

Robust Design Method for Adjustable Mechanisms

Takeo Kato and Masatoshi Muramatsu

Department of Mechanical Engineering, Tokai University, Hiratsuka 259-1292, Japan

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Abstract: Most conventional robust design methods assume design solutions are fixed values. Using these methods, designers set each control factor to a fixed value to maximize the robustness of objective characteristics. However, fluctuations in the objective characteristic often exceed the allowable range in a design problem. Consequently, it is difficult to obtain sufficient robustness using conventional methods. This research defines adjustable control factors whose values can be adjusted within a given range to increase robustness and proposes a method to calculate robustness, including factors to adjust the objective characteristic values are within the tolerance by the adjustment, is calculated by the Monte Carlo method, while the range of adjustable control factors is optimized using the Vector evaluated particle swarm optimization. Finally, an engineering example is presented to demonstrate the applicability of the proposed method.

Key words: Robust design, design theory and methodology, seat design, particle swarm optimization.

1. Introduction

Robust design aims to ensure product performance robustness against fluctuant factors, such as user characteristics and material properties, by deriving the optimum (unique) value of the design parameter (design solution). Due to globalized markets and material procurement, robust design has received much attention, and many robust design methods have been proposed [1]. Some methods evaluate robustness of the objective characteristic using an orthogonal array for efficiency [2-5], while others derive robustness using the objective characteristic values calculated via a Taylor series approximation [6-12]. Additionally robustness has been calculated as the feasibility of the objective characteristic being within the tolerance to consider the objective characteristic distribution [13, 14].

In most conventional methods, designers set control factors to fixed values to maximize the robustness. In cases where the objective characteristic distribution is smaller than the tolerance (Fig. 1a), these methods can derive a design solution (optimized control factor values) x_0 with sufficient robustness. However, in cases where the objective characteristic distribution is larger than the tolerance (Fig. 1b), a solution to sufficiently maximize robustness cannot be obtained. In such cases, the control factors must be adjusted to ensure robustness. In other words, as the values of the control factors are varied, the whole of the objective characteristic distribution should be located within the tolerance (Fig. 1c).

The concept of adjusting the factors originates from Taguchi's method [3]. In this method, the control factors are set to minimize the objective characteristic fluctuation, and then the designer selects a tuning factor, which has a negligible effect on the fluctuation to minimize the difference between the nominal value of the objective characteristic and its target value. Otto [15] assumed the tuning factor is adjusted after the fluctuation of objective characteristic. Hence, after the objective characteristic fluctuates, the factors are altered to minimize the difference between the fluctuated objective characteristic values and their target

Corresponding author: Takeo Kato, Ph.D., junior associate professor, research fields: design theory and methodology and robust design. E-mail: t.kato@tokai-u.jp.



Fig. 1 Conceptual illustration of a design problem that includes a factor whose value is adjustable.

values. Otto proposed a method to evaluate the robustness using the expected value of the objective characteristic adjusted by the tuning factors. This method provided a new concept of robustness in which an adjustment improves the robustness and it helps relax the design requirements such as dimension tolerance and material property tolerance. However, Otto's method is not applicable to design problems where the designer chooses the tuning factors and their adjustable ranges because these parameters are preliminarily set in Otto's method. This is a common problem in mechanical design. For example, to design a seat, the designer must determine which adjustable mechanism, such as seat reclining mechanism, and its adjustable range to apply to the seat. Because a method to address such a design problem has yet to be proposed, the designer must determine these parameters using his/her personal design experience.

This research proposes a method to derive the optimum range for the adjustable factors which are chosen adequately to improve the robustness of the objective characteristics. This paper is divided as follows: Section 2 presents definitions and terminologies; the proposed robustness index, calculation method, and range optimization of the factors using the VEPSO (vector evaluated particle swarm optimization) are described in Section 3; Section 4 illustrates an application of the proposed method to a seat design problem; while Section 5

provides conclusions and the future research direction.

2. Definitions and Terminologies

In robust design, objective characteristic (product performance) y fluctuates according to fluctuant factors (control factors x and noise factors z). Although the values of control factors fluctuate, designers can set their nominal values, but not those of noise factors. In this paper, ACFs (adjustable control factors) whose values t can be adjusted in the adjustable range $[t_1, t_u]$ anytime while using or manufacturing the product to maintain the objective characteristic are newly defined. t_1 and t_u are the lower and upper values of ACFs, respectively. ACFs are similar to the tuning factors defined in Otto's method [15] with respect to their adjustment, but the adjustable ranges of ACFs and tuning factors differ (i.e., the designer defines the ranges of the ACFs).

