Empirical Development of the Rock Mass Deformation Modulus

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Abstract: Rock mass deformation modulus is a fundamental factor for a safe and economical design of rock structures like large underground openings, tunneling, and open pit mine as well as foundations in both the initial state of stresses act on rock mass and its strength characteristics. The rock mass deformation modulus recently has been measured by in-situ loading tests and has been estimated by use of empirical equation based on classification systems and data of laboratory tests. In-situ tests to measure modulus directly are so expensive, times consuming and the reliability of the results of these tests is sometimes doubtful; subsequently, many researches have been carried out to estimate this parameter based on classification systems. In this study, a new empirical equation was proposed by use of statistical analyses based on a database of more than 142 in-situ tests, like plate load tests, dilatometer tests, flat jack tests, and classification systems; in addition, properties of the intact rock.

Key words: Rock mass deformation modulus, in-situ tests, classification systems, empirical equation.

1. Introduction

The deformation modulus is the most representative parameter describing the pre-failure mechanical behavior of any engineering material. Also, deformation modulus is an important parameter used in order to estimate various rock mass properties in a rock engineering projects. Many methods are available to estimate the rock mass deformation modulus such as in-situ tests, empirical equations between deformation modulus and rock mass classification system, and geophysical (usually seismic) methods as well as estimating from laboratory test results [1]. In rock mass, the existence of discontinuities makes the mechanical properties of rock mass differ greatly from that of intact rocks. Although the laboratory tests have existed for estimating deformation modulus; however, the results are not very suitable for this purpose since the specimen is too small. Thus, the in-situ tests are presently employed in major rock engineering projects, to provide more rational data for the design of rock structures which are available for the prediction of rock mass behavior under different loading conditions [2, 3].

Furthermore, the rock mass deformation modulus is currently evaluated by means of different types of in-situ loading tests such as the plate loading test, the flat jack test, the borehole loading tests, and the radial jacking test as well as the water chamber test and so on [4]. In-situ tests to measure modulus directly are so expensive, times consuming and the reliability of the results of these tests is sometimes doubtful; consequently, many authors have proposed empirical relations for estimating deformation modulus indirectly by using various rock mass classification systems such as the RMR (rock mass rating), the GSI (geological strength index), the Q (tunneling quality index), the RMI (rock mass index) and the RQD (rock mass quality designation) like Bieniawski [5], Serafim and Pereira [6], Nicholson and Bieniawski [7], Mehrotra [8], Grimstad and Barton [9], Mitri et al. [10], Hoek and Brown [11], Read et al. [12], Palmstrom and Singh [13], Barton [14-16], Galera et al. [17], Shen et al. [18], and Sanei et al. [19].

Moreover, Zhang and Einstein [20] estimated the
deformation modulus of rock masses by using of RQD. Sonmez et al. [21] developed an empirical model to determine the deformation modulus of rock masses indirectly based on the GSI system. Also, Hoek and Diederichs [22] proposed an empirical equation to estimate rock mass modulus by use of a sigmoid function. On the other hand, Chun et al. [23] presented a model in order to predict deformation modulus based on multiple and polynomial regression analyses of the RMR systems. Furthermore, Sanei et al. [24] developed the second-order polynomial equation for estimating of rock mass deformation modulus by use of JH (Japan Highway) corporation classification system.

In this study, the values of RMR, Q, GSI and JH classification systems were determined and properties of the intact rock like modulus of elasticity from laboratory tests was calculated; in addition, according to ISRM [25] the values of rock mass deformation modulus from in-situ tests (plate load, dilatometer, and flat jack test) were measured. Then, four new empirical equations for estimating the rock mass deformation modulus were developed by use of a database of 47 plate load tests, 86 dilatometer tests, and 9 flat jack test at the Bakhtiary Dam site in Iran. All in all, by use of the statistical analyses among them, the suitable equation was select to estimate the rock mass deformation modulus.

2. Bakhtiary Dam Site

The site of Bakhtiary Dam and Hydroelectric Power plant project include the design and construction of a 315 m high, double curvature, concrete dam, and an underground powerhouse, with a nominal capacity of 1,500 MW, are located in Lorestan Province, in the southwest of Iran, northeast of the Tang-e-Panj railway station on the Tehran-Ahwaz railway, with the following coordinates: 48°46'50" E/32°57'41" N (Fig. 1).

