Identification of Suspension Bridge Parameters under Exploitational Conditions

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Abstract: The paper concerns a research into dynamic properties of the steel suspension bridge across Opolska Street in Krakow, Poland. Parameter identification was carried out with the application of the nonlinear system identification method on the basis of system responses to exploitational excitation resulting from pedestrian traffic. In order to verify obtained results, on the basis of the geometrical and material properties of the considered system, the FEM (finite elements model) was created. Created FEM model was updated through the comparison with the model determined by the use of experimental modal analysis method and then applied to analytical evaluation of the considered suspension bridge natural frequencies.

Key words: Steel bridge, nonlinear system identification, exploitational excitation.

1. Introduction

While analyzing properties of mechanical systems working under operational loads, the measurement of forces exciting system vibrations is difficult or impossible to carry out directly. This issue is especially important in case of systems of significant mass or stiffness, which demand for energy necessary to excite motion with many mode shapes is too high to be realized with the use of standard equipment for structural dynamics testing. Moreover, the output-only identification methods known from literature and used in the engineering practice, find application to research into properties of linear systems only [1].

The paper concerns parameter identification of the suspension bridge on the basis of measured system responses to exploitational excitation with pedestrian traffic. The research was carried out with the use of method consisting in sequential application of restoring

force, boundary perturbation and direct parameter identification techniques [2, 3]. On the contrary to classical nonlinear system identification methods, the method requires neither the knowledge of excitation acting on the system nor system linear behaviour in the broad frequency range around an operating point. The method can be applied to both linearity testing and parameter identification of linear and nonlinear systems. Correctness of the experimental research was verified by analytical determination of natural frequencies and corresponding mode shapes with the use of the finite elements method.

The paper is organized as follows: Section 2 provides the theory of the applied methods; Section 3 concerns the results of the research into dynamic properties of the steel suspension bridge in Krakow, Poland; Section 4 gives conclusions.

2. Identification of Nonlinear System Parameters

Identification of the considered suspension bridge

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was performed by means of the output-only method combining restoring force, boundary perturbation and direct parameter estimation techniques (Fig. 1).

In the first step [2, 3], system dynamic motion equations are formulated and nonlinear restoring forces are estimated on the basis of measured system responses. In the following step, identified nonlinear restoring forces are eliminated from the formulated dynamic motion equations that describe the balance of forces acting on the considered system. Since the excitation remains unknown, the number of unknown parameters is greater than the number of dynamic motion equations that can be formulated. In order to provide an additional dynamic motion equation and determine absolute values of system parameters, the boundary perturbation method is used. The method consists in introducing an additional mass (inertia) altering dynamic behaviour of the considered system and retaking measurements of modified system masses accelerations. In the final step, values of system parameters are estimated.

Presented above method of nonlinear system parameters identification on the basis of measured system responses is a convenient parameter identification method for strongly nonlinear systems working under operational loads, the measurement of which is difficult or impossible to carry out [5, 6].

Fig. 1 Applied nonlinear system identification method [4].

2.1 Restoring Force Method

Restoring force method algorithm for single degree-of-freedom systems was formulated by Masri and Caughey in 1979 [7]. In the following years, the modified algorithms of the method were developed. These variations are referred to as force-state mapping [8, 9] and local restoring force method [10].

The method is based on the dynamic motion equation specified by the Newton's second law:

$$M \cdot \{\ddot{x}(t)\} + \{f(\{x(t)\}, \{\dot{x}(t)\})\} = \{u(t)\} \quad (1)$$

where, M denotes (effective) system mass; $\{u(t)\}$: exciting force vector; $\{x(t)\}$, $\{\dot{x}(t)\}$: displacement and velocity of mass M ; and $\{f(\{x(t)\}, \{\dot{x}(t)\})\}$: vector of internal restoring forces.