The concept of robustness in this research is defined below. If the tolerance of objective characteristic $[y_1, y_u]$ exists as shown in Fig. 1, then ACFs can be adjusted to locate each fluctuation value of the objective characteristic within the tolerance. Hence, the robustness index for ACFs (R_A) is defined as the feasibility that the objective characteristic values are within the tolerance at least once by the adjustment of ACFs. Using ACFs and R_A , design problems in this research are expressed as shown in Eq. (1):

Find
$$[t_1, t_u]$$
, x
to maximize $R_A(y(=f(x, z, t)))$ (1)
to minimize $|t_u - t_l|$

where, f is the objective function. To prevent an unnecessary expansion of the ACF ranges, which increases the production costs and failure rate, this formulation does not only maximize R_A . Minimizing the size of the range described in Eq. (1) is an example of preventing an unnecessary expansion because other factors (e.g., the form and location of the range) can lead to the aforementioned issues.

Below are definitions and descriptions of the terminologies used in this paper:

• Objective characteristic $(y = f(\mathbf{x}, \mathbf{z}, t))$: The characteristic to express the function of the design objective, and is calculated by objective function f.

• Control factors ($x = \{x_i\}, i = 1, 2, ..., n_x$): Factors whose nominal values are set by the designer, but fluctuate the objective characteristic. n_x represents the total number of control factors.

• Noise factors ($z = \{z_i\}$, $i = 1, 2, ..., n_z$): Factors that fluctuate the objective characteristic, but their nominal values cannot be set by designers. n_z expresses the total number of noise factors.

• ACFs ($t = \{t_i\}, i = 1, 2, ..., n_t$): Control factors with nominal values that can be adjusted within their adjustable ranges. n_t denotes the total number of ACFs.

• Adjustable range of ACFs ($[t_u, t_1]$): The range defined by the designer, where the ACFs are adjustable.

• Assignable points of ACFs ($\{t_j\}, j = 1, 2, ..., n_{ap}$): The combinations of ACFs' values that can be varied to be within the adjustable range. n_{ap} denotes the number of ACFs' assignable points.

• Robustness index (R_A) : Index to evaluate the robustness of the objective characteristics with regards to the ACFs adjustment.

3. Robust Design Method for ACFs

3.1 Robustness Index for ACFs

In conventional robust design methods, robustness indices are approximated to improve the calculated efficiency. For example, the index in Ramakrishnan's method [11], which is the weighted sum of the mean value and standard deviation of the objective characteristic, is calculated by using a Taylor series However, approximated values approximation. significantly differ from the actual values or cannot be derived in the cases where the followings are not satisfied: (1)The objective characteristics monotonically increase or decrease with respect to the factors; (2) The objective function is differentiable; (3) The fluctuations in the factors are sufficiently small; and (4) The factors are independent from each other.

The proposed robustness index must be calculated accurately because the adjustable range must be minimized as shown in Eq. (1). In other words, by using an accurate robustness index, the designers must set the adjustable range as small as possible. Consequently, the Monte Carlo method, which derives accurate values but is time consuming, is applied to calculate the index. The calculation methods is described below.

All ACFs' assignable point values should be used to calculate R_A . Specifically, sets of objective characteristic fluctuations that satisfy the tolerance are derived with respect to all assignable point values t_j , as shown in Fig. 2. R_A is calculated as the ratio of the sum of the sets of fluctuant combinations of x and z where at least one of the objective characteristic values y_j derived from t_j is within the tolerance shown in Eq. (2):

$$R_{\rm A} = P \left[\bigcup_{j=1}^{n_{\rm ap}} \left\{ C(\boldsymbol{x}, \boldsymbol{z}) \middle| y_{\rm l} \le f(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{t}_j) \le y_{\rm u} \right\} \right]$$
(2)



Fig. 2 Set of the objective characteristic fluctuations used to calculate the robustness index.