2.1 Geological Characterization of the Dam Site

Limestone layers of Sarvak formation, which are Mid-Cretaceous marine sediments, form the foundation of the dam, powerhouse and other appurtenant structures. These layers are generally tightly folded. The bed rock consists of limestone and marly limestone that contains nodules of siliceous limestone (or Chert). These deposits, which were sedimented between the formations of Garau (at the bottom) and Gurpi (at the top), are marked as Bangestan Group (Kazhdomi, Sarvak, Surgah and Ilam formations) of Cretaceous age. The limestone of the Bakhtiary Dam reservoir, which is overlying Garau formation and underlying Gurpi formation, has been

Fig. 1  Location of Bakhtiary Dam site.
considered to be Sarvak formation. The Sarvak formation is divided into 7 geological units. The 6 units, which are within the dam area, can be considered as Sarvak formation. The units Sv2 to Sv7 have outcrops in dam site, the appurtenant structures and the powerhouse area. Sv1 has no outcrops in this area as it is completely covered by Sv2. The Geology longitudinal profile of the left bank of Bakhtiary Dam site is shown in Fig. 2.

2.2 Intact Rock Properties

Based on laboratory tests on core samples of 6 units, namely Sv2-Sv7, (162 unconfined compression tests, 125 triaxial compression tests, and 47 Brazilian tests), the mechanical characteristics of the intact rocks from different geological units were measured. Also, Table 1 shows the physical and mechanical properties of intact rock specimens.

2.3 Rock Mass Properties

Based on geophysical surveys of the dam site by using the petite seismic method, and using different rock mass classification systems, the rock mass properties were determined. The values of RMR, Q, GSI and JH classification systems are given in Table 2.

2.4 Discontinuities

The rock mass of the dam site is intersected by four main discontinuity types which consist of bedding planes and three joint sets (major joint set J1, joint set J2, and joint set J3), which affect its stability and the bearing capacity. The characteristics of bedding planes and joint sets are summarized in Table 3.

![Geology longitudinal profile of the left bank of Bakhtiary Dam site](image-url)
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Table 1  Physical and mechanical properties of intact rock specimens.

<table>
<thead>
<tr>
<th>Physical and mechanical properties</th>
<th>Index</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>$\rho$</td>
<td>g/cm³</td>
<td>2.61-2.74</td>
</tr>
<tr>
<td>Uniaxial compressive strength</td>
<td>$\sigma_u$</td>
<td>MPa</td>
<td>77-133</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
<td>$E$</td>
<td>GPa</td>
<td>55-73</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu$</td>
<td></td>
<td>0.3</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>$\sigma_t$</td>
<td>MPa</td>
<td>6.3-11.2</td>
</tr>
<tr>
<td>Cohesion</td>
<td>$C$</td>
<td>MPa</td>
<td>29-36</td>
</tr>
<tr>
<td>Friction angle</td>
<td>$\phi$</td>
<td></td>
<td>35-45</td>
</tr>
</tbody>
</table>

Table 2  Estimated RMR, Q, GSI and JH values for different rock mass classifications at Bakhtiary Dam site.

<table>
<thead>
<tr>
<th>Lithology</th>
<th>RMR89</th>
<th>Q</th>
<th>GSI</th>
<th>JH</th>
</tr>
</thead>
<tbody>
<tr>
<td>SV2 + SV3</td>
<td>30-51</td>
<td>0.002-80</td>
<td>35-50</td>
<td>67-83</td>
</tr>
<tr>
<td>SV2 + SV3 + SV4</td>
<td>48-66</td>
<td>0.002-100</td>
<td>45-60</td>
<td>75-89</td>
</tr>
<tr>
<td>SV5 + SV6</td>
<td>41-72</td>
<td>0.01-100</td>
<td>45-65</td>
<td>71-91</td>
</tr>
<tr>
<td>SV7</td>
<td>46-56</td>
<td>0.002-100</td>
<td>45-55</td>
<td>75-85</td>
</tr>
</tbody>
</table>

Table 3  Characteristics of bedding planes and joint sets.