Since $\{f\}$ only depends on $\{x(t)\}$ and $\{\dot{x}(t)\}$, it can be represented by a surface over the $(x, \dot{x})$ state space:

$$\{f(\{x(t)\}, \{\dot{x}(t)\})\} = \{u(t)\} - M \cdot \{\ddot{x}(t)\} \quad (2)$$

In practice, the measurements of vibration accelerations are performed while vibration velocities and displacements are determined by numerical time-domain integration of measured accelerations. If the measurements are taken at time intervals $t_i = (i - 1) \Delta t$, Eq. (2) can be written as follows:

$$\{f_i\} = \{f(\{x(t_i)\}, \{\dot{x}(t_i)\})\} = \{u(t_i)\} - M \cdot \{\ddot{x}(t_i)\} \quad (3)$$

At each instant $i = 1, \dots, N$, the triplet $\{x(t_i)\}$, $\{\dot{x}(t_i)\}$, f_i is specified. The first two values indicate the point in the phase plane while the third value specifies the distance between this point and the restoring force surface.

Masri and Caughey [11] proposed the method consisting in restoring force modelling by the use of the Chebyshev series:

$$\{f(\{x\}, \{\dot{x}\})\} = \sum_{i=0}^m \sum_{j=0}^n C_{ij} T_i(\{x\}) T_j(\{\dot{x}\}) \quad (4)$$

where, $T_i(x)$ denotes Chebyshev polynomial of the i th order.

For hysteretic systems, the internal restoring force does not depend entirely on system position in the phase plane. As an example, the Bouc-Wen model [12, 13] can be considered:

$$\begin{cases} M\{\ddot{x}\} + \{f(\{x\}, \{\dot{x}\})\} + \{z\} = \{u(t)\} \\ \{\dot{z}\} = -\alpha|\{\dot{x}\}|\{z\}|\{z\}|^{n-1} - \beta\{\dot{x}\}|\{z\}| + [A]\{\dot{x}\} \end{cases} \quad (5)$$

In this case, the Chebyshev polynomials method cannot be used since the system internal restoring force is a function not only of $\{x\}$ and $\{\dot{x}\}$ but also $\{z\}$, so that a force surface over a phase plane is multivalued.

In Refs. [14, 15], the authors proposed to represent the hysteretic restoring force by the following equation:

$$\{\dot{f}\} = g(\{f\}, \{\dot{x}\}) \quad (6)$$

In such a case, the restoring force first derivative is represented by the surface over the plane $(f, \dot{x})$.

3. Results of Operational Identification of Suspension Bridge Parameters

Suspension bridge over Opolska Street in Krakow (Fig. 2) is a welded massive construction that could easily withstand motor traffic but is used as a footbridge only. Bridge platform of 4.64 m in width, consists of orthotropic reinforced concrete plate on the steel grate and two steel beams of dimensions $0.5 \times 0.86 \text{ m}^2$ is hung from the steel A-type pylon 34.5 m high. The overall bridge span amounts to 120 m.

Modal parameters of the considered suspension bridge estimated on the basis of transfer functions between excitation signal and system responses measured with the use of impulse test, which was carried out in the absence of pedestrian and vehicle traffic [17], are presented in Table 1 and Fig. 3.

In the course of the identification experiment, according to the scheme assumed during modal tests, the network of 14 measurement points (Fig. 4) was assumed: seven points along the left and seven points along the right footbridge edge (localized at the balustrade base).

In Table 2, there are presented coordinates of the consecutive measurement points.

Fig. 2 Considered suspension bridge [16].

Table 1 Considered system natural frequencies and corresponding damping factors [17].

No.	$f(\text{Hz})$	$\zeta(\%)$	No.	$f(\text{Hz})$	$\zeta(\%)$
1	2.78	0.94	10	9.64	0.77
2	2.96	0.62	11	10.21	0.15
3	3.25	0.86	12	11.20	0.19
4	5.02	1.09	13	13.87	0.31
5	5.64	0.93	14	16.03	0.61
6	6.06	0.44	15	16.99	0.21
7	6.27	1.03	16	18.49	0.34
8	7.07	0.49	17	21.04	0.25
9	8.92	0.10	18	23.33	0.13

Fig. 3 Selected mode shapes of the considered suspension bridge [17] estimated with the application of the operational modal analysis.

Fig. 4 Assumed network of measurement points.

Table 2 Location of the assumed measurement points.