where, the square bracket expresses a set of C(x, z)where the objective characteristic is located within the tolerance by adjusting ACFs. This means R_A is the rate of the set and the entire set. The assignable point values are expressed as a finite number of discontinuous values t_i because R_A is calculated by using the Monte Carlo method. The number of the assignable (discontinuous) values should be sufficient to assume the ACF is continuous. However, the number should be decreased if the calculation amount is too large. To calculate R_A , first, s random combinations of the control and noise factors are generated based on their probability density functions. Second, objective characteristic y_i is calculated by using the generated random combinations $\{x_i, z_i\}$ (i = 1, 2, ..., s) and all the assignable point values. That is, the number of calculating objective characteristic values is the product of the random combination number s and the assignable points numbers of ACFs. Finally, the values calculated from each random combinations of x_i and z_i are assessed to determine whether at least one of the calculated values is within the tolerance (i.e., at least one assignable point which consists an objective characteristic value that satisfies the tolerance). Then $R_{\rm A}$ is calculated as

$$R_{A} = \frac{1}{s} \sum_{i=1}^{s} M_{i}$$

$$\begin{pmatrix} M_{i} = \begin{cases} 1 & (\exists t \in \{t_{j}\}; y_{1} \le f(\boldsymbol{x}_{i}, \boldsymbol{z}_{i}, t) \le y_{u}) \\ 0 & (\text{otherwise}) \\ j = 1, 2, ..., n_{ap} \end{pmatrix}$$
(3)

3.2 Adjustable Range Optimization

This study proposed an optimization algorithm using the VEPSO in order to solve the design problem of Eq. (1). An outline of the VEPSO and the algorithm using are described below.

3.2.1 Outline of VEPSO

The VEPSO [16, 17] is an improved method of the PSO [18] that is one of the representative metaheuristics in order to handle the multi objective optimization problems. The PSO imitates the movement of organisms in a bird flock or fish school

and searches a solution by using the information both from the individuals (particles) and their swarm. The VEPSO assigns an objective to each of swarms and searches a solution using the information inside or between swarms. The location vector (i.e., design variables) of the *i*-th particle in the *j*-th swarm $\mathbf{x}_i^{[j]}$ is updated as follows:

$$\boldsymbol{x}_{i}^{[j]}(T+1) = \boldsymbol{x}_{i}^{[j]}(T) + \boldsymbol{v}_{i}^{[j]}(T)$$
(4)

where, T is the number of iterations; v is the velocity vector to direct the particles to the updated locations and is calculated as

$$\mathbf{v}_{i}^{[J]}(T+1) = k \left[w \mathbf{v}_{i}^{[J]}(T) + c_{1} r_{i} \left\{ \mathbf{x}_{pb,i}^{[J]}(T) - \mathbf{x}_{i}^{[J]}(T) \right\} + c_{2} r_{2} \left\{ \mathbf{x}_{gb,i}^{[P]}(T) - \mathbf{x}_{i}^{[J]}(T) \right\} \right]$$

$$\left(p = \begin{cases} M & \text{if } j = 1, \\ j - 1 & \text{if } j = 2, 3, ..., M \end{cases} \right)$$
(5)

where, *M* is the number of swarms. c_1 and c_2 are the parameters to express the degree of incidence of the private best location of each particle \mathbf{x}_{pb} and the global best location \mathbf{x}_{gb} , respectively; r_1 and r_2 denote the random numbers uniformly distributed in [0, 1]. *w* is the parameter to define the effect of the current velocity vector and decreases based on *T* as shown in Eq. (6):

$$w(T) = w_{\max} - \frac{w_{\max} - w_{\min}}{T_{\max}} \cdot T$$
(6)

where, w_{max} and w_{min} are the maximum and minimum value of w, respectively. T_{max} is the maximum number of the iterations. k denotes the parameter relating the convergence performance and expressed as Eq. (7):

$$k = \frac{2}{\left|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}\right|} \quad (\varphi = c_1 + c_2)$$
(7)

As shown in Eq. (5), the velocity vectors are defined by using the global best locations of the different swarms. This enables the solution search based on the information from the other swarms and the global locations (solutions) of the swarms to approach each other. Therefore, the solution search of the VEPSO has the following features: assigning a objective to each of swarms and searching the solutions located close to each other. These features generate the following merits to solve the robust design problem (Eq. (1)): (1) The design solution (assignable points) to ensure the robustness can be efficiently derived by assigning the robustness regarding the part of the fluctuation of the factors to each of the swarm; (2) The distance between assignable points (the adjustable range) can be smaller.

