_i -dimensional vector a_i , scalars $\alpha_{1i}, \alpha_{2i}, \alpha_{3i}, \varepsilon_{1i}$ will be denoted

$$\tilde{A}_i = A_i + b_i a_i^*, \tilde{q}_i = q_i + \alpha_{1i} b_i, l_{1i} = \alpha_{2i} b_i, l_{2i} = \alpha_{3i} b_i,$$

$$\xi_i = \text{colon}(S_i, \delta_i, f_i(\delta_i)), B_i = \{\tilde{q}_i, l_{1i}, l_{2i}\},$$

$$\Gamma_i = \begin{pmatrix} \varepsilon_{1i} D_i & 0 & \frac{\varepsilon_{1i}}{2} \\ 0 & 0 & 0 \\ \frac{\varepsilon_{1i}}{2} & 0 & 0 \end{pmatrix}, g_i = \begin{pmatrix} (1 + \varepsilon_{1i}) C_i^* \\ \alpha_i D_i C_i^* \\ O_{ni}^* \end{pmatrix},$$

$$Y_i(x_i, \xi_i) = -\text{Re} \xi_i^* g_i x_i + \xi_i^* \Gamma_i \xi_i.$$

$$\Pi_i(j\omega) = \text{Reg}_i(\tilde{A}_i - j\omega E_{n_i})^{-1} B_i + \Gamma_i,$$

where g_i - matrix $3 \times n_i$, B_i -matrix of order $n_i \times 3$, Γ_i - matrix of order 3×3 , $\Pi_i(j\omega)$ - Hermitian (3×3) matrix, Y_i - quadratic form of vectors x_i, ξ_i .

Using this notations, we'll formulate and prove the

following theorem.

Theorem. Let there exist a vector a_i , scalars $\alpha_{1i}, \alpha_{2i}, \alpha_{3i}, \varepsilon_{1i} > 0, \alpha_i \in (0, 1), D_i > 0$ such that

1. matrix $\tilde{A}_i = A_i + b_i a_i^*$ Hurwitz,
2. pair $(\tilde{A}_i, B_i)$ is completely controllable, $B_i = \{\tilde{q}_i, l_{1i}, l_{2i}\}$, where

$$\tilde{q}_i = q_i + \alpha_{1i} b_i, l_{1i} = \alpha_{2i} b_i, l_{2i} = \alpha_{3i} b_i,$$

3. pair $(\tilde{A}_i, g_i)$ is completely observable, where

$$g_i = \begin{pmatrix} (1 + \varepsilon_{1i}) C_i^* \\ \alpha_i D_i C_i^* \\ O_{ni}^* \end{pmatrix}$$

4. $\Pi_i(j\omega) > 0 \quad (\forall \omega \in (-\infty, +\infty))$, where $\Pi_i(j\omega) = \text{Reg}_i(\tilde{A}_i - j\omega E_{n_i})^{-1} B_i + \Gamma_i$

5. $f_i'(0) \neq \alpha_i D_i^2 (1 - \alpha_i)$

Then, when controlling $u_i = a_i^* x_i + \alpha_{1i} S_i + \alpha_{2i} \delta_i + \alpha_{3i} f_i(\delta_i)$, the domain of attraction of the coordinate origin in the band G_{0i} , which can be estimated by using the Lyapunov function (3), is given by the inequality of the form $\tilde{v}_{0i}(\delta_i, S_i, x_i) < \tilde{v}_{0i}$, where $\tilde{v}_{0i}$ critical value is determined by the following ratio.

$$\tilde{v}_{0i} = \min\{\tilde{\rho}_{0i}, \tilde{\rho}_{-1i}\}, \tilde{\rho}_{0i} = \tilde{v}_{0i} \Big|_{\substack{\delta_i = \delta_{0i} \\ S_i = 0, x_i = 0}},$$

$$\tilde{\rho}_{-1i} = \tilde{v}_{0i} \Big|_{\substack{\delta_i = \delta_{-1i} \\ S_i = 0, x_i = 0}} \quad (4)$$