No.	Point name	X (m)	Y (m)	Z (m)	Description
1	Lewy: 1	0.0	5.0	0.0	Point of the 1st rope fastening
2	Lewy: 2	9.5	5.0	0.0	Between the 1st and the 2nd rope
3	Lewy: 3	19.5	5.0	0.0	Point of the 2nd rope fastening
4	Lewy: 4	28.5	5.0	0.0	Between the 2nd rope and pylon
5	Lewy: 5	39.0	5.0	0.0	Pylon
6	Lewy: 6	48.5	5.0	0.0	Between pylon and the 3rd rope
7	Lewy: 7	58.5	5.0	0.0	Point of the 3rd rope fastening
8	Praw: 1	0.0	0.0	0.0	Point of the 1st rope fastening
9	Praw: 2	9.5	0.0	0.0	Between the 1st and the 2nd rope
10	Praw: 3	19.5	0.0	0.0	Point of the 2nd rope fastening
11	Praw: 4	28.5	0.0	0.0	Between the 2nd rope and pylon
12	Praw: 5	39.0	0.0	0.0	Pylon
13	Praw: 6	48.5	0.0	0.0	Between pylon and the 3rd rope
14	Praw: 7	58.5	0.0	0.0	Point of the 3rd rope fastening
15	Base	0.0	0.0	0.0	-

Measurements of vibration accelerations in vertical direction were carried out in the points “praw: 1”, “praw: 3”, “praw: 5” and “base” by means of the uniaxial piezoelectric accelerometer of the PCB 393A03 type.

Measured time histories were registered with the use of the 8-channel SCADAS Mobile analyzer. It was also assumed that estimation of the considered system parameters will be carried out with the use of the operational identification method presented in the second paragraph of this paper. Therefore, two measurement sessions were carried out. In the second

session, the bridge was loaded with the additional mass ΔM_Z . Acceleration time histories measured in the considered measurement points are presented in Fig. 5.

In the course of the considered system parameters identification process, discrete 2-degrees-of-freedom model (Fig. 6) was used. The following notation was assumed: M_1 denotes reduced mass of bridge support; M_Z : reduced mass in the point of rope fastening; K_2 : stiffness of bridge span; K_1 : stiffness of bridge support; C_2 : bridge span damping coefficient; C_1 : bridge support damping coefficient; x_1, x_2, x_b : displacements of M_1, M_Z and base; N_1, N_2 : nonlinear forces.

Dynamic motion equations formulated for system reduced masses M_1 and M_Z are as follows:

(a)

(b)

Fig. 5 Acceleration time histories in points (a) “praw: 3” and (b) “base” measured in the first (—) and the second (—) measurement session.

Fig. 6 Discrete model of the considered system.

$$\begin{cases} M_1 \{\ddot{x}_1\} + (C_1 + C_2)\{\dot{x}_1\} - C_2\{\dot{x}_2\} + K_1\{x_1\} + \\ + K_2\{x_1\} - K_2\{x_2\} + N_1 + N_2 = C_1\{\dot{x}_b\} + K_1\{x_b\} \\ M_Z\{\ddot{x}_2\} - C_2\{\dot{x}_1 - \dot{x}_2\} - K_2\{x_1\} + \\ + (K_2 + K_3)\{x_2\} = N_1 + \{F_Z\} \\ \{x_1\} = \{x_1(t)\}, \{x_2\} = \{x_2(t)\}, \\ N_1 = N_1(\{x_1\}, \{x_2\}, \{\dot{x}_1\}, \{\dot{x}_2\}), \\ N_2 = N_2(\{x_1\}, \{x_b\}, \{\dot{x}_1\}, \{\dot{x}_b\}) \end{cases} \quad (7)$$

Dynamic motion equation for M_Z , is transformed to the following form:

$$M_Z\{\ddot{x}_2\} = -C_2\{\dot{x}_2 - \dot{x}_1\} - K_2(\{x_2\} - \{x_1\}) + N_1 + \{F_Z\} \quad (8)$$

Relations between acceleration of mass M_Z and relative velocity; between masses M_1 and M_Z ; between acceleration of the mass M_Z and relative displacement; as well as between masses M_1 and M_Z , were determined.

In order to reconstruct restoring forces acting on the considered system, on the basis of measured acceleration time histories, time histories of velocities and displacements in the considered measurement points were determined. On the basis of relation between acceleration of M_Z in the function of relative velocity of masses M_1 and M_Z , and relation between acceleration of M_Z and relative displacement of masses M_1 and M_Z , it was assessed that the character of damping and stiffness restoring forces is linear (Fig. 7).

