A Statistical Evaluation of the Frictional Pressure Losses Correlations for Slurry Flow

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Abstract: Hydraulic fracturing has increased immensely in recent years. An accurate prediction of frictional pressure losses of fracturing slurries is crucial for successful treatment and avoiding premature screen-out or even treatment failure. Scarce data and lack of theoretical basis of slurry flow, especially in coiled tubing, has led to very limited number of correlations that are available to predict slurry frictional pressure losses. Yet, the accuracy of the available correlations is still questionable. The current paper presents a statistical comparative analysis of the available frictional pressure losses correlations for slurry flow in straight and coiled tubing employing the recently introduced math modeling technique giving weight for the models known as AIC (Akaike information criterion). With the help of AIC, the authors evaluated the available correlations to examine their accuracy. The results show that none of the available correlations can accurately predict friction pressure losses of slurries. The correlations show some reasonable accuracy within a very limited data range. However, they failed outside this range indicative of their poor applicability. AIC shows how much information is lost when using these correlations which can lead to erroneous results, and even job failure. This fact keeps the gates widely opened for more in-depth experimental, analytical, and theoretical analysis for better understanding of flow behavior with fracturing slurries aiming at developing a more realistic correlation to predict their frictional pressure losses. This paper represents the authors' first step toward developing such correlation, with the application of information theory and AIC.

Key words: Slurry flow, friction pressure, information theory, straight tubing, coiled tubing, multiplayer.

Nomenclature

a, z	Constants in Eq. 3.b
CT	Coiled tubing
c_v	Volumetric solid concentration
d	Pipe diameter, in.
dp/dl	Friction pressure gradient, psi/ft
e	Proppant friction exponent
i	Any models in the set
l	Liquid
M	Multiplayer
MSE	Mean square error
n	Flow behavior index, dimensionless
OD	Outside diameter, in.
p	Proppant
sl	Slurry
ST	Straight tubing
v'	Mean velocity, ft/s

w	Any models in the set
γ	Specific gravity
Δ	Information lost
ϕ	Proppant volume fraction
ϕ_m	Maximum proppant volume fraction
ϕv	Mean velocity (ft/s)
ω	Akaike weight

1. Introduction

In hydraulic fracturing treatment, bottom hole treating pressure has to be calculated in order to define the net pressure generated in the fracture. Inaccurate net pressure, especially when pumping proppant, may result in an early termination of the treatment and not achieve the design goals. Proppant/solids-laden fluids are often called slurries or suspensions or dispersions. Their flow is very complex to model due to the existence of two-phase flow consisting of base gel and solid proppant. In coiled tubing, it is even more complex due to complex flow regimes and possible

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separation of solids from the suspending medium under the effect of centrifugal forces in the curved flow geometry [1, 2]. This complexity may lead erroneous results when calculating friction pressure losses for slurry flow to calculate the resulted bottom hole treating pressure and hence the net pressure. Several researchers [3-6] provided a comprehensive review and background on slurry flow. However, much of the previous research be focused on the flow of solids in Newtonian fluids and numerous correlations were made available for predicting the slurry relative viscosity which can be used for frictional pressure losses estimation.

For non-Newtonian fracturing slurries, several papers reported studies of frictional pressure losses. Hannah et al. [7] presented a multiplier approach to predict the increased friction pressure caused by proppant. Shah and Lee [8] presented a detailed theory and empirical master curve for predicting the effect of proppant on friction pressure losses. Keck et al. [9] proposed a Dodge-Metzner type equation for the base gel and recommended the use of multiplier to calculate the slurry frictional pressure losses. Recently, Azouz et al. [10] reported experimental results of friction pressure of linear guar and crosslinked guar gum and HPG (hydroxypropyl guar) in 1½-in. coiled tubing. McCann and Islas [11] also reported some full-scale test results using coiled tubing of diameters of 1.75, 2 and 2.375 in. on a 98 in. diameter reel.

Overall, literature reviewed clearly shows two distinct approach to estimate the frictional pressure losses for slurries. The first approach is through using friction multiplier while the second approach is attempting to define the total pressure drop as a sum of base gel friction and additional pressure drop due to proppant.