3.2.2 Procedure to Derive Optimum Adjustable Range Using VEPSO

This study amended the robustness index (Eq. (2)) in order to evaluate each assignable point that assures the robustness regarding the part of the fluctuation of the factors. The robustness assured by the *j*-th assignable point is expressed in Eq. (8):

$$R_{j} = P\left[C(\boldsymbol{x}, \boldsymbol{z})_{j} \mid y_{1} \leq f(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{t}_{j}) \leq y_{u}\right]$$
(8)

where, $C(\mathbf{x}, \mathbf{z})_j$ ($\subseteq C(\mathbf{x}, \mathbf{z})$) is a part of the fluctuation of the factors assigned to the *j*-th assignable point and holds Eq. (9):

$$\sum C(\boldsymbol{x}, \boldsymbol{z})_j = C(\boldsymbol{x}, \boldsymbol{z}) \tag{9}$$

When considering the four assignable points, the four swarms are defined and search for the adjustable range (assignable points) based on the VEPSO procedure. The optimization algorithm using the VEPSO is described in Fig. 3. In this algorithm, the parameters of the VEPSO (e.g., c, w, T_{max} , etc.) are firstly set. Next, the number of the assignable points is decided and the same number of the swarms is set. The



Fig. 3 Proposed algorithm of robust design method.

locations of the particles are updated based on the objective (robustness) R_j ($j = 1, 2, ..., n_{ap}$). The update of the locations iterates until $T = T_{max}$, and the global best location of the swarms are derived as an design solution (adjustable range).

4. Illustrative Example

4.1 Problem Description

To demonstrate the proposed robust design method, we applied it to a seat design for railway vehicles because numerous people with diverse physiques and sitting postures use these seats. However, the conventional seat design typically assumes an average physique and posture. Thus, designing a seat that is robust for various physiques and postures is desirable. Herein the design objective focused on the hip-sliding force, which is generated on the buttocks by the static instability of the upper and lower body masses, causing discomfort when sitting [19]. Therefore, the design objective is to inhibit the hip-sliding force for various physiques and postures.

Table 1 defines the objective characteristic and factors of this design. The control factors, seat cushion angle θ_C , seat back angle θ_B , and forward tilt angle of the upper seat back θ_F , can be adjusted by the mechanisms for the seat cushion forefront lifting function, reclining function, and forward tilt function, respectively. Previous research [20] has demonstrated the influence of these angles on the hip-sliding force. Therefore,

 Table 1
 Definition of objective characteristic and factors.

| Items | Definition |
|---|---|
| Hip-sliding force (as objective characteristic) | Tolerance of Hip-sliding force: -10-20 N |
| Seat cushion angle $\theta_{\rm C}$ Seat back angle $\theta_{\rm B}$ Forward tilt angle $\theta_{\rm F}$ (as ACFs) | $\begin{array}{l} 10 \leq \theta_C \leq 25 \\ 20 \leq \theta_B \leq 35, \ \theta_C + 10 \leq \theta_B \\ 0 \leq \theta_F \leq 30 \end{array}$ |
| Body height <i>L</i> Body height <i>M</i> Sitting posture (as noise factors) | L and M are normal distributions Mean value of L: 1.65 m, standard deviation of L: 0.08 m Mean value of M: 58.1 kg , standard deviation of M: 9.09 kg Ratio of standard, stretched waist and bent waist sitting postures: 3:1:6 |

these angles are considered ACFs. Noise factors include users' physiques and sitting postures. The physiques are defined based on actual measurements of Japanese citizens [21]. Additionally, we considered three sitting postures: a standard sitting posture where the lumbar region is in contact with the seat back, a stretched waist sitting posture where the waist is stretched and slid forward from the standard sitting posture, and a bent waist sitting posture where the waist is bent and slid forward from the standard sitting posture. The ratio of these sitting postures is 3:1:6 [19].

Sagittal plane model of the human body and the seat (Fig. 4) was used to derive the objective function. The human model assumes that the movements of the low momentum joints are zero and is consequently, composed of four high momentum joints. On the other hand, the seat model, constructed based on the existing seat found on a 485 train (Hatsukari), is divided into three parts: seat cushion, upper seat back and lower seat back, which are rigidly linked. Figs. 5a-5c show the objective functions (the formula to compute the hip-sliding force for each posture) of standard, stretched waist and bent waist sitting posture derived based on these models. The robustness index (R_A) is derived as the weighted sum of the indices calculated by using these formulae.