Therefore, the next step of the assumed identification method (consisting in estimation of parametric models f_N of identified nonlinear forces and their elimination

Fig. 7 Example reconstructed (a) stiffness and (b) damping restoring forces.

from dynamic motion equation formulated for mass M_Z was neglected:

$$\{\ddot{x}_2\} - f_N = \frac{C_2}{M_Z}(\{\dot{x}_1\} - \{\dot{x}_2\}) + \frac{K_2}{M_Z}(\{x_1\} - \{x_2\}) + \frac{\{F_Z\}}{M_Z} \quad (9)$$

On the basis of the considered system discrete model, with the use of the direct parameter identification method, the following equations were formulated:

$$\begin{cases} K_2 \left(1 - \frac{1}{T_{21}(\omega_k)} \right) = \omega_k^2 M_Z \\ T_{21}(0) = K_2 (K_2 + K_1)^{-1}, \quad k = 1, 2, \dots, N_f \end{cases} \quad (10)$$

where, $\{T_{21}(j\omega)\} = \{X_2(j\omega)\} \{X_1(j\omega)\}^{-1}$: transmissibility function between displacements of masses M_Z and M_1 (determined after elimination of nonlinear restoring forces f_N); $X_1(j\omega)$, $X_2(j\omega)$: Fourier transforms of signals $x_1(t)$ and $x_2(t)$; N_f : number of identified degrees of freedom; and $T_{21}(0)$: value of $\{T_{21}(j\omega)\}$ for $f = 0$ Hz.

Since the measurement of exact bridge elements mass values is impossible to carry out, the system in Eq. (10) is a system of two equations with three unknowns: M_Z , K_2 and K_1 .

In the second measurement session, system dynamic behaviour was modified by loading the bridge platform with an additional mass. For the modified system, according to the algorithm of the boundary perturbation method, the measurements of system masses vibration accelerations were repeated. As a result of system modification, Eq. (10) can be written as follows:

$$K_2 \left(1 - \frac{1}{T_{21}'(\omega_p)} \right) = \omega_p^2 (M_Z + \Delta M_Z) \quad (11)$$

where, $\{T_{21}'(j\omega)\}$: transmissibility function between

displacements of masses ($M_Z + \Delta M_Z$) and M_1 , $p = 1, 2, \dots, N_f$.

Eqs. (10) and (11) form a system of three equations with three unknowns, which makes it possible to estimate absolute values of the required system parameters.

In Fig. 8, there is presented comparison of determined transmissibility functions $\{T_{21}\}$ and $\{T_{21}'\}$ between displacements in points “praw: 3” and “base”.

The results presented below were estimated for transfer functions $\{T_{21}\}$ and $\{T_{21}'\}$ determined between points “praw: 3” (point of the second line fastening) and “base”. For the purposes of determining maxima of transfer functions $\{T_{21}\}$ and $\{T_{21}'\}$, the stabilization diagrams method (Fig. 9) realized with the use of the ERA (eigensystem realisation algorithm) method implemented in the VIOMA (virtual in-operation modal analysis toolbox) was used. Magnitudes corresponding to the consecutive maxima of the considered transfer functions were determined with the application of the Signal Processing toolbox. Obtained results were used as a basis for computations of the considered system required parameters.

As the measure of system parameters estimation accuracy, the relative percentage error of the reduced mass M_Z estimation was assumed:

$$Err = (M_Z^{MES})^{-1} \cdot |M_Z^{MES} - M_Z^e| \cdot 100 \cdot [\%] \quad (12)$$

where, M_Z^{MES} denotes mass determined on the basis of the formulated FEM model; and M_Z^e : mass estimated with the use of the operational identification method.

In Table 3, there are presented values of the considered suspension bridge parameters estimated with the application of the operational identification method and relative percentage errors of mass M_Z estimated for the consecutive extremes of transfer functions. Parameters corresponding to the extremes of transfer functions $\{T_{21}\}$ and $\{T_{21}'\}$, for which pole lines

determined with the use of stabilization diagrams (Fig. 9) are stabilizing, are marked in grey.

Fig. 8 Comparison of transfer functions: (a) $\{T_{21}\}_{2_line-base}$ and $\{T_{21}'\}_{2_line-base}$, (b) $\{T_{21}\}_{pylon-base}$ and $\{T_{21}'\}_{pylon-base}$.