Proof. Computing in the band G_{0i} the time derivative of the function $\tilde{v}_{0i}(\delta_i, S_i, x_i)$ (3) along the trajectory of system (1), (2) ($K_i = D_i, R_i = 0$), we obtain

$$\dot{\tilde{v}}_{0i}(\delta_i, S_i, x_i) = x_i^* H_i (\tilde{A}_i x_i + \tilde{q}_i S_i + l_{1i} \delta_i + l_{2i} f_i(\delta_i)) - \varepsilon_{1i} D_i S_i^2 - \varepsilon_{1i} S_i f_i(\delta_i) + (1 + \varepsilon_{1i}) S_i C_i^* x_i + \alpha_i D_i C_i^* x_i + \dot{v}_{0i}(\delta_i, S_i) \Big|_{(1.4)} =$$

$$S_i(x_i, \xi_i) + \dot{v}_{0i}(\delta_i, S_i) \Big|_{(1.4)},$$

where

$$S_i(x_i, \xi_i) = x_i^* H_i (\tilde{A}_i x_i + B_i \xi_i) - Y_i(x_i, \xi_i),$$

$$\dot{v}_{0i}(\delta_i, S_i) \Big|_{(1.4)} =$$

$$-D_i [\sqrt{\alpha_i(1 - \alpha_i)} S_i \sqrt{\alpha_i \delta_i f_i(\delta_i)}]^2 \leq 0.$$

According to the frequency theorem for the degenerate case $S_i(x_i, \xi_i) \leq 0$ ($\forall x_i \in R_i^{n_i}, \forall \xi_i \in R_i^3$) if and only if the pair is completely controllable and

$$Y_i \left[(j\omega E_{n_i} - \tilde{A}_i)^{-1} B_i \xi_i, \xi_i \right] \geq 0 (\forall \omega \in R_i^1).$$

The conditions 1) -4) imply the existence of the positive definite $(n_i \times n_i)$ -matrix H_i , satisfying the ratio $S_i(x_i, \xi_i) \leq 0$ ($\forall x_i \in R_i^{n_i}, \forall \xi_i \in R_i^3$). The matrix H_i and $(n_i \times 3)h_i$ matrix can be found as the solution of the following equations

$$\tilde{A}_i^* H_i + H_i \tilde{A}_i + h_i h_i^* = 0,$$

$$H_i b_i + g_i^* = \chi_i h_i, 2\bar{\Gamma}_i = \chi_i^* \chi_i.$$

derived from the identity

$$\text{Re } x_i^* H_i (\tilde{A}_i x_i + B_i \xi_i) - Y_i(x_i, \xi_i) = -|h_i^* \chi_i - \chi_i^* \xi_i|^2.$$

Thus, in the band G_{0i} the function $\tilde{v}_{0i}(\delta_i, S_i, x_i)$ is positive definite $\dot{\tilde{v}}_{0i}(\delta_i, S_i, x_i)$ - negative sign. We'll note that the variety $\dot{\tilde{v}}_{0i}(\delta_i, S_i, x_i) = 0$ does not contain the positive half-trajectories, except the position of stable equilibrium $O(0, 0, 0_{n_i})$. Indeed, from the equation

$$\dot{\tilde{v}}_{0i}(\delta_i, S_i, x_i) = 0 \quad \text{it follows that}$$

$\delta_i = 0, S_i = 0, h_i x_i = 0$, since $\dot{\tilde{v}}_{0i}(\delta_i, S_i) = 0$ only at $\delta_i = 0, S_i = 0$. If we assume that $x_i = \text{const} \neq 0$, then from the system of equations (1) and (2) ($K_i = D_i, R_i = 0$) we obtain the equation $\tilde{A}_i x_i = 0$ det $\tilde{A}_i \neq 0$ because $\tilde{A}_i$ - Hurwitz.

Therefore, it should be $x_i = 0$. Further, according to the theorem on asymptotic stability of N.N. Krasovskii [5], we obtain the asymptotic stability of the zero equilibrium position $O(0, 0, 0_{n_i})$. The domain of attraction is defined by the inequality $\tilde{v}_{0i}(\delta_i, S_i, x_i) < \bar{\tilde{v}}_{0i}$, where the critical value $\bar{\tilde{v}}_{0i}$ will be found from the ratio (4). The theorem is proved.