<table>
<thead>
<tr>
<th>Value or description</th>
<th>Bedding</th>
<th>J1</th>
<th>J2</th>
<th>J3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aperture (mm) 0.1-1</td>
<td>90</td>
<td>90</td>
<td>95</td>
<td>95</td>
</tr>
<tr>
<td>Aperture (mm) 1-5</td>
<td>10</td>
<td>10</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Spacing (cm) 2-6</td>
<td>1.5</td>
<td>25</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Spacing (cm) 6-20</td>
<td>47</td>
<td>48.5</td>
<td>52</td>
<td>25</td>
</tr>
<tr>
<td>Spacing (cm) 20-60</td>
<td>44.5</td>
<td>45</td>
<td>43.5</td>
<td>62.5</td>
</tr>
<tr>
<td>Spacing (cm) 60-200</td>
<td>3.5</td>
<td>4</td>
<td>1.5</td>
<td>6.5</td>
</tr>
<tr>
<td>Spacing (cm) 200-600</td>
<td>3.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Infilling Clay</td>
<td>42.5</td>
<td>10</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Infilling Calcite</td>
<td>46</td>
<td>67.5</td>
<td>55</td>
<td>43</td>
</tr>
<tr>
<td>Infilling Bitumen</td>
<td>4.5</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Infilling FeO</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Infilling Tight</td>
<td>5</td>
<td>21.5</td>
<td>43</td>
<td>57</td>
</tr>
<tr>
<td>Roughness Undulating—Ss</td>
<td>19</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Roughness Planar—Ss</td>
<td>41</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Roughness Undulating—Sm</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Roughness Planar—Sm</td>
<td>18</td>
<td>47</td>
<td>54.5</td>
<td>72.5</td>
</tr>
<tr>
<td>Roughness Undulating—Ro</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Roughness Planar—Ro</td>
<td>16</td>
<td>48</td>
<td>43.5</td>
<td>27.5</td>
</tr>
</tbody>
</table>

Ss—Slickensided, Sm—Smooth, Ro—Rough.

3. Evaluations of Rock Mass Classification Systems at the Bakhtiary Dam

Rock mass classification systems were used for various engineering design and stability analysis; therefore, based on empirical relations between rock mass parameters and engineering applications, such as tunnels, slopes, and foundations, the values of rock mass classification systems were calculated that are given in Table 2.

3.1 Rock Mass Classification by JH Method

Akagi et al. [27] presented a new method for the classification of rock masses that the JH rating system was obtained from the study of 6,101 tunnel sections constructed by the JH Corporation. It should be noted that the range of JH system is from 0 to 100. Moreover, in JH method rocks are divided into four classes according to compressive strength in the fresh state and the modes of subsequent weathering and deterioration.
as well as investigated the degrees of contribution of observation items in each class. Also, grouping of rocks in JH classification method are given in Table 4.

### 4. In-situ Measurements of Deformation Modulus

In-situ deformation tests are expensive and difficult to conduct and they are mostly conducted in special works; in addition, initial preparations at each test site are particularly time consuming. Today, several types of in-situ tests are used to determine the modulus of deformation. Therefore, in this study, totally, 47 PLT (plate load tests), 86 DLT (dilatometer tests), and 9 FJT (flat jack tests) were performed at Bakhtiari Dam site. Also, locations of plate load, dilatometer and flat jack tests are shown in Fig. 3.

#### 4.1 Interpretation of Plate Load Test Results

PLT were performed in the 6 exploratory galleries at Bakhtiari Dam site (Fig. 3). In fact, plate load testing using rigid bearing plates, were performed in accordance ISRM method for determining of rock mass deformability. Also, data of all PLTs at dam site were analyzed and the values of deformation modulus were measured based on the method proposed by ISRM [25]. According to ISRM, for rigid plate tests, the basic formula for calculating the rock mass deformation modulus is:

\[
E_a = q_{av} \times \frac{a}{2w_0} \left[ 2(1-v^2) \cot \phi \ln \left( 1 + \frac{z}{z_w} \right) \right] \quad (1)
\]

where \(E_a\), \(q_{av}\), \(a\), \(w_0\), \(v\) and \(z\) are modulus of deformation (GPa), average stress on the loaded surface (MPa), radius of the loaded circular area (m), displacement in the direction of the applied load beneath the center of circle (at the depth = 0) (mm), Poisson’s ratio of rock mass (taken as 0.3 that measured from petite seismic tests in exploratory galleries) and depth from the surface (m), respectively. Also, the values of in-situ deformation modulus of all performed PLTs are given in Table 5.