Additionally, in the PSO, the parameters (e.g., c, w, etc.) are important for the convergence or the computational efficiency. Therefore, this study implemented some optimizations regarding the recommended values of the parameters in the conventional studies and compared the results to clarify the proper values of them. This study focused on c_1 and c_2 and conducted the four analyses by using the parameter combinations: $(c_1 = 2.80, c_2 = 1.30)$ recommended by Carlisle [22]; (2.05, 2.05) suggested by Kennedy [23]; (1.55, 2.55) and (1.05, 3.05) that are smaller values of c_1 . This study also implemented the optimization using the traditional GA (genetic algorithm). The definition of the parameters is summarized in Table 2.



Fig. 4 Model of the human body and seat.

Table 2Definition of parameters.

| | r | | | | | |
|---------------------------------------|--|----------------------|------|------|------|--|
| Items | Set value | | | | | |
| | GA | Proposed method | | | | |
| | | (Analysis) | | | | |
| | | 1 | 2 | 3 | 4 | |
| Tolerance of <i>y</i> | $-10 \le y \le 20$ | | | | | |
| Feasible area of t_1 | $10 \le \theta_{\rm C} \le 25$ | | | | | |
| Feasible area of t_2 | $20 \le \theta_{\rm B} \le 35, \ \theta_{\rm B} \le \theta_{\rm C} + 10$ | | | | | |
| Feasible area of t_3 | $0 \le \theta_{\rm F} \le 30$ | | | | | |
| Max iteration number T_{max} | 10,000 | 100 | | | | |
| c_1 | - | 2.8 | 2.05 | 1.55 | 1.05 | |
| c_2 | - | 1.3 | 2.05 | 2.55 | 3.05 | |
| w _{min} | - | 0.4 | | | | |
| w _{max} | - | 0.9 | | | | |
| Number of assignable points | 2 | 2 (number of swarms) | | | | |
| Swarm size | - | 20 | | | | |
| Solution number | 5 | | | | | |

4.2 Results

The design solutions (adjustable ranges) derived by the proposed method using the different parameters and by the GA are shown in Fig. 6. Additionally, the Euclidean distance between assignable points D is calculated in order to compare the size of the adjustable range. The average and the standard deviation of the distance are indicated as shown in Table 3. Fig. 6 and Table 3 show the adjustable ranges derived by the proposed method are smaller than and assure as same robustness as those by the GA. Particularly, the mean value and the standard deviation of the adjustable ranges derived in analysis 2 are small. This means the values of the parameter ($c_1 = c_2 = 2.05$) are suitable to minimize the adjustable range. This is caused by the two features of the VEPSO: (1) The larger c_2 prevents the global solution search same as the PSO; (2) The

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$$\begin{aligned} & H_{HS} = -F_{h} \cos\theta_{C} - F_{v} \sin\theta_{C} - k(-F_{h} \sin\theta_{C} + F_{v} \cos\theta_{C}) \\ & F_{h} = F_{2} \cos\theta_{C} - F_{3} \sin(\theta_{Hi} + \theta_{C}) \\ & F_{v} = F_{2} \sin\theta_{C} + F_{3} \sin(\theta_{Hi} - \theta_{C}) + M_{2}l_{2b}g + M_{3}l_{3a}g \\ & F_{2} = \frac{M_{1}l_{1b}g + M_{2}l_{2a}g}{\sin\theta_{C} - \cos\theta_{C} \tan\theta_{An}}, \quad F_{3} = \frac{F_{4-5} + (M_{4}l_{4a}g + M_{3}l_{3b}g)(\cos\theta_{B} - \kappa \sin\theta_{B})}{-\cos\theta_{Ab} - \kappa \sin\theta_{Ab}} \\ & F_{4} = (M_{5}l_{5a}g + M_{4}l_{4b}g)(\cos\theta_{B} - \kappa \sin\theta_{B}) + F_{5}(\cos\theta_{F} - \kappa \sin\theta_{F}), \\ & F_{5} = M_{5}l_{5b}g\{\cos(\theta_{B} - \theta_{F}) - \kappa \sin(\theta_{B} - \theta_{F})\} \\ & \theta_{An} = \sin^{-1}(H/L_{1}), \quad \theta_{Hi} = 180^{\circ} - \phi, \quad \theta_{Ab} = \phi + 90^{\circ} - \theta_{C} + \theta_{B} - \theta_{F}, \\ & \phi = \sin^{-1}\{(L'/L_{3})\sin(90^{\circ} + \theta_{B} - \theta_{C})\} \\ & L' = L_{h} \cos(90^{\circ} + \theta_{B} - \theta_{C}) + \sqrt{L_{3}^{2} - L_{h}^{2}} \sin^{2}(90^{\circ} + \theta_{B} - \theta_{C}) \end{aligned}$$