The use of coiled tubing for hydraulic fracturing has increased tremendously in recent years. This is mainly due to the numerous benefits coiled tubing offers over conventional straight tubing. The primary benefits of the coiled tubing are the simpler logistics and faster

time for putting a well on production. Because of these advantages, the coiled tubing is being evaluated for wider fracturing applications. Even though the advantages of coiled tubing are understood, there are very few studies available on the friction pressure losses for slurry flow through coiled tubing. As a result, the industry finds it difficult to predict these losses during fracturing treatments. Recently, investigation of the flow behavior of fracturing slurries has attracted great interest, especially in 2⅝-in. coiled tubing as reported by Gavin [12].

Lack of proper slurry flow modeling and theoretical understanding has, however, contributed to the limited data available in this field of investigation. This has led to limited number of correlation that can really describe flow complexity of fracturing slurries and give an accurate estimation of frictional pressure losses in both straight and coiled tubing.

This paper is the authors' first step to develop a more accurate correlation to predict frictional pressure losses of slurries. In this paper, the authors will examine the most common and most recent multiplayer based correlations that are available in literature and evaluate their applicability using the newly introduced model selection technique, that is information theoretic based. This can open the gates further for more modeling techniques to improve model selection to accurately predict flow behavior of fracturing slurries as well as various other parameters that are very crucial in the oil and gas industry.

2. Multiplayer Based Friction Pressure Losses Correlations

The authors limited their evaluation and model analysis to the most common multiplayer based correlations that can be used to predict friction pressure losses for slurries in both straight and coiled tubing. Two of the correlations adopted in this analysis were developed using the state of the art facility at the WCTC, Well Construction Technology Center at the University of Oklahoma.

WCTC is unique in that it is an advanced technology research center that incorporates high pressure, high temperature fluid flow applications using both field scale and lab scale equipment for the oil industry. Flow loops vary in terms of geometry between straight and coiled tubing with various curvature ratios and concentric annuli with pumping units that can achieve very high flow rates (10 bbl/min) under high pumping pressure (5,000 psi). In developing the considered correlations, data from different flow loops, steel grades, geometries, curvature ratios, base fluids and proppant concentrations were used as listed in Table 1 [13].

The other correlation adopted here was developed by Pandey and Robert [14] using surface pressure data of real frac treatment jobs.

2.1 WCTC #1

The first preliminary equation was developed in 2002 where the multiplier was a function of both flow behavior index, n and volumetric proppant concentration, c_v as given below [14]:

$$\left(\frac{dp}{dl}\right)_{sl} = M \left(\frac{dp}{dl}\right)_l \quad (1)$$

For coiled tubing:

$$M = 2.17454 - \frac{0.515844}{n} + 10.336485c_v^2 \quad (1a)$$

For straight tubing:

$$M = 1.69271 - \frac{0.233666}{n} + 15.804344c_v^2 \quad (1b)$$

2.2 WCTC #2

The second equation was as an improvement and modification of the first preliminary equation [13]. However, the multiplier is a function of proppant

concentration, c_v only as given below:

$$\left(\frac{dp}{dl}\right)_{sl} = M \left(\frac{dp}{dl}\right)_l \quad (2)$$

For coiled tubing:

$$M = 0.98893 + 4.50287c_v^{1.63596} \quad (2a)$$

For straight tubing:

$$M = 0.992 + 3.30132c_v^{1.20539} \quad (2b)$$

The second correlation was compared with field data of two wells (nine stages for the first well and seven stages for the second well). The predicted and measured data for the first well showed a good agreement with an average percent deviation of 4.8% and a maximum deviation of 8%. For the second well, similar agreements were noticed between measured and predicted data with 3.5% and 6.2% as an average and maximum deviation, respectively [13].

WCTC #1 and #2 correlations were compared with the previously published correlations of Durand and Condolios [7-9, 15, 16] and their superiority was clear. Thus, these two equations are employed in the analysis.

2.3 PR Correlation

The other correlation considered in this research was developed in 2002 using surface pressure data [14]. In their study, the surface pressures and bottom hole treating pressures recorded during a frac treatment was utilized to compute the frictional pressure drops during proppant stages and hence to develop a multiplier based correlation for slurry flow in straight tubing. To develop their correlation, Pandey and Robert [14] used data from 168 frac treatment jobs with cross linked CMHPG (carboxy methyl hydroxyl propyl guar) using

Table 1 Specifications of experimental set-up and slurry data for WCTC correlations.