#### 4.2 Interpretation of Dilatometer Test Results

DLT were performed in 21 boreholes drilled at Bakhtiari Dam site that they were performed in accordance with ISRM method for measuring deformability using a flexible dilatometer with radial displacement measurements. Also, data of all DLTs at dam site were analyzed and the values of deformation modulus were measured based on the method proposed by ISRM [25]. The basic formula for calculating the rock mass deformation modulus is:

\[
E_d = (1+\nu) \frac{D \Delta P}{\Delta D} \quad (2)
\]

where, \(E_d\), \(\nu\), \(D\), \(\Delta P\) and \(\Delta D\) are modulus of deformation (MPa), Poisson’s ratio of rock mass (taken
4.3 Interpretation of Flat Jack Test Results

FJT were performed in 3 exploratory galleries at Bakhtiary Dam site that they were performed in accordance with ISRM method for deformability determination using flat jack technique. Also, data of

as 0.3), diameter of borehole (mm), pressure increment within the considered segment (MPa) and corresponding average change in drill hole diameter $D$ (mm), respectively. Also, the values of in-situ deformation modulus of all performed DLTs are given in Table 5.

![Fig. 3](image)

**Fig. 3** Locations of plate load, dilatometer and FJT in the exploratory galleries in Bakhtiary Dam site.

**Table 5** Values of deformation modulus in Bakhtiary Dam site.

<table>
<thead>
<tr>
<th>In-situ test</th>
<th>PLT</th>
<th>DLT</th>
<th>FJT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deformation modulus (GPa)</td>
<td>1.7-35.5</td>
<td>1-22</td>
<td>3-19</td>
</tr>
</tbody>
</table>
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all FJTs at dam site were analyzed and the values of deformation modulus were measured based on the following formula suggested by ISRM [25]:

\[ E_d = K_i(1 - \nu^2)\frac{\Delta P}{\Delta D} \]  
where, \( E_d \), \( \nu \), \( D \), \( \Delta P \) and \( K_i \) are modulus of deformation (MPa), Poisson's ratio of rock mass (taken as 0.3), slot opening (mm), pressure increment within the considered segment (Bar) and a constant (related to the stiffness, shape, configuration, and number of flat jacks), respectively. Also, the values of in-situ deformation modulus of all performed FJTs are given in Table 5.

5. Results and Discussion

In-situ tests are expensive and difficult to conduct, therefore, the value of the deformation modulus is often estimated indirectly from the empirical equations. In this study, by statistical analyses, four new empirical equations were developed to estimate the rock mass modulus by using a database of 142 in-situ test results at the Bakhtiary Dam site; then, by use of statistical analyses among them, the suitable equation would be select to estimate deformation modulus.

5.1 Estimation of Deformation Modulus from RMR System

In the first step, the values of RMR classification system at the Bakhtiary Dam were determined and properties of the intact rock like modulus of elasticity from laboratory tests was calculated; in addition, the values of the rock mass deformation modulus were measured from more than 142 in-situ tests. Then, non-linear regression analysis was used between the values of relation \( E_m/E_i \) and the values of RMR. Eq. (4) shows that the second-order polynomial with \( R^2 = 0.70 \) is optimum. Also, plot of the values of relation \( (E_m/E_i) \) versus RMR classification system is shown in Fig. 4.

\[ E_m = E_i(0.0003RMR^2 - 0.0363RMR + 1.0199) \]  

where, \( E_m \), \( E_i \) and RMR are rock mass deformation modulus, modulus of elasticity and rock mass rating classification system, respectively.

5.2 Estimation of Deformation Modulus from Q System

In the first step, the values of Q classification system at the Bakhtiary Dam were determined and properties of the intact rock like modulus of elasticity from laboratory tests was calculated; in addition, the values of the rock mass deformation modulus were measured from more than 142 in-situ tests. Then, non-linear regression analysis was used between the values of relation \( E_m/E_i \) and the values of Q. Eq. (5) shows that the second-order polynomial with \( R^2 = 0.69 \) is optimum. Also, plot of the values of relation \( (E_m/E_i) \) versus Q classification system is shown in Fig. 5.

\[ E_m = E_i(0.00002Q^2 + 0.0044Q + 0.0595) \]  

where, \( E_m \), \( E_i \) and \( Q \) are rock mass deformation modulus, modulus of elasticity, and tunneling quality index classification system, respectively.

5.3 Estimation of Deformation Modulus from GSI System

In the first step, the values of GSI classification system at the Bakhtiary Dam were determined and properties of the intact rock like modulus of elasticity from laboratory tests was calculated; in addition, the values of the rock mass deformation modulus were measured from more than 142 in-situ tests. Then, non-linear regression analysis was used between the values of relation \( E_m/E_i \) and the values of GSI. Eq. (6) shows that the second-order polynomial with \( R^2 = 0.69 \) is optimum. Also, plot of the values of relation \( (E_m/E_i) \) versus GSI classification system is shown in Fig. 6.