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$$\begin{aligned} F_{HS} &= -F_{h} \cos\theta_{C} - F_{v} \sin\theta_{C} - \kappa \left(-F_{h} \sin\theta_{C} + F_{v} \cos\theta_{C}\right) \\ F_{h} &= F_{2} \cos\theta_{C} + F_{3+4} \cos(\theta_{Hi} + \theta_{C}), \\ F_{v} &= F_{2} \sin\theta_{C} + F_{3+4} \sin(\theta_{Hi} + \theta_{C}) + M_{2}l_{2b}g + (M_{3} + M_{4})l_{ma}g \\ F_{2} &= \frac{M_{1}l_{1b}g + M_{2}l_{2a}g}{\sin\theta_{C} - \cos\theta_{C} \tan\theta_{An}}, F_{3+4} = \frac{F_{5} + (M_{3}l_{5a}g + (M_{3} + M_{4})l_{mb}g)(\cos(\theta_{B} - \theta_{F}) - \kappa \sin(\theta_{B} - \theta_{F}))}{-\cos\theta_{T} + \kappa \sin\theta_{T}} \\ F_{5} &= M_{5}l_{5b}g(\cos(\theta_{B} - \theta_{F}) - \kappa \sin(\theta_{B} - \theta_{F})), \\ \theta_{An} &= \sin^{-1}(H/L_{1}), \theta_{Hi} = 180^{\circ} - \phi, \theta_{T} = \phi + 90^{\circ} - \theta_{C} + \theta_{B} - \theta_{F}, \\ \phi &= \sin^{-1}[\{L'/(L_{3} + L_{4})\}\sin(90^{\circ} + \theta_{B} - \theta_{C})] \\ L' &= L_{h} \cos(90^{\circ} + \theta_{B} - \theta_{C}) + \sqrt{(L_{3} + L_{4})^{2} - L_{h}^{2} \sin^{2}(90^{\circ} + \theta_{B} - \theta_{C})} \end{aligned}$$

(b)

$$F_{HS} = -F_{h} \cos\theta_{C} - F_{v} \sin\theta_{C} - \kappa(-F_{h} \sin\theta_{C} + F_{v} \cos\theta_{C})$$

$$F_{HS} = -F_{h} \cos\theta_{C} - F_{v} \sin\theta_{C} - \kappa(-F_{h} \sin\theta_{C} + F_{v} \cos\theta_{C})$$

$$F_{h} = F_{2} \cos\theta_{C} + F_{3+4} \cos(\theta_{Hi} + \theta_{C} - \tau),$$

$$F_{v} = F_{2} \sin\theta_{C} + F_{3+4} \sin(\theta_{Hi} + \theta_{C} - \tau) + M_{2}l_{2b}g + (M_{3} + M_{4})l_{mb}g)(\cos(\theta_{B} - \theta_{F}) - \kappa \sin(\theta_{B} - \theta_{F}))$$

$$F_{2} = \frac{M_{1}l_{1b}g + M_{2}l_{2a}g}{\sin\theta_{C} - \cos\theta_{C} \tan\theta_{An}}, \quad F_{3+4} = \frac{F_{5} + (M_{5}l_{5a}g + (M_{3} + M_{4})l_{mb}g)(\cos(\theta_{B} - \theta_{F}) - \kappa \sin(\theta_{B} - \theta_{F}))}{-\cos(\theta_{T} - \omega) + \kappa \sin(\theta_{T} - \omega)}$$

$$F_{5} = M_{5}l_{5b}g(\cos(\theta_{B} - \theta_{F}) - \kappa \sin(\theta_{B} - \theta_{F})),$$

$$\theta_{An} = \sin^{-1}(H/L_{1}), \quad \theta_{Hi} = 180^{\circ} - \phi + \tau, \quad \theta_{T} = \phi + 90^{\circ} - \theta_{C} + \theta_{B} - \theta_{F} + \omega,$$

$$\phi = \sin^{-1}\{(L'/L''')\sin(90^{\circ} + \theta_{B} - \theta_{C})\}$$

$$L' = L_{h} \cos(90^{\circ} + \theta_{B} - \theta_{C}) + \sqrt{(L''')^{2} - L_{h}^{2} \sin^{2}(90^{\circ} + \theta_{B} - \theta_{C})},$$

$$L''' = \sqrt{L_{3}^{2} + L_{4}^{2} - 2L_{3}L_{4} \cos(180^{\circ} - 24^{\circ})}$$
(c)