Parameters	Coil No.	OD (in.)	d (in.)	Reel diameter (in.)
Loop specifications	1	0.5	0.435	22.5
	2	1.5	1.188	72
	3	1¾	1.482	82
	4	1¾	1.532	82
	5	2¾	2.063	112
Base fluids	25 and 35 lb/Mgal linear and cross-linked guar			
Proppant concentrations	2-10 ppg			

zirconate as a cross linker. Proppant with 20/40 mesh size and various specific gravities with a concentration of up to 10 lb/Mgal were used. They analyzed their multiplayer in light of key parameters that could affect the proppant friction, namely, gel concentration, proppant density, tubular inside diameter, and average flow velocity. PR correlation is given below:

$$\left(\frac{dp}{dl}\right)_{sl} = M \left(\frac{dp}{dl}\right)_l \quad (3)$$

$$M = \left[1 - \frac{\phi}{\phi_m}\right]^{-e} \quad (3a)$$

$$e = \left(0.9035 - 0.0091v\right) \left(\gamma_p - \gamma_w\right)^a d^z \quad (3b)$$

This correlation was generated using an average velocity in range of 20 fps to 80 fps which is the practical range for most frac treatments nowadays. An average deviation of about 0.23% was observed when the results obtained from correlation were compared with frictional pressure losses calculated in cases where bottom hole pressures were measured. Furthermore, proppant friction exponents obtained from correlation were used in several other jobs that were not part of the initial study and reasonably good pressure history matches were obtained.

3. Information Theoretic Based Model Selection Techniques

Often, in applications, researchers are faced with a need to model a certain phenomenon so that prediction can be made in future. There are typically three steps to model a system [16]. First, a set of parameters is introduced to describe the system. Then, a model is developed by employing some physical principles. This model enables researchers to make predictions of measurements of observable parameters. Once the set of candidate models has been chosen, a mathematical analysis is needed to select the best model among the candidate models. Burnham and Anderson [17] emphasized the importance of choosing models based on sound scientific principles. Models, by definition, are only approximations to unknown reality; there are

no true models that perfectly reflect full reality. George Box made the famous statement “All models are wrong but some are useful.” [18]. A good model selection technique should balance goodness of fit with simplicity. The simplicity of the model is generally measured by counting the number of parameters in the model (also the nature of the model plays a role in the model simplicity).

In this work, an information theoretic approach is used to compare with various models for prediction of slurry frictional pressure losses in straight and coiled tubing. Model selection techniques that are information theoretic based can be considered as estimators of the probability of the model producing the given data. The mathematical derivations of the criterion will not be discussed in details as this can be found in many references [17, 18]. However, we will briefly explain two models can be distinguished by their correspondence value of the information criterion.

The information criterion, C is defined as:

$$C = -2 \log \left[l \left(\hat{\theta} | y \right) \right] + 2k \quad (4)$$

where, k is the number of the estimated parameters in the model including σ (standard deviation) and the intercept. $l(\hat{\theta}|y)$ is the numerical value of the likelihood at its maximum [19-21]. The value of this criterion gives the information lost if a particular model is used to approximate the truth model (that is usually unknown). Consequently, the model with the smallest value is considered the best among the set of candidate models [17]. It is very helpful to find the weight of the model compared to the rest of the models. The weight for a model is defined to be:

$$\omega_i = \frac{\exp\left(-\frac{\Delta_i}{2}\right)}{\sum_{r=1}^m \exp\left(-\frac{\Delta_r}{2}\right)} \pi^{r^2} \quad (5)$$

ω_i is considered as the weight of evidence in favor of model i being the best model of the two candidate models. This weight can also be interpreted as the

probability that particular model is the best model and hence it can be used. In other words, the model with higher value of ω is a better model for future prediction [22].

It is clear that the information theoretic approach is theoretically sound, in other words, it is a formula that is mathematically derived and proved to minimize the information lost when we use the best chosen model to approximate the truth model. Furthermore, the weight of the model allows us to see how strong the fit of a model compared to the second model is. As stated earlier, the simplicity of the model is a very essential issue in model selection and the information theoretic approach takes the number of parameters in the model under account and hence protects the over fitting issue. Among many other modeling technique, this one shows many evidences of stability and it is more sound mathematically and practically [23-24].