Fig. 9 Example of stabilization diagram (estimated for transfer functions $\{T_{21}\}_{2_rope-base}$).

In order to verify the correctness of the obtained results, on the basis of the geometrical and material properties [16], the FEM (finite elements model) of the considered suspension bridge was created (Figs. 10 and 11). Computations were carried out in the Strand7 software [18]. Created FEM was updated through the comparison with model determined by the use of experimental modal analysis method [17] and then applied to analytical evaluation of the considered suspension bridge natural frequencies.

Table 3 Parameters of transfer functions, estimated model parameters, percentage relative error.

No.	ω (Hz)	$T_{21}(\omega)$	ω' (Hz)	$T_{21}'(\omega)$	M_Z^e (kg)	10^3	Err (%)
1	1.50	36.1574	1.50	45.1786	683.8892	269.7	
2	2.75	30.8497	2.75	30.6031	12621.76	6,722.6	
3	3.75	21.0406	3.75	20.1296	192.0703	3.8	
4	4.75	27.6815	4.25	20.4249	234.849	26.9	
5	5.75	28.2772	5.5	26.3638	193.3022	4.5	
6	6.25	30.7916	6.25	37.3069	169.3744	-8.4	
7	7.00	17.2791	7.00	18.5328	173.5164	-6.2	
8	9.50	27.5478	9.75	25.0533	196.869	6.4	
9	10.25	16.0256	10.25	22.7646	140.4449	-24.1	
10	13.50	13.9085	13.50	10.9267	281.953	52.4	
11	16.25	12.2613	16.00	14.1146	155.4952	-15.9	
12	18.00	8.8135	18.00	9.6580	155.9759	-15.7	
13	18.50	6.0253	18.50	9.2229	84.16074	-54.5	
14	22.75	19.538	22.75	19.4115	186.178	0.6	
15	28.50	14.1120	28.50	16.8444	154.1666	-16.7	
16	30.75	14.2798	30.75	13.5544	198.0801	7.1	
17	31.75	11.2387	31.75	11.1093	188.6531	2.0	
18	35.50	12.8411	35.25	10.4371	271.1228	46.6	
19	39.25	7.0916	39.25	9.1178	114.1498	-38.3	
20	42.25	3.1264	41.75	2.3841	-91.5741	-149.5	
21	51.25	2.7156	51.25	2.7680	148.4777	-19.7	
22	61.75	5.7512	61.00	4.9006	513.5548	177.6	
23	68.75	6.8484	68.50	5.8130	386.5977	109.0	
24	69.25	7.2044	69.25	5.2234	-4613.79	-2,593.9	
25	71.25	3.7376	71.25	3.3946	639.1044	245.5	
26	75.00	10.5100	75.50	8.4672	318.2946	72.1	
27	77.75	15.8281	77.25	14.2626	210.2761	13.7	
28	81.75	4.6713	80.75	3.8536	-3722.85	-2112.4	
29	87.25	10.6337	87.00	8.9851	269.8113	45.8	
30	91.50	7.3948	91.25	4.5596	-284.781	-253.9	

In the first step, the nonlinear static problem was solved by modelling the cables with the use of tension only cut-off bars. Two types of loads were considered: the bridge without any additional mass and after introduction of discrete mass near the point of the first cable fastening (in the middle of span). Taking into account initial static stresses, the first natural

Fig. 10 Created finite elements model of the suspension bridge.

Fig. 11 Bridge span-model created with the use of the finite elements method.

Table 4 Parameters of transfer function and system parameters determined on the basis of the FEM.