\[ E_m = E_i(0.0004GSI^2 - 0.0345GSI + 0.8889) \]  

where, \( E_m \), \( E_i \) and GSI are rock mass deformation modulus, modulus of elasticity, and geological strength index classification system, respectively.
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Fig. 4  The values of relation $E_m/E_i$ vs. RMR classification system.

$$E_m/E_i = 0.0003\text{RMR}^2 - 0.0363\text{RMR} + 1.0199$$

$R^2 = 0.70$

Fig. 5  The values of relation $E_m/E_i$ vs. $Q$ classification system.

$$E_m/E_i = 0.00002Q^2 + 0.0044Q + 0.0595$$

$R^2 = 0.69$
5.4 Estimation of Deformation Modulus from JH System

In the first step, the values of JH classification system at the Bakhtiary Dam were determined and properties of the intact rock like modulus of elasticity from laboratory tests was calculated; in addition, the values of the rock mass deformation modulus were measured from more than 142 in-situ tests. Then, non-linear regression analysis was used between the values of relation \( \frac{E_m}{E_i} \) and the values of JH. Eq. (7) shows that the exponential equation with \( R^2 = 0.67 \) is optimum. Also, plot of the values of relation \( \frac{E_m}{E_i} \) versus JH classification system is shown in Fig. 7.

\[
E_m = E_i \left(0.000002 e^{0.1392JH}\right)
\]

where, \( E_m \), \( E_i \) and JH are rock mass deformation modulus, modulus of elasticity, and JH corporation classification system, respectively.

5.5 Proposed a New Empirical Equation to Estimate the Rock Mass Deformation Modulus

In this study, the rock mass deformation modulus in Bakhtiary Dam site was obtained from in-situ tests and four new equations. Moreover, in a similar study, the deformation modulus in dam site from different empirical equations had been determined and the results had compared with measured results; subsequently, the results showed that the first equation of Hoek and Diederichs [22] had the most coincidence with measured results. On the other hand, the results of in-situ test were compared with four new equations and Hoek and Diederichs empirical equation in order to choose the eligible empirical equation to estimate deformation modulus. Hoek and Diederichs [22] proposed the following empirical equation for estimation of rock mass deformation modulus:

\[
E_m/E_i = 0.0004GSI^2 - 0.0345GSI + 0.8889
\]

\[ R^2 = 0.69 \]
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where, D, GSI and $E_m$ are distribution factor, geological strength index classification system and rock mass deformation modulus, respectively.

In this study, the standard descriptive measure of goodness-of-fit was employed to evaluate the accuracy of deformation modulus calculated from empirical equations. RMSE (root mean squared error) is:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (E_{m,i} - E_{E,i})^2} \quad 1 = 1, 2, \ldots, n$$

where, $n$, $E_m$ and $E_{E,i}$ are the number of data $i$, measured value of deformation modulus, and estimated value of deformation modulus by empirical equations, respectively. Also, MARPE (mean absolute relative prediction error) is:

$$\text{MARPE} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{E_{m,i} - E_{E,i}}{E_{E,i}} \right| \quad i=1, 2, \ldots, n$$

Furthermore, the ER (error ratio) or MER (mean error ratio) is:

$$\text{ER} = \frac{E_{m,i}}{E_{E,i}} \quad i=1, 2, \ldots, n$$

In this study, based on the above equations, the values of RMSE and MARPE equations which are close to zero and the value of MER equation which is close to one, the suitable empirical equation was suggested in order that it can carefully estimate the values of deformation modulus. A list of the equations for selecting the suitable equation in order to estimate the deformation modulus is given in Table 6.

In Table 6, the validation of four new equation by use of statistical analyses were examined; subsequently, the results showed that Eq. (7) was highly suitable to estimate deformation modulus owing to $R^2 = 0.67$, RMSE = 0.07, and MARPE = 0.46 as well as MER = 1.28.
Moreover, the best results can be considered when the values of modulus were predicted by Eq. (7). Also, the comparison between the measured and estimated deformation modulus from Eq. (7) is shown in Fig. 8.

6. Conclusions

In this paper, the rock mass deformation modulus by actual data collected from Bakhtiary Dam site was measured. Then, four new empirical equations to estimate rock mass deformation modulus based on modulus of elasticity and the RMR, the Q and the GSI as well as the corporation JH classification systems was developed.

Then, through statistical analysis (RMSE, MARPE, ER and $R^2$, the rock mass deformation modulus was estimated by empirical equations and the results were compared with the results of measured deformation modulus. All in all, the results showed that Eq. (7) was the suitable equation for estimating the rock mass deformation modulus.

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References