4. Model Analysis

Data used in this analysis include two main sets. The first set includes the experimentally measured values of friction pressure losses of slurry in both straight and coiled tubing that were generated and gathered at WCTC. For each geometry in WCTC set, data are divided into three subsets for different tubing sizes, base fluid type, and proppant concentration where AIC is employed. Furthermore, all WCTC data are grouped in one set for more broad analysis. The other data set was gathered by Pandey and Robert [14].

Afterwards, both WCTC data and PR data are

combined and used as a base for statistical analysis for more pragmatic evaluation.

4.1 Coiled Tubing

Two equations are considered for coiled tubing case as the other equation developed by Pandey and Robert [14] was only for straight tubing. WCTC #1 has two variables, n and c_v and hence its k value is 3 while WCTC #2 has one variable, c_v and its k value is 2. To get a better picture of prediction, three subsets of data are combined and AIC is employed to see which model is better for the whole range.

Table 2 summarizes the statistical results for both WCTC equations in the coiled tubing case when comparing their predictions with data measured at WCTC. Using both the experimental and predicted values, the MSE (mean square error) of each equation was calculated and then the information criterion stated above was employed to calculate the weight of the same equation. As seen from Table 2 below, for various tubing sizes, WCTC #1 is preferred to WCTC #2 with a probability of 63% versus 37%. Probably with these percentages one can use either equation with a preference for WCTC #1.

When considering different base fluids, WCTC #2 is the best with no chance, what so ever, of having WCTC #1 as a possibility. In other words, this analysis suggests using WCTC #2. For different proppant concentrations, it is evident that WCTC #1 is a much better option as its probability is 98% versus only 2% for WCTC #2.

Table 2 Statistical results for coiled tubing using WCTC data.

Parameters	Eq.	k	MSE	$\log(l)$	AIC	ω
Tubing size	WCTC #1	3	0.0333	34.0092	-65.0184	0.63
	WCTC #2	2	0.0371	32.9514	-63.9028	0.37
Base fluid	WCTC #1	3	0.2020	26.3908	-49.7815	0.00
	WCTC #2	2	0.1022	37.6217	-73.2434	1.00
Proppant concentration	WCTC #1	3	0.0184	23.9722	-44.9444	0.98
	WCTC #2	2	0.0371	19.7614	-37.5229	0.02
All data set	WCTC #1	3	0.1453	62.6831	-122.3664	0.00
	WCTC #2	2	0.0774	83.1689	-164.3384	1.00

Table 3 Statistical results for straight tubing using WCTC data.

Parameters	Eq.	k	MSE	$\log(l)$	AIC	ω
Tubing size	WCTC #1	3	0.0186	39.8562	-76.7124	0.00
	WCTC #2	2	0.0534	29.2992	-56.5983	1.00
	PR	7	0.7511	78.7864	-191.8975	0.00
Base fluid	WCTC #1	3	0.2514	22.7838	-42.5677	1.00
	WCTC #2	2	0.0511	49.0721	-96.1441	0.00
	PR	7	0.6591	69.0975	-170.0785	0.00
Proppant concentration	WCTC #1	3	0.5647	3.4284	-3.8569	0.00
	WCTC #2	2	0.0282	21.4224	-40.8448	1.00
	PR	7	0.6780	52.8432	-132.1192	0.00
All data set	WCTC #1	3	0.3018	38.9384	-74.8767	0.00
	WCTC #2	2	0.0485	98.3728	-194.7453	1.00
	PR	7	0.9799	85.0719	-231.1892	0.00

It is apparent that different models behave differently for different ranges of data; and hence to see more broad behavior of these two equations, the three subsets of data are combined as one set where the weights of both equations are calculated which shows WCTC #2 to be the best in predicting friction pressure losses of slurries in coiled tubing.

In addition to the weight factor, ω and the values of MSE are listed which clearly agree with our conclusions using weight factor. WCTC #1 shows lower MSE in some cases while WCTC #2 shows lower MSE in other cases as well as when combining the whole data.

4.2 Straight Tubing

For the straight tubing case, the three equations are statistically examined using the data measured at WCTC and Table 3 summarizes the results. Table 4 shows the results of the same three equations based on data gathered by Pandey and Robert [14].

Table 3 shows that WCTC #2 is best when considering different tubing sizes and different proppant concentration with a probability of 100%. On the other hand, WCTC #1 is better when considering various base fluids. For the whole data, WCTC #2 is the best with a probability of 100 %.