3. Result

3.1 Numerical Example

We are going to consider the model of the "synchronous generator- steam turbine system," which

is described by equations of the form:

$$\begin{aligned} \frac{d\delta_i}{dt} &= S_i, \\ T_{ji} \frac{dS_i}{dt} &= P_{Ti} - D_i S_i - \\ &\left[\frac{E_i^2}{z_{11i}} \sin \alpha_{11i} + \frac{E_i U}{z_{12i}} \sin(\delta_i - \alpha_{12i}) \right] \\ T_{Pi} \frac{dP_{Ti}}{dt} &= -P_{Ti} + \rho_{0i} P_{0i} - \frac{P_{0i}}{\sigma_{0i}} S_i + u_i \end{aligned} \quad (5)$$

Here δ_i - angle of EMF of the generator; S_i - sliding of the generator; T_{ji} - constant of inertia of the rotating masses; P_{Ti} - power of steam turbine; $D_i > 0$ - coefficient of dumping; E_i - estimated EMF of the generator; U - voltage on the busbars of infinite capacity; z_{11i} - intrinsic resistance of the generator; z_{12i} - mutual resistance between the generator and busbars; α_{11i} - additional angle of intrinsic resistance; α_{12i} - additional angle of mutual resistance, ρ_{0i}, P_{0i} - the set constant values, u_i - the control action of the mechanism of the turbine control.

Suppose that the following values of the coefficients of the equation (5) are set: $T_{ji} = 2400 \cdot 10^4$, $P_{Ti} = 10357.48$, $D_i = 16 \cdot 10^4$, $E_i = 588$, $U = 525$, $z_{11i} = 38$, $z_{12i} = 85$, $\alpha_{11i} = 1.53$, $\alpha_{12i} = 0.33$, $T_{Pi} = 251.2$, $\rho_{0i} = 0.994$, $P_{0i} = 11263$, $\sigma_{0i} = 0.06$. Through the transfer of the origin to the point $(\delta_i, S_i, P_{Ti}) = (0.686, 0, 10357.48)$, the equation (5) can be written as:

$$\begin{aligned} \frac{d\delta_i}{dt} &= S_i, \\ \frac{dS_i}{dt} &= c_i P_{Ti} - D_i S_i - f_i(\delta_i), \\ \frac{dP_{Ti}}{dt} &= A_i P_{Ti} + u_i, \end{aligned} \quad (6)$$

where $D_i = 50.5 \cdot 10^4$, $f_i(\delta_i) = f_{0i} [\sin(\delta_i + \theta_{0i}) - \sin \theta_{0i}]$, $f_{0i} = 1.513 \cdot 10^{-4}$, $\theta_{0i} = 0.3562$, $c_i = 4.167 \cdot 10^{-8}$, $A_i = -39.81 \cdot 10^{-4}$. The function $f_i(\delta_i)$ is continuously differentiable 2π - periodic function with the mean value on the period $\gamma_0^i = -0.5277 \cdot 10^{-4} < 0$. We can also show that the function $f_i(\delta_i)$ vanishes on the set $[0, 2\pi)$ in the points $\delta_i = 0$ and $\delta_i = \delta_{0i} = 2.429$.

Fig. 1

To investigate the stability "in the large" of the system (6) we use the theorem, where the Lyapunov function has the form:

$$\tilde{v}_{0i}(\delta_i, S_i, x_i) = \frac{1}{2} x_i^* H_i x_i + \frac{\varepsilon_{1i}}{2} S_i^2 + v_{0i}(\delta_i, S_i)$$

defined on the phase plane $R_i^3(\delta_i, S_i, P_{Ti})$ in the band

$$G_{0i} = \left\{ \frac{\delta_i, S_i, P_{Ti}}{\delta_{-1i}} < \delta_i < \delta_{0i}, S_i \in R_i^1, P_{Ti} \in R_i^1 \right\} \quad \text{We}$$

find the domain of attraction of the origin in the band G_{0i} at the control of the type: $u_i = \alpha_i^* x_i + \alpha_{1i} S_i + \alpha_{2i} \delta_i + \alpha_{3i} f_i(\delta_i)$. The initial conditions: $\delta_i = 0, S_i = 2.5 \cdot 10^{-2}, P_{Ti} = 0$.

Figure 1 shows that the proposed method allows to estimate accurately the domain of attraction, which is defined by the inequality $\tilde{v}_{0i}(\delta_i, S_i, x_i) < \tilde{v}_{0i}$ where $\tilde{v}_{0i}$ - critical value.

4. Conclusion

Based on the theoretical results obtained in the work, the computational experiments that

demonstrated the sufficient efficacy of the proposed method for the considered phase system were conducted.

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