 $\begin{array}{l} \theta_{\rm C}: \text{Seat cushion angle, } \theta_{\rm B}: \text{Seat back angle, } \theta_{\rm T}: \text{Forward tilt angle, } \theta_{\rm Hi}: \text{Hip angle, } \theta_{\rm Ab}: \text{Abdomen angle, } \theta_{\rm An}: \text{Ankle angle, } \kappa: \text{Coefficient of frictional resistance, } H: \text{Seat cushion height (400mm), } F_i: \text{Force on } i \text{ th human body section, } F_{\rm HS}: \text{Hip sliding force, } F_h: \text{Horizontal force on trochanter major, } F_v: \text{Vertical force on trochanter major, } L: \text{Body height, } L_i: \text{Length of } i \text{th body section, } L_h: \text{Buttock-trochanterion length, } M: \text{Body weight, } M_i: \text{Weight of } i \text{th body section, } L_i = \text{Composite ratio of } I_i \text{ and the length from } i \text{th body section upper-edge to gravity-center, } l_{ib}: 1 - l_{ia}, \\ I_ma: \text{Composite ratio of 3rd and 4th body section in stretched waist sitting posture, } l_{m'a}: l_ma \text{ in bent waist sitting posture } \\ \left(\begin{array}{c} L_1 = 0.2880L - 0.0424, \ L_2 = 0.0027L + 0.4057, \ L_3 = 0.3274L - 0.2908, \ L_4 = 0.0609L + 0.0356, \ L_5 = 0.0930L - 0.0549, \\ L_h = 0.3118L - 0.4113, \ M_1 = 0.12M, \ M_2 = 0.2M, \ M_3 = 0.14M, \ M_4 = 0.18M, \ M_5 = 0.36M \\ l_{ia} = 0.61, \ l_{2a} = 0.43, \ l_{3a} = 0.11, \ l_{4a} = 0.11, \ l_{5a} = 0.35, \ l_{ma} = 0.329, \ l_{m'a} = -0.608(L_3 + L_4) + 0.579 \end{array} \right) \end{array}$

Fig. 5 Hip-sliding force estimation equations.





Fig. 6 Solution candidates of each analysis.

Table 3 Results of analyses.

| Method | | (c_1, c_2) | R _A | μ_D | σ_D |
|----------------------------------|---|--------------|----------------|---------|------------|
| GA | | - | 0.996 | 17.7 | 4.07 |
| Proposed method (analysis) | 1 | (2.8, 1.3) | 0.998 | 3.92 | 9.66 |
| | 2 | (2.05, 2.05) | 0.999 | 0.23 | 0.79 |
| | 3 | (1.55, 2.55) | 0.999 | 0.49 | 1.41 |
| | 4 | (1.05, 3.05) | 0.999 | 1.21 | 4.36 |

larger c_2 encourages the swarms to search the area close to each other. Because of the trade-off relationship between the two features, the same degree of incidence is compromised to be an optimum value in the proposed method.

5. Conclusions

In this research, ACFs which can be adjusted within a given range to increase the robustness were defined. Additionally, a method to calculate the robustness R_A , including the objective characteristics adjustment by the factors and derive an optimum range of the factors is proposed. R_A indicates the feasibility that the objective characteristic values are within the tolerance at least once by the adjustment of ACFs. A calculation method for the index, which uses the Monte Carlo method, are proposed. In contrast, the range of ACFs is optimized by the vector evaluated particle swarm optimization. In the procedure, R_A is used to evaluate the particles in several swarms, and each particle searches for the optimum adjustable range of ACFs.

The proposed method was applied to an engineering example (seat design problem). In this application, it was confirmed that the proposed method can derive the design solution with high robustness and small adjustable range.

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