Same conclusions can be drawn from MSE values. Overall, WCTC #2 shows better performance when considering the whole data with probability of 100% and very small MSE value.

Meanwhile, PR does not show good agreement considering both weight factor and MSE. Its probability is 0% with larger MSE values which makes it a non-viable option to predict friction pressure losses of slurries in straight tubing. Table 4 shows the statistical results of the three equations when comparing their predictions with data gathered by Pandey and Robert [14].

As it can be seen in Table 4, PR correlation has somehow a better performance than the other two equations indicated by small value of MSE, 0.1051 and larger probability, 65%. The other two equations developed at WCTC show very poor performance indicated by higher MSE values and lower probability or weight factor. However, WCTC #2 shows a better performance than WCTC #1.

As an overall conclusion, data gathered by WCTC team and by Pandey and Robert [14] are combined as one general set and used to generally evaluate the three equations' performance. The results are shown in Table 5.

As it can be seen from Table 5, there is no preference of any equation to the others as they all fail to predict friction pressure losses of slurry in straight tubing with a reasonable accuracy as indicated by MSE values and weight factor. Again, WCTC #2 still has small MSE value and high probability when compared with the others. Yet, it is not a viable option to predict accurate

Table 4 Statistical results for straight tubing using PR data.

Eq.	k	MSE	log(l)	AIC	ω
WCTC #1	3	0.7816	79.1199	-77.6695	0.11
WCTC #2	2	0.3376	62.9977	-54.3359	0.24
PR	7	0.1051	46.5564	-19.7712	0.65

Table 5 Statistical results for straight tubing using both WCTC and PR data.

Eq.	k	MSE	log(l)	AIC	ω
WCTC #1	3	0.9178	88.0774	-88.9973	0.31
WCTC #2	2	0.7447	53.7296	-55.8109	0.47
PR	7	0.9985	102.9980	-91.0919	0.22

values that are comparable to the measured data as it shows high MSE values and low probability or weight factor.

5. Discussion

In this paper, the most recent multiplayer based correlations for friction pressure losses of slurries in straight and coiled tubing are evaluated. Old correlations like Hannah, Keck, etc. are not considered as the comparison between their performance and new correlations' performance shows their poor accuracy [24].

As it can be inferred from the results of statistical analysis listed in Tables 2-5, there is no equation that can be used to predict friction pressure losses of slurries in either straight or coiled tubing with reasonable accuracy. Based on weight factor and mean square error, certain equations may perform reasonably within certain ranges. Outside this range, it fails.

Also, both WCTC equations and PR equation give reasonable values when comparing them with their own data set, as expected, but both fail to yield comparable values to the opposite data set. When combining both WCTC data with PR data, no equation shows success in predicting accurate results.

Among the three equations, WCTC #2 seems to be more reasonable than the other two equations. Yet, its accuracy is still questionable, especially when comparing it with PR data. In addition, WCTC #2 is a function of proppant concentration only and neglects the effect of base fluid rheological properties. This may be mainly due to the use of two base fluids only, 25

lb/Mgal and 35 lb/Mgal guar where there may be no distinctive difference between their rheological properties in terms of flow behavior index.

Finally, the scarce data available for slurry flow particularly in coiled tubing make it difficult to develop an equation that can be used to predict their flow behavior with reasonable accuracy. This fact keeps the doors opened for more in-depth experimental and theoretical analysis of flow behavior of slurries in straight and coiled tubing aiming at developing a more realistic correlation.

6. Conclusions and Recommendations

Sometimes in the oil and gas industry, the real problem is the large number of correlations that can be used to predict specific parameter. In such case, the question is which correlation one should use. This paper introduces a new concept of model selection that can alleviate this dilemma, especially when we know that true models are very difficult to get. With that being said, this paper presents a comprehensive statistical comparison among the available models used to calculate friction pressure losses of slurries in straight and coiled tubing during frac treatment job.

Three multiplayer based models are included in the analysis. All models were generated using either field data or large scale experimental setup and were validated with field data. It is evidently clear that none of the models works for all ranges of data. However, WCTC #2 seems to be more realistic than other models, although it only considers the effect of proppant concentration and ignores the effect of base fluid rheology. This justifies the need for a more accurate correlation that can describe flow behavior of slurries in both and straight tubing. Therefore, more in-depth experimental and theoretical analysis of flow behavior of slurries in straight and coiled tubing is recommended.

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