No.	f (Hz)	f' (Hz)	$\{T_{21}\}$	$\{T_{21}'\}$	M_Z (kg)	10^3
1	0.054564	0.054564	18.327210	-612,300.00	0.00	
2	0.054567	0.054567	18.326213	3,271,650.38	185.13	
3	0.054568	0.054568	18.325849	-16,754,632	185.13	
4	0.054569	0.054569	18.325288	-812,560.25	185.13	
5	0.054570	0.054570	18.325087	-1.5172736	34.53	
6	0.054570	0.054570	18.324976	-13.4567	130.85	
7	0.054570	0.054570	18.324975	-98,224.299	185.12	
8	0.054570	0.054570	18.324967	-32,608.135	185.09	
9	0.075208	0.075208	13.296501	-10,518.666	185.03	
10	0.075208	0.075208	13.296474	-8046.4568	184.99	
...						
101	1.353986	1.353986	0.7385600	-3,253.7522	184.81	
102	1.400780	1.400771	0.7138926	35,267.9765	185.12	
103	1.401700	1.401700	0.7134192	-8,157,955.2	185.13	
104	1.401779	1.401778	0.7133796	-375.37476	182.21	
105	1.401785	1.401785	0.7133761	-22.755641	148.30	
106	1.401797	1.401797	0.7133700	-412,225.87	185.13	
107	1.401802	1.401802	0.7133673	1,617.30992	185.75	
108	1.401818	1.401818	0.7133594	-926,770.3	185.13	
109	1.401819	1.401819	0.7133588	-22,648.137	185.08	
110	1.527711	1.527391	0.6547113	-0.0012792	1.29	
...						
151	2.653387	2.653387	0.3768768	-2,233.5721	184.67	
152	2.65339	2.65339	0.3768764	2,517,379.2	185.13	
153	2.653396	2.653396	0.3768755	3,739,935.86	185.13	
154	2.653457	2.653457	0.3768669	-45.822044	165.01	
155	2.655375	2.655358	0.3765971	-3207.7116	184.82	
156	2.659093	2.659081	0.3760698	261.479729	189.47	
157	2.83449	2.828334	0.353565	276.620019	180.99	
158	2.890432	2.885138	0.3466038	-0.1181863	3.55	
159	2.894665	2.894501	0.3454827	-0.0293499	4.51	
160	2.912882	2.911404	0.3434769	10.8197695	407.69	
...						
301	9.021716	9.021693	0.1108439	-254,672.12	185.09	
302	9.023906	9.023906	0.1108168	937,368.285	185.14	
303	9.024288	9.024284	0.1108121	-48,321.618	185.10	
304	9.129262	9.129265	0.1095378	-4,500.5966	184.90	
305	9.129447	9.129459	0.1095355	-24,488.108	185.03	
306	9.129512	9.129513	0.1095349	601.146432	187.00	
307	9.129514	9.129515	0.1095348	2,234.45402	185.62	
308	9.12957	9.12957	0.1095342	29,081.9064	185.17	
309	9.130841	9.1308	0.1095194	-26,284.538	185.08	
310	9.131321	9.131345	0.1095129	-411.05741	182.50

Fig. 12 Selected mode shapes of the considered suspension bridge estimated with the use of the finite elements method.

frequencies of construction without and with an additional mass introduced were estimated.

Having applied exciting force of spectral density constant in the frequency range 0-12 Hz, transfer functions of construction before $\{T_{21}\}$ and after introduction of an additional mass $\{T_{21}'\}$ were determined. Values of resonant frequencies and corresponding magnitudes of the considered transfer functions are gathered in Table 4. In the last column, the values of the estimated reduced mass M_Z are presented. Selected mode shapes of the considered suspension bridge, estimated with the use of the finite elements method, are gathered in Figs. 12 and 13.

Fig. 13 Selected mode shapes of the considered suspension bridge estimated with the use of the finite elements method.

4. Conclusions and Final Remarks

The paper concerns research into dynamic properties of the steel suspension bridge in Krakow, Poland. Identification of system parameters was carried out by means of the output-only nonlinear system identification method, on the basis of system responses to exploitational excitation, resulting from pedestrian traffic. In order to verify the correctness of the obtained results, on the basis of the geometrical and material properties of the considered system, the FEM was created. Created FEM was updated through the comparison with model determined by the use of experimental modal analysis method and then applied to analytical evaluation of the considered suspension bridge natural frequencies.

Taking into account considerable bridge mass and stiffness, exploitational excitation resulting from pedestrian traffic seems to be insufficient for the purposes of analysis of the considered system dynamic properties. Proper excitation of system mode shapes requires application of an additional excitation provided by the shaker.

Restoring force method can be used for the purposes of nonlinearity detection in systems of continuous mass distribution.

Linear properties of the considered bridge prove its good technical states (e.g., lack of clearances, cracks